

## Flow induced microstructure in Newtonian and Viscoelastic Fluids.

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### ABSTRACT

Pair interactions between neighboring particles and turning couples on long bodies formed from touching bodies give rise to flow induced microstructures. In Newtonian fluids, pair interactions in a fluidized suspension lead to dispersions with particles arranged in lines across the stream. In viscoelastic fluids, sedimenting particles aggregate into chained bodies parallel to the stream when the flow is slow and normal stresses dominate, and into across the stream arrays again when the flow is supercritical and dominated by inertia. The microstructural arrangements in Newtonian and viscoelastic fluids are maximally different. Simple mathematical arguments are given here which identify the forces and couples that give rise to all of the main observed microstructures. A mechanism for intensifying normal stresses by shear thinning is proposed.

### INTRODUCTION

We are going to carry out an analysis of our recent experiments on the sedimentation of particles in Newtonian and viscoelastic fluids. The results to be described are generic and not results for one special fluid. Of course, the differences between fluids is important, but we shall not focus on these differences which are well documented in our original papers: [1,2,3,4,5]. A review of all these results can be found in [6]. The main themes in our interpretations were not proposed by prior authors; these themes rest on the competition between inertia and normal stresses, on the importance of turning couples on long bodies in determining the stable configurations of suspension of spherical bodies, on the contrary behaviors of particles in viscoelastic and Newtonian liquids, on the all-important contrary effects of compressive normal stresses [7] and inertia. We are presently doing high performance direct simulations of the motion of particles in Newtonian and viscoelastic liquids[8,9].

### TURNING COUPLES ON LONG BODIES

It is surprising at first sight that turning couples on long bodies determine the stable configurations of suspensions of spherical bodies. A long body is an ellipsoid or a cylinder; a broad body is a flat plate. When such bodies are dropped in Newtonian fluids, they turn and put their long or broadside perpendicular to the stream. This is an effect of inertia which is usually explained by turning couples at points of stagnation. The mechanism is the same one that causes an aircraft at a high angle of attack to stall.

It is not possible to get long particles to turn broadside in a Stokes flow; bodies with fore-aft symmetry do not experience torques. The settling orientation is indeterminate in Stokes flow; however, no matter how small the Reynolds number may be, the body will turn its broadside to the stream; inertia will eventually have its way. When the same long bodies fall slowly in a viscoelastic liquid, they do not put their broadside perpendicular to the stream; they do the opposite, aligning the long side parallel to the stream.

The difference in the orientation of long bodies falling in Newtonian and viscoelastic fluids is very dramatic; basically the flow orientations in the two fluids are orthogonal. Of course, in very dilute polymeric liquids, the effects of inertia and viscoelasticity will compete and the competition will be resolved by a tilt angle away along the stream. For cylinders with sharp corners, normal stress effects produce the "shape tilting" observed by Liu and Joseph [2] and explained here. Another, much more dramatic change in the tilting of a long cylinder or flat plate is associated with the way that inertia comes to dominate high-speed flows of viscoelastic fluids.

#### PARTICLE-PARTICLE INTERACTIONS

The flow-induced anisotropy of a sedimenting or fluidized suspension of spheres is determined by the pair interactions between neighboring spheres. The principal interactions can be described as *drafting*, *kissing* and *tumbling* in Newtonian liquids and as *drafting*, *kissing* and *chaining* in viscoelastic liquids. The drafting and kissing scenarios are surely different, despite appearances. Kissing spheres align with the stream; they are then momentarily long bodies.

The long bodies momentarily formed by kissing spheres are unstable in Newtonian liquids to the same turning couples that turn long bodies broadside-on. Therefore, they tumble. This is a local mechanism which implies that globally, the only stable configuration is the one in which the most probable orientation between any pair of neighboring spheres is across the stream. The consequence of this microstructural property is a flow-induced anisotropy, which leads ubiquitously to lines of spheres across the stream; these are always in evidence in two-dimensional fluidized beds of finite size spheres. Though they are less stable, planes of spheres in three-dimensional beds can also be found by anyone who cares to look.

The drafting of spheres in a Newtonian liquid is governed by the same mechanism by which one cyclist is aided by the low pressure in the wake of another. The spheres certainly do not follow streamlines since they are big and heavy. If a part of one sphere enters the wake of another, there will be a pressure difference to impel the second sphere all the way into the wake where it experiences a reduced pressure at its front and not so reduced pressure at the rear. This increased pressure difference impels the trailing sphere into kissing contact with the leading sphere. The motion of the trailing sphere relative to the leading one is in the same sense as in the undisturbed case, into the rear pole of the leading sphere.

