

FLOW INDUCED MICROSTRUCTURE OF PARTICLES IN FLUIDIZED SUSPENSIONS

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Pair interactions between neighboring particles and turning couples on long bodies formed from touching bodies give rise to flow induced microstructures. In Newtonian liquids, pair interactions in a fluidized suspension lead to dispersions with particles arranged in lines across the stream. Similar microstructures in a modified form can be observed in fluidized suspensions of particles in turbulent gas. In viscoelastic liquids, sedimenting particles aggregate into chained bodies parallel to the stream when the flow is slow and normal stresses dominate, and into across the stream arrays again when the flow is supercritical and dominated by inertia. The microstructural arrangements in Newtonian and viscoelastic fluids are maximally different. Simple arguments are given here which identify the forces and couples that give rise to all of the main observed microstructures.

Keywords: suspension, fluidization, microstructure, Newtonian, viscoelastic

INTRODUCTION

We are going to describe and carry out a qualitative analysis of our recent experiments on the sedimentation of particles in Newtonian and viscoelastic liquids. The results to be described are generic and not results for one special liquid. Of course, the differences between liquids is important, but we shall not focus on these differences which are well documented in our original papers¹⁻⁶. The main themes in our interpretations rest on the competition between inertia and normal stresses, on the importance of turning couples on long bodies in determining the stable configurations of suspension of spherical bodies, on the contrary behaviors of particles in viscoelastic and Newtonian liquids, on the all-important contrary effects of viscoelastic “pressures”⁷ and inertia. We are presently doing high performance direct simulations of the motion of particles in Newtonian and viscoelastic liquids⁸⁻¹⁴. Gas-solid flows are usually turbulent; they appear to give rise to microstructural features like those in turbulent liquids which are dominated by wakes and turning couples on long bodies.

TURNING COUPLES ON LONG BODIES

It is surprising at first sight that turning couples on long bodies determine the stable configurations of suspensions of spherical bodies. A long body is an ellipsoid or a cylinder; a broad body is a flat plate. When such bodies are dropped in Newtonian fluids, they turn and put their long or broadside perpendicular to the stream. This is an effect of inertia which is usually explained by turning couples at points of stagnation. The mechanism is the same one that causes an aircraft at a high angle of attack to stall.

It is not possible to get long particles to turn broadside in a Stokes flow; bodies with fore-aft symmetry do not experience torques. The settling orientation is indeterminate in Stokes flow; however, no matter how small the Reynolds number may be, the body will turn its broadside to the stream; inertia will eventually have its way. When the same long bodies fall slowly in a viscoelastic liquid, they do not put their broadside perpendicular to the stream; they do the opposite, aligning the long side parallel to the stream.

The difference in the orientation of long bodies falling in Newtonian and viscoelastic liquids is very dramatic; basically the flow orientations in the two liquids are orthogonal. Of course, in very dilute polymeric liquids, the effects of inertia and viscoelasticity will compete and the competition will be resolved by a tilt angle away along the stream. For cylinders with sharp corners, normal stress effects produce the “shape tilting” observed by Liu and Joseph² and explained here. Another, much more dramatic change in the tilting of a long cylinder or flat plate

is associated with the way that inertia comes to dominate high-speed flows of viscoelastic liquids.

PARTICLE-PARTICLE INTERACTIONS

The flow-induced anisotropy of a sedimenting or fluidized suspension of spheres is determined by the pair interactions between neighboring spheres. The principal interactions can be described as *drafting*, *kissing* and *tumbling* in Newtonian liquids and as *drafting*, *kissing* and *chaining* in viscoelastic liquids. The drafting and kissing scenarios are surely different, despite appearances. Kissing spheres align with the stream; they are then momentarily long bodies.

The long bodies momentarily formed by kissing spheres are unstable in Newtonian liquids to the same turning couples that turn long bodies broadside-on. Therefore, they tumble. This is a local mechanism which implies that globally, the only stable configuration is the one in which the most probable orientation between any pair of neighboring spheres is across the stream. The consequence of this microstructural property is a flow-induced anisotropy, which leads ubiquitously to lines of spheres across the stream; these are always in evidence in two-dimensional fluidized beds of finite size spheres. Though they are less stable, planes of spheres in three-dimensional beds can also be found by anyone who cares to look.

The drafting of spheres in a Newtonian liquid is governed by the same mechanism by which one cyclist is aided by the low pressure in the wake of another. The spheres certainly do not follow streamlines since they are big and heavy. If a part of one sphere enters in the wake of another, there will be a pressure difference to impel the second sphere all the way into the wake where it experiences a reduced pressure at its front and not so reduced pressure at the rear. This increased pressure difference impels the trailing sphere into kissing contact with the leading sphere. The motion of the trailing sphere relative to the leading one is in the same sense as in the undisturbed case, into the rear pole of the leading sphere.