Riddle, Navarez and Bird [10] presented an experimental investigation in which the distance between the two identical spheres falling along their line of centers in a viscoelastic fluid was a function of time. They found that for all five fluids used in the experiments that the spheres attract if they are initially close and separate if they are not close; there is a critical separation distance. This looks like a competition between normal stresses and inertia, which is decided by a critical distance which may vary with latitude. Competition of normal stresses and inertia is more typical than rare, and for flows slow enough to enter into the second order region the critical distance scales with  $\sqrt{\Psi_1/\rho}$ , where  $\Psi_1$  is the coefficient of the first normal stress difference and  $\rho$  is the density. The property that chaining tensions are short range is also put in evidence in Figure 1b, which shows that spheres can also detach from the trailing end of a chain when the distance between the last two spheres exceeds a critical value, as in the experiments of Riddle et al. [10].

If two touching spheres are launched side-by-side in a Newtonian fluid, they will be pushed apart until a stable separation distance between centers across the stream is established; then the spheres fall together without further lateral migrations (see Figure 2a).

On the other hand, if the same two spheres are launched from an initial side-by-side configuration in which the two spheres are separated by a smaller than critical gap, as in Figure 2b, the spheres will attract, turn and chain. One might say that we get dispersion in the Newtonian liquid and aggregation in the viscoelastic liquid.

SPHERE-WALL INTERACTIONS

If a sphere is launched near a vertical wall in a Newtonian liquid, it will be forced away from the wall to an equilibrium distance at which lateral migrations stop (see figure 3a); in the course of its migration it will acquire a counter-clockwise rotation (see Figure 7) which appears to stop when the sphere stops migrating. The rotation is anomalous in that clockwise rotation would be induced from shear at the wall. The anomalous rotation seems to be generated by blockage in which high stagnation

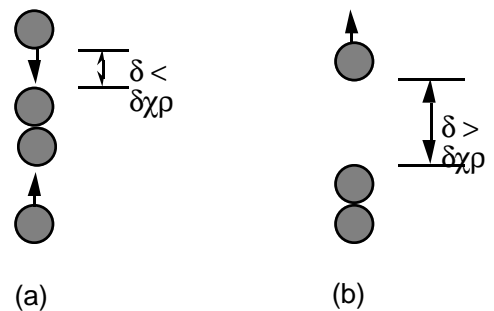


Figure 1. The falling chained spheres are viewed in a frame in which they are at rest. Particles may link to the chain from the bottom or top. If  $\delta > \delta_{cr}$  the chained spheres will fall away faster than the trailing sphere.

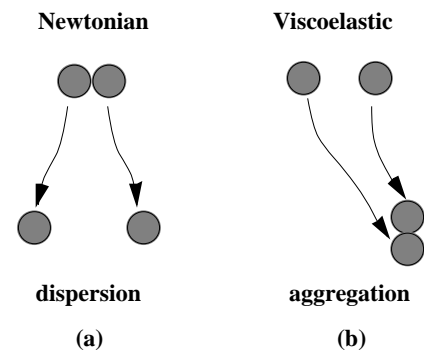


Figure 2. Side-by-side sphere-sphere interactions

pressures force the fluid to flow around the outside of the sphere, as shown in figure 7.

If the same sphere is launched near a vertical wall in a viscoelastic liquid, it will be sucked all the way to the wall (see Figure 3b). It rotates anomalously as it falls. This is very strange since the sphere appears to touch the wall where friction would make it rotate in the other sense. Closer consideration shows that there is a gap between the sphere and the wall. The anomalous rotation is again due to blocking which forces liquid to flow around the outside of the sphere (see figure 7).

The pulling action of the wall can be so strong that even if the wall is slightly tilted from the vertical so that the sphere should fall away, it will still be sucked to the wall (see Figure 4).

If the launching distance between the sphere and a vertical wall is large enough, the wall will not attract a sphere falling in a viscoelastic fluid. This means that there is a critical distance  $\hat{\delta}$  for attraction. Of course, this distance is smaller when the wall is tilted as in Figure 4. In this case, if the sphere is launched at a distance greater than the critical one, it will fall away from the wall.

The effect of two closely-spaced walls on the migration of particles is not completely understood. We have just said that spheres which fall near a wall in a viscoelastic liquid will be pulled to the wall, but not if the launching distance from the wall is larger than a critical one. On the other hand, we noted that spheres and cylinders dropped between closely-spaced walls do center. We may think that if a sphere is launched between widely-spaced walls at a distance farther than the critical one, it will not be attracted to the near wall and certainly not to the far one. So the equilibrium position will depend on the initial distance, or it is more likely from symmetry to seek the

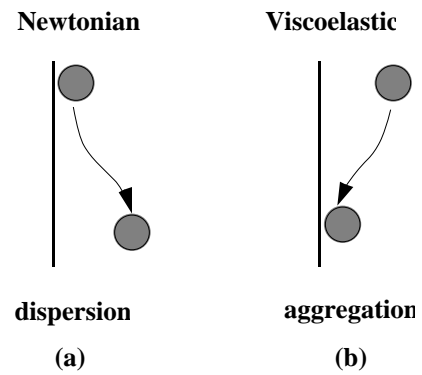


Figure 3: Sphere-wall interactions.