Riddle, Navarez and Bird¹⁰ presented an experimental investigation in which the distance between the two identical spheres falling along their line of centers in a viscoelastic fluid was a function of time. They found that for all five fluids used in the experiments that the spheres attract if they are initially close and separate if they are not close; there is a critical separation distance. This looks like a competition between normal stresses and inertia, which is decided by a critical distance which may vary with latitude. Competition of normal stresses and inertia is more typical than rare, and for flows slow enough to enter into the second order region the critical distance scales with $\sqrt{\Psi_1/\rho}$, where $\Psi_1 > 0$ is the coefficient of the first normal stress difference and ρ is the density. The property that chaining tensions are short range is also put in evidence in Figure 1b, which shows that spheres can also detach from the trailing end of a chain when the distance between the last two spheres exceeds a critical value, as in the experiments of Riddle et al.¹⁶

If two touching spheres are launched side-by-side in a Newtonian fluid, they will be pushed apart until a stable separation distance between centers across the stream is established; then the spheres fall together without further lateral migrations (see Figure 2a).

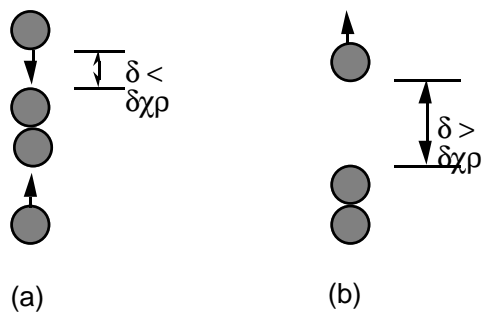


Figure 1. The falling chained spheres are viewed in a frame in which they are at rest. Particles may link to the chain from the bottom or top. If $\delta > \delta_{cr}$ the chained spheres will fall away faster than the trailing sphere.

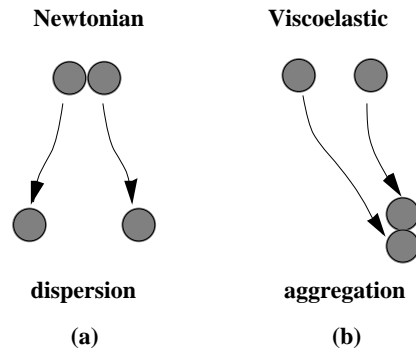


Figure 2. Side-by-side sphere-sphere interactions

On the other hand, if the same two spheres are launched from an initial side-by-side configuration in which the two spheres are separated by a smaller than critical gap, as in Figure 2b, the spheres will attract, turn and chain. One might say that we get dispersion in the Newtonian liquid and aggregation in the viscoelastic liquid.

SPHERE-WALL INTERACTIONS

If a sphere is launched near a vertical wall in a Newtonian liquid, it will be forced away from the wall to an equilibrium distance at which lateral migrations stop (see figure 3a); in the course of its migration it will acquire a counter-clockwise rotation (see Figure 7) which appears to stop when the sphere stops migrating. The rotation is anomalous in that clockwise rotation would be induced from shear at the wall. The anomalous rotation seems to be generated by blockage in which high stagnation pressures force the fluid to flow around the outside of the sphere, as shown in figure 7.

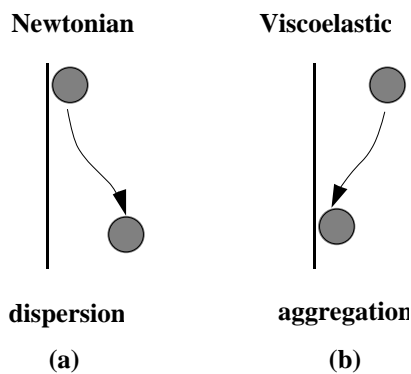


Figure 3: Sphere-wall interactions.

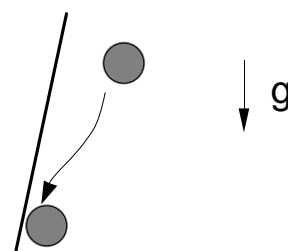


Figure 4: A Sphere in viscoelastic liquid is sucked to a tilted wall.

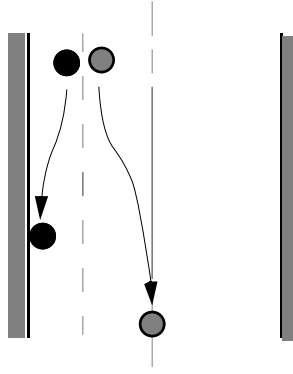


Figure 5: Spheres dropped between widely-spaced walls. The dotted line is the critical distance d_{cr} for wall-sphere inter-action. When $d < d_{cr}$, the sphere goes to the wall. When $d > d_{cr}$, the sphere seeks the center.

If the same sphere is launched near a vertical wall in a viscoelastic liquid, it will be sucked all the way to the wall (see Figure 3b). It rotates anomalously as it falls. This is very strange since the sphere appears to touch the wall where friction would make it rotate in the other sense. Closer consideration shows that there is a gap between the sphere and the wall. The anomalous rotation is again due to blocking which forces liquid to flow around the outside of the sphere (see figure 7).