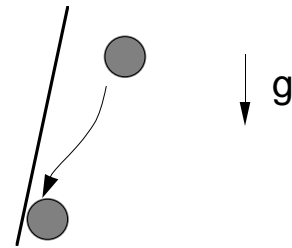


Figure 4: A Sphere in viscoelastic liquid is sucked to a tilted wall.

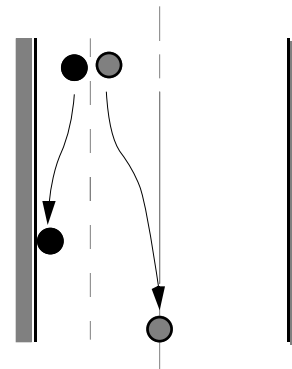


Figure 5: Spheres dropped between widely-spaced walls. The dotted line is the critical distance  $d_{cr}$  for wall-sphere interaction. When  $d < d_{cr}$ , the sphere goes to the wall. When  $d > d_{cr}$ , the sphere seeks the center.

center, as shown in Figure 5. We do not know the answer yet.

If the walls are so closely spaced that the distance  $d$  between walls is equal to or smaller than the critical one  $\hat{\delta}$  for migration, then both walls will attract the sphere, though perhaps not equally. Experiments suggest centering in this case.

### FORCES AND TORQUES

Let  $\mathbf{n}$  be the outward normal at a point on the boundary  $\partial\Omega$  of a rigid body. The forces and torques on that body are the integrated resultants of the traction vector:

$$\mathbf{T} \cdot \mathbf{n} |_{\partial\Omega} = -p\mathbf{n} + \mathbf{S} \cdot \mathbf{n} \quad (1)$$

and its moments over the whole body, where  $p$  is the pressure and  $\mathbf{S}$  is the extra stress which is modeled by a constitutive equation. In general  $p$  is not related to the deformation in a universal way; there is no constitutive equation for  $p$  and it must be determined from the solution of dynamical problems, a new  $p$  each time. There are two cases in which we can relate  $p$  to the velocity and its gradients universally, in potential flow and in creeping flow in two-dimensions. From these two cases we can find good understanding of the forces and torques that move and turn rigid bodies in Newtonian and viscoelastic fluids.

Associated with (1) are three traction components, a normal component

$$T_{nn} = -p + \mathbf{n} \cdot \mathbf{S} \cdot \mathbf{n} \quad (2)$$

and two independent shear components

$$T_{nt} = \mathbf{t} \cdot \mathbf{T} \cdot \mathbf{n} = \mathbf{t} \cdot \mathbf{S} \cdot \mathbf{n} = S_{nt} \quad (3)$$

where  $\mathbf{t}$  is one of two independent unit vectors tangent to  $\partial\Omega$ . The shear tractions are associated with the resistance of the fluid to the motions of the body; the normal tractions turn long bodies and push spherical bodies across streamlines.

A great simplification of analysis comes from the fact that at a point on the boundary of a rigid body

$$\mathbf{n} \cdot \mathbf{A}_1 \cdot \mathbf{n} |_{\partial\Omega} = \mathbf{n} \cdot [\nabla \mathbf{u} + \nabla \mathbf{u}^T] \cdot \mathbf{n} |_{\partial\Omega} = 0 \quad (4)$$

where  $\mathbf{A}_1[\mathbf{u}]$  is twice the rate of strain for the velocity field  $\mathbf{u}$ , provided only that  $\text{div} \mathbf{u} = 0$ ; the fluid is incompressible as is the case here.

### NEWTONIAN FLUIDS AND INERTIA

For Newtonian fluids the extra stress is given by  $S = \eta A_1[\mathbf{u}]$  where  $\eta$  is the viscosity and (2) and (4) imply that

$$T_{nn}^{\text{Newt}} = -p^{\text{Newt}} = -p^{\text{Inertia}} - p^{\text{Viscous}} \quad (5)$$

When inertia is negligible,  $p^{\text{viscous}} = p_N$  given by Stokes flow with

$$\nabla p_N = \text{div} \eta A_1 = \eta \nabla^2 \mathbf{u} \quad (6)$$

when  $\eta$  is constant. Spherical bodies do not migrate and bodies with fore-aft symmetry do not experience torques in Stokes flow. On the other hand it is well known that

$$T_{nn}^{\text{inertia}} = -p^{\text{inertia}} \quad (7)$$

is quadratic in velocity and gives rise to forces which produce lateral drift and turn long and broad bodies perpendicular to the stream.

#### VISCOELASTIC FLUIDS

It is necessary to propose a constitutive equation for the extra stress. No single constitutive equation works for all motions. So many constitutive equations have been proposed based on molecular cartoons of real fluids. Fortunately aggregations of particles with chained spheres along the stream occur in very slow flows in which all those models collapse into a universal form, the second order fluid which is quadratic in the shear rate and represents the recent memory of the fluid by a time derivative; it is valid asymptotically only for slow and slowly varying motions, but for these motions the rheology is general and not model dependent. The equations of motion for a second order fluid are

$$\rho \left[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right] = \text{div} \mathbf{T}, \quad (8)$$

$$\mathbf{T} = -p \mathbf{1} + \eta A_1 + \alpha_1 A_2 + \alpha_2 A_1^2, \quad (9)$$

$$A_2 = \left( \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) A_1 + A_1 \nabla \mathbf{u} + \nabla \mathbf{u}^T A_1$$

where  $\alpha_1$  and  $\alpha_2$  are the quadratic constants and

$$\Psi_1 = -2\alpha_1 > 0, \quad \Psi_2 = 2\alpha_1 + \alpha_2 \quad (10)$$

are the coefficients of  $\dot{\gamma}^2$  (where  $\dot{\gamma}$  is the shear rate) in the expansion of the first and second normal stress functions in powers of  $\dot{\gamma}^2$ .  $\Psi_2$  is usually negative and much smaller than  $\Psi_1$ ; it vanishes in popular models like Oldroyd B.

Analysis of motions of a second order fluid may be greatly simplified in two dimensions [11] or when  $\alpha_1 + \alpha_2 = (\Psi_1 + 2\Psi_2)/2 = 0$  [12]. In either case, the velocity field is the same as that of the Stokes flow while the pressure is given by

$$p = p_N + \frac{\alpha_1}{\eta} \frac{Dp_N}{Dt} + \left(\frac{\alpha_2}{2} + \frac{3\alpha_1}{4}\right) \mathbf{A}_1 : \mathbf{A}_1, \quad (11)$$

where  $p_N$  is defined by (6).

For plane flows, the stress can be written as [11]

$$\begin{bmatrix} T_{xx} & T_{xy} \\ T_{xy} & T_{yy} \end{bmatrix} = -\left(p_N + \frac{\alpha_1}{\eta} \frac{Dp_N}{Dt} + \frac{\alpha_1}{2} \Gamma\right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\eta + \alpha_1 \frac{D}{Dt}\right) \begin{bmatrix} 2a & b+c \\ b+c & 2a \end{bmatrix} + \alpha_1(b-c) \begin{bmatrix} -b-c & 2a \\ 2a & b+c \end{bmatrix} \quad (12)$$

where  $D/Dt$  is the substantial derivative,  $a = \partial u / \partial x = -\partial v / \partial y$ ,  $b = \partial u / \partial y$ ,  $c = \partial v / \partial x$ ,  $\alpha_1 = -\Psi_1/2$  and  $\Gamma = 4a^2 + (b+c)^2$ . Now choose a generic point  $P$  on the boundary  $\partial\Omega$  of the body  $\Omega$  and define local coordinates  $(x, y)$  with velocity  $(u, v)$  where  $x$  is tangential and  $y$  normal to  $\partial\Omega$ . Since  $\Omega$  is a rigid body, there is no variation of  $u$  or  $v$  along  $\partial\Omega$ . Hence,  $a = c = 0$ ,  $b = \dot{\gamma}$ , and the stress at  $P$  is

$$\begin{bmatrix} T_{xx} & T_{xy} \\ T_{xy} & T_{yy} \end{bmatrix} = -\left(p_N - \frac{\Psi_1}{2\eta} \frac{Dp_N}{Dt} - \frac{\Psi_1}{4} \dot{\gamma}^2\right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\eta - \frac{\Psi_1}{4} \frac{D}{Dt}\right) \begin{bmatrix} 0 & \dot{\gamma} \\ \dot{\gamma} & 0 \end{bmatrix} - \frac{\Psi_1}{2} \dot{\gamma}^2 \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad (13)$$

It follows that the normal component of the stress  $T_{yy}$  is given by [13]:

$$T_{yy} = -p_N + \frac{\Psi_1}{2\eta} \frac{Dp_N}{Dt} - \frac{\Psi_1}{4} \dot{\gamma}^2. \quad (14)$$

The foregoing analysis works also in three dimensions when  $\alpha_1 = -\alpha_2$  ( $\Psi_1 = -2\Psi_2$ ) [4]. In either case, the total stress depends on  $\Psi_1$  but the pressure is given by (11).

In determining the contribution of the pressure to the total normal stress in the plane case where  $\alpha_2$  is actually irrelevant, it is necessary to assign a value to  $\alpha_2$ . The irrelevance of  $\alpha_2$  stems from the

fact that in the reduction of (8) to (12),  $\alpha_2$  in the expression for the pressure

$$p = p_N - \frac{\Psi_1}{2\eta} \frac{Dp_N}{Dt} + (\alpha_2 + \frac{3}{2}\alpha_1)\dot{\gamma}^2 \quad (15)$$

cancel an identical contribution in the extra stress

$$T_{yy} + p = (2\alpha_1 + \alpha_2)\dot{\gamma}^2. \quad (16)$$

The decomposition of the total normal stress into a "pressure" and extra stress is unique because of (11), but the decision to call (11) a pressure is arbitrary. Since

$$\left(\alpha_2 + \frac{3}{2}\alpha_1\right) = \hat{\beta}/2$$

where  $\hat{\beta} > 0$  is the climbing constant, and

$$2\alpha_1 + \alpha_2 = \Psi_2 < 0$$

for nearly all solutions and melts, both quadratic contributions to (15) and (16) are compressive. Moreover, in many cases  $\hat{\beta}$  is large and  $\Psi_2$  is small, so that the main compressive stresses are generated by the normal stresses in the pressure (15) [14].

#### FLOW MICROSTRUCTURES ASSOCIATED WITH COMPRESSIVE NORMAL STRESSES

The time derivative of  $p_N$  vanishes in steady flows over stationary bodies. The form of normal stresses in (15) and (16) informs intuition about how particles move and turn in a slow flow of a viscoelastic liquid; one has only to look for crowded streamlines in the Stokes flow near the body to see how the normal stresses are distributed over the body. If the particle has fore-aft symmetry, the Stokes pressure and viscous shear stress each yield a zero torque on the body; thus the normal stresses will turn the body into the stream [14,15] as in figure 6(a). The argument just given suggests that the longest line of less regular bodies ought to align parallel to the stream; a cube actually does fall slowly with the line through opposite vertices parallel to gravity. For two identical spheres or circular cylinders settling side by side (figure 6b), strong shears occur on the outside and the resulting compressive stresses push the particles together; they then act like a long body and are turned into the stream by torques like those in figure 6(a). Two particles settling in tandem experience imbalanced compressive normal stresses at the bottom of the leading particle and the top of the trailing particle, causing them to chain as in figure 6(c). The lateral attraction of a particle to a nearby wall can be explained by a similar mechanism (figure 6b). Experimental evi-



dence of particle-particle and particle-wall interactions has been documented in [5].

The compressive stresses which are generated by the motion of particles in plane flow of a second-order fluid produce aggregation rather than dispersion; they align long bodies with the stream and produce chains of particles aligned with the stream.

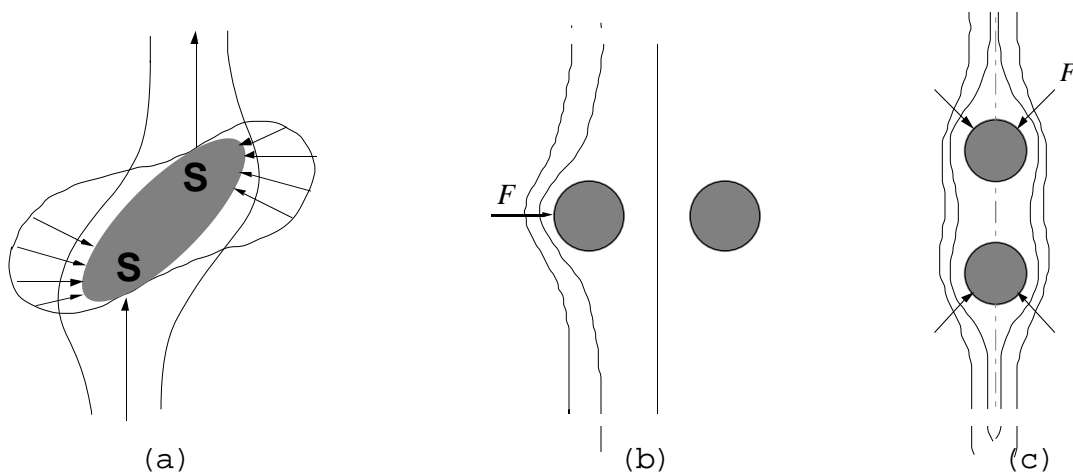


Figure 6 Cartoons of streamlines around bodies settling in Stokes flow. The normal stresses are negative and proportional to  $\dot{\gamma}^2$ ; they are large and compressive where the streamlines are crowded, basically where the flow is fast. Inertial pressures are large at stagnation point  $\dot{\gamma} = 0$ , where normal stresses vanish. (a) Normal stresses turn the major axis of the ellipse into the stream. For slow flows inertial forces are smaller than normal stresses. (b) The normal stresses force side by side particles together and they urge particles to the wall. (c) Compressive forces cause particles in tandem to chain.

#### SHEAR THINNING

Now I give a heuristic argument which suggests that the effects of compressive normal stresses are intensified by shear thinning because larger values of  $\dot{\gamma}$  are produced where the streamlines are crowded. In the case of an Oldroyd B fluid

$$\Psi_1 = \eta(\lambda_1 - \lambda_2) \quad (17)$$

where  $\lambda_1$  and  $\lambda_2$  are the relaxation and retardation times. Shear thinning does not appear at second-order in the asymptotic analysis leading to the second-order form of the Oldroyd-B model. Therefore we are taking liberties with mathematical rigor by writing the shear thinning form of

$$\Psi_1 = \eta(\lambda_1 - \lambda_2)\dot{\gamma}^{n-1} \quad (18)$$

where  $0 < n < 1$ . The effect of shear thinning is to decrease the viscosity and increase the shear rate at places of high  $\dot{\gamma}$  on the

body. In a pipe flow with a prescribed pressure gradient, the pressure force balances the shear force at the wall so the shear stress  $\tau_w = \eta(\dot{\gamma})\dot{\gamma}$  is the same for all viscosity functions. If the fluid thins in shear, the viscosity  $\eta$  goes down and the shear-rate  $\dot{\gamma}$  goes up, keeping the product constant. Then  $\eta(\dot{\gamma})\dot{\gamma}^2 = \tau_w\dot{\gamma}$  is larger than what it would be if the fluid did not shear thin because  $\dot{\gamma}$  is larger.

The increase in the intensity of compressive normal stresses means that the turning couples which turn long bodies into the stream and the pushing stresses which cause spherical particles to aggregate are all increased. For example, the standoff distance of circular cylinder sedimenting near a wall (figure 7) will be much less in the shear thinning form of the Oldroyd B fluid as is shown by direct numerical simulation [9].

REVERSAL OF THE NORMAL STRESS AT A POINT OF STAGNATION

A point of stagnation on a stationary body in potential flow is a unique point at the end of a dividing streamline at which the velocity vanishes. In a viscous fluid all the points on the boundary of a stationary body have a zero velocity but the dividing streamline can be found and it marks the place of zero stress near which the velocity is small. The stagnation pressure makes sense even in a viscous fluid where the high pressure of the potential flow outside the boundary layer is transmitted right through the boundary layer to the body. It is a good idea to look for the dividing streamlines where the shear stress vanishes in any analysis of the flow pattern around the body.

The points marked S on Figure 7 are points of stagnation for a real no-slip fluid marking the place where  $\dot{\gamma} = 0$ . In the slow flow analysis just given, these points where  $\dot{\gamma} = 0$  have no viscoelastic normal stress. For faster subcritical flows which are

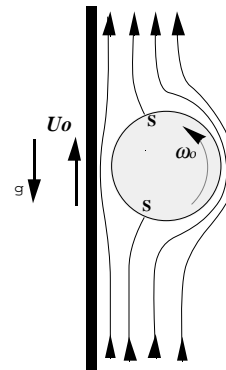


Figure 7. Cartoon of the settling of a circular particle in a Newtonian fluid at a vertical wall in a coordinate system in which the center of the particle is at rest, so the wall moves up with speed  $U_0$ . If the particle is dropped at the wall, the fluid will go around the outside and turn the particle in the anomalous sense as shown. There are two "stagnation" points S on the circle where the shear stress vanishes associated with high positive pressure on the bottom and a smaller negative pressure near the top. The positive pressure "lifts" the particle away from the wall and it seeks an equilibrium in the channel center. Normal stress effects are greatest at the outside of the cylinder where the streamlines are crowded and the shear rates are large. The effect of shear thinning is to increase these shear rates and the forces which now push the cylinder closer to the wall.

still dominated by viscoelastic stresses, we can imagine viscoelastic effects to be transmitted through a boundary layer which will reverse the sign of the normal stress there. Some of the principal causes for the reversal can be seen in the equations governing the potential flow of a second order fluid given in [16]. Not all models of a viscoelastic fluid admit potential flow as a solution irrespective of the boundary conditions [17], but a second order fluid does, and for these, there is a Bernoulli equation which is in the form

$$\rho\phi_{,t} + \frac{\rho|\mathbf{u}|^2}{2} + p - \hat{\beta}\nabla\mathbf{u}:\nabla\mathbf{u} = c \quad (19)$$

where  $\hat{\beta}$  is the climbing constant which is positive in nearly all viscoelastic liquids and can even be large. Obviously there is a competition between inertia  $\rho|\mathbf{u}|^2$  and normal stresses  $\hat{\beta}|\nabla\mathbf{u}|^2$  with the latter dominating for slow speeds and large gradients. *Inertia scales with the square of the velocity and normal stresses scale with the square  $U^2/L^2$  of the rate of shear or extension.* Carrying out analysis of potential flow a little further, we find that at a stagnation point, the stress  $\sigma_{11}$  in the direction  $x_1$  of stretching is given by

$$\sigma_{11} = -\frac{\rho}{2}U^2 + 4\eta\frac{U}{L}\dot{s} + \hat{\gamma}\frac{U^2}{L^2}\dot{s}^2 \quad (20)$$

where  $\hat{\gamma}$  is a positive combination of first and second normal stress coefficients and  $\dot{s}$  is a dimensionless rate of stretching [1]. Equation (20) shows clearly how the sign of the normal stress at a point of stagnation can be reversed by high rates of stretching in a viscoelastic fluid.

The reversal of the extensional normal stresses at points of stagnation would pull long bodies into the stream, reversing the tendency of inertia to push them across the stream. In this case the turning couples of high compressive normal stresses due to shear compete with high tension at points of stagnation due to extension..

### Tilt Transition

Liu and Joseph [2] have done experiments on the settling of long cylinders in aqueous solutions of polyox and polyacrylamide, and in solutions of polyox in glycerin and water. The tilt angles of long cylinders and flat plates falling in these viscoelastic liquids were measured. The effects of particle length, particle weight, particle shape, liquid properties and liquid temperature were determined. In some experiments, the cylinders fall under gravity in a bed with closely-spaced walls. No matter how or where a cylinder is released, the axis of the cylinder centers itself between the close walls and falls steadily at a fixed angle of tilt with the horizontal. A discussion of the tilt angle may be framed in terms of competition between viscous effects, viscoelastic effects and inertia. When inertia is small, viscoelasticity dominates and the particles settle with their broadside parallel or nearly parallel to the direction of

fall. When inertia is large, the particles settle with their broadside perpendicular to the direction of fall. The tilt angle varies continuously from  $90^\circ$ , when viscoelasticity dominates, to  $0^\circ$ , when inertia dominates. The balance between inertia and viscoelasticity was controlled by systematic variation of the weight of the particles and the composition and the temperature of the solution. Particles will turn broadside-on when the inertia forces are larger than viscous and viscoelastic forces. This orientation occurred when the Reynolds number  $R$  was greater than some number not much greater than one in any case, and less than 0.1 in Newtonian liquids and very dilute solutions. In principle, a long particle will eventually turn its broadside perpendicular to the stream in a Newtonian liquid for any  $R > 0$ , but in a viscoelastic liquid this turning cannot occur unless  $R > 1$ . Another condition for inertial tilting is that the elastic length  $\lambda U$  should be longer than the viscous length  $\nu/U$  where  $U$  is the terminal velocity,  $\nu$  is the kinematic viscosity and  $\lambda = \nu/c^2$  is a relaxation time where  $c$  is the shear wave speed measured with the shear wave speed meter (Joseph [11]). The condition  $M = U/c > 1$  was provisionally interpreted by Liu and Joseph [2] as a hyperbolic transition of solution of the vorticity equation analogous to transonic flow. They showed that strong departures of the tilt angle from  $q = 90^\circ$  begin at about  $M = 1$  and end with  $q = 0^\circ$  when  $1 < M < 4$  (see figures 8 and 9 for some representative results).

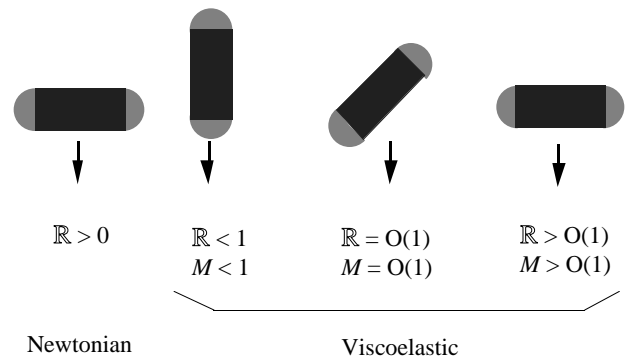


Figure 8: Orientation of cylinders falling in Newtonian and viscoelastic liquids.

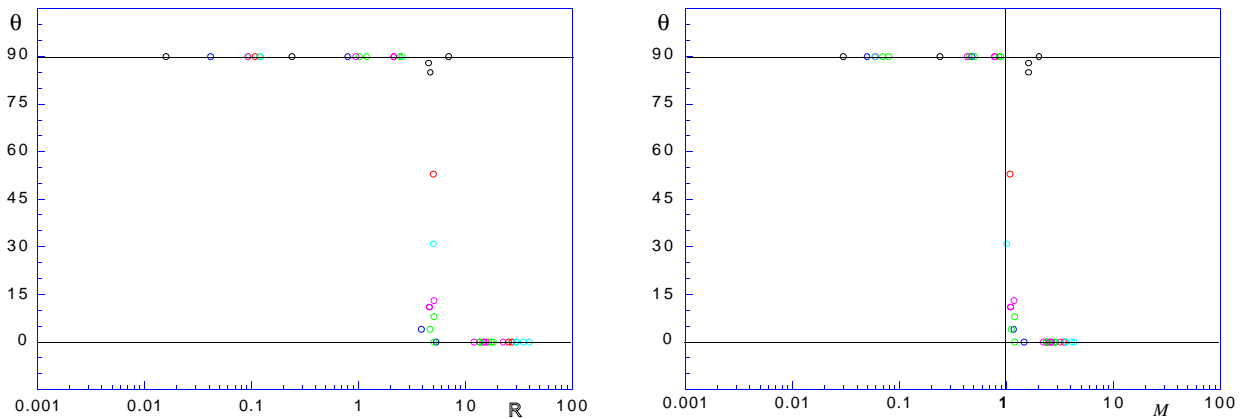


Figure 9. Tilt angle vs. Reynolds number and Mach number. Cylinders (length 0.8 in., diameters 0.1 - 0.4 in.) falling in 2% polyacrylamide/water solution. The data are taken from cylinders with round ends only. (Reproduced from *J. Fluid Mech.* 255, 1993, with permission.)

It is perhaps helpful to frame the criteria for the tilt transition in terms of a comparison between the fall speed  $U$  and the other speeds which depend on material and not on  $U$ ; long and broad objects falling in a viscoelastic liquid will turn broadside to the stream when the fall velocity  $U$  is greater than the diffusion speed  $v/d$  and the shear wave speed  $c$ . The reason is that under these conditions, signals cannot reach the fluid before the falling body and the body feels the pressures of potential flow at its front side. Such pressures turn the body broadside-on.

## CONCLUSIONS

- Fluidized suspensions have an *anisotropic structure* which is determined by the *microstructure*.
- The microstructure depends on the dynamics of *pair particle interactions*.
- Pair particle interactions are dominated by *wakes* and *turning couples* on long and broad bodies.
- Pair particle interactions in Newtonian and viscoelastic fluids are *maximally different*.
- Stagnation pressures due to inertia turn long bodies across the stream. Spherical particles draft, kiss and tumble into cross stream arrays. Inertial forces force particles to separate; they are dispersive.
- Normal stresses are compressive and in plane flows are proportional to the square of the shear rate. Normal stresses vanish at stagnation points and are large where the flow is fast and inertia is small. Spherical particles draft, kiss and chain, linked along rather than across the stream. Normal stresses force particles together; they are aggregative rather than dispersive.
- Shear thinning intensifies the effects of compressive normal stresses by increasing the shear rate at the places where the shear rates are greatest.
- Normal stress effects dominate flows for which the Reynolds number and viscoelastic Mach numbers are less than one.
- Viscous effects and normal stress effects are suppressed when the relative speed  $U$  between particle and fluid is greater than the speed of *diffusion*

$$U > v/d,$$

$$(\text{Re} = Ud/v > 1)$$

and the speed of shear waves

$$U > c \left( = \sqrt{\eta/\lambda\rho} \right).$$

$$(M = U/c > 1).$$

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Literature cited:

1. D.D. Joseph, J. Nelson, H. Hu and Y.J. Liu, "Competition between inertial pressures in the flow induced anisotropy of solid particles," in *Theoretical and Applied Rheology*, P. Moldenaers and R. Keunings (eds.), Elsevier, Amsterdam, 1992, 60-64.
2. Y.J. Liu and D.D. Joseph, *J. Fluid Mech.*, 255 (1993) 565-595.
3. D.D. Joseph and Y.J. Liu, *J. Rheol.*, 37 (1993) 1-22.
4. Y.J. Liu, J. Nelson, J. Feng and D.D. Joseph, *J. Non-Newtonian Fluid Mech.*, 50 (1993) 305-329.
5. D.D. Joseph, Y.J. Liu, M. Poletto, and J. Feng, *J. Non-Newtonian Fluid Mech.*, 54 (1994), 45-86.
6. D.D. Joseph, "Motions of particles settling in a viscoelastic fluid," *Proceedings of the Second International Conference on Multiphase Flow*, Kyoto, Japan, April 3-7, 1995. PD1 (1-8)
7. D.D. Joseph, J. Feng, Accepted for publication in *J. Non-Newtonian Fluid Mech.* (1996).
8. D.D. Joseph, "Interrogation of numerical simulations for the modeling of flow induced microstructure," *ASME FED 189 (Liquid-Solid Flows)*, 31-40 (1994).
9. D.D. Joseph, J. Feng, and P.Y. Huang, Accepted for publication in *J. Non-Newtonian Fluid Mech.* (1996).
10. M.J. Riddle, C. Narvarez and R.B. Bird, *J. Non-Newtonian Fluid Mech.*, 2 (1977) 23-25.
11. A.C. Pipkin and R. Tanner, "A survey of theory and experiment in viscometric flows of viscoelastic liquids," *Mechanics Today*, 1 (1972) 262-321.
12. H. Giesekus, *Rheol. Acta*, 3 (1963) 59-71.
13. R.B. Bird, R.C. Armstrong and O. Hassager, *Dynamics of Polymeric Liquids*, Vol 1. (1987) John Wiley & Sons.
14. J. Feng, D.D. Joseph, R. Glowinski and T.W. Pan, *J. Fluid Mech.*, 283 (1995) 1-16.
15. Y.J. Liu and D.D. Joseph, *J. Fluid Mech.*, 255 (1993) 565-595.
16. D.D. Joseph, *J. Non-Newtonian Fluid Mech.*, 42 (1992) 385-389.
17. D.D. Joseph and T. Liao, *J. Fluid Mech.*, 265 (1994) 1-23.