The pulling action of the wall can be so strong that even if the wall is slightly tilted from the vertical so that the sphere should fall away, it will still be sucked to the wall (see Figure 4).

If the launching distance between the sphere and a vertical wall is large enough, the wall will not attract a sphere falling in a viscoelastic fluid. This means that there is a critical distance $\hat{\delta}$ for attraction. Of course, this distance is smaller when the wall is tilted as in Figure 4. In this case, if the sphere is launched at a distance greater than the critical one, it will fall away from the wall.

The effect of two closely-spaced walls on the migration of particles is not completely understood. We have just said that spheres which fall near a wall in a viscoelastic liquid will be pulled to the wall, but not if the launching distance from the wall is larger than a critical one. On the other hand, we noted that spheres and cylinders dropped between closely-spaced walls do center. We may think that if a sphere is launched between widely-spaced walls at a distance farther than the critical one, it will not be attracted to the near wall and certainly not to the far one. So the equilibrium position will depend on the initial distance, or it is more likely from symmetry to seek the center, as shown in Figure 5. We do not know the answer yet.

If the walls are so closely spaced that the distance d between walls is equal to or smaller than the critical one $\hat{\delta}$ for migration, then both walls will attract the sphere, though perhaps not equally. Experiments suggest centering in this case.

FORCES AND TORQUES

The forces and torques which control the microstructural properties of fluidized suspensions have been analyzed and computed in some papers listed in the references. Here we describe the fluid mechanics in qualitative terms. The forces and torques on the boundary of a solid are the integrated resultant of the traction vector and its moments over the whole body. The traction vector has a tangential (shear stress) and normal component. The shear tractions are associated with the resistance of the fluid to the motions of the body; the normal tractions turn long bodies and push spherical bodies across streamlines. The shear tractions are of only minor importance for flow induced microstructure. Moreover the viscous part of the normal stress is usually small; it is zero in Newtonian fluids for which $\text{div } \mathbf{u} = 0$ ¹². Of course, in gases $\text{div } \mathbf{u} \neq 0$ but it is probable that the effects of the gas compressibility on the motion of solids is minor.

It follows from what we just said that the main action in determining microstructure is the action of the pressure, to which we have already alluded. The pressure in Newtonian liquids

and gases is determined by inertial effects in flow; we may think of the Bernoulli equation and high pressures are at points where the flow is slow and the highest pressures are at stagnation points.

A point of stagnation on a stationary body in potential flow is a unique point at the end of a dividing streamline at which the velocity vanishes. In a viscous fluid all the points on the boundary of a stationary body have a zero velocity but the dividing streamline can be found and it marks the place of zero stress near which the velocity is small. The stagnation pressure makes sense even in a viscous fluid where the high pressure of the potential flow outside the boundary layer is transmitted right through the boundary layer to the body. It is a good idea to look for the dividing streamlines where the shear stress vanishes in any analysis of the flow pattern around the body.

The high pressures at stagnation points are at the front of bodies where the stream impinges. Behind bodies in viscous liquids and gases are “dead water” or wake regions associated with the separation of the boundary layer. Points of separation are like reverse stagnation points and they are close to the points where the flow speed is highest; such points are associated with adverse pressure gradients.

Wake regions behind bodies and high pressure regions in front of bodies produce a large pressure gradient between bodies falling in tandem, pulling the rearward body into the forward body, like debris behind a fast moving truck.

The foregoing description is not appropriate for flows without inertia; Stokes flows are basically boring since the forces which cause symmetric bodies to turn and which impel spherical bodies to cross streamlines are absent.

Slow flows of viscoelastic liquids are quite different; since inertia in such flows is unimportant and normal stresses proportional to the square of the shear rate $\dot{\gamma}$ on the boundary of a solid dominate. For the slow steady flow of a body in a viscoelastic liquid the normal stress at each point on the boundary of the solid is given by

$$T_{nn} = -p_N - \frac{\Psi_1}{4} \dot{\gamma}^2 \quad (1)$$

where p_N is the pressure for a Stokes flow over the same body and $\Psi_1 > 0$. We can call $-\Psi_1 \dot{\gamma}^2 / 4 < 0$ a viscoelastic “pressure”. It’s like a pressure because it is always compressive.

The viscoelastic pressure is great where $\dot{\gamma}$ is large and it is large at points on the body where the flow is fastest; just the opposite of inertia. The viscoelastic pressure opposes inertia and overcomes it when the flow is slow. The form of normal stresses in (1) informs intuition about how particles move and turn in a slow flow of a viscoelastic liquid; one has only to look for crowded streamlines in the Stokes flow near the body to see how the normal stresses are distributed over the body. If the particle has fore-aft symmetry, the Stokes pressure and viscous shear stress each yield a zero torque on the body; thus the normal stresses will turn the body into the stream^{14,15} as in figure 6(a). The argument just given suggests that the longest line of less regular bodies ought to align parallel to the stream; a cube actually does fall slowly with the line through opposite vertices parallel to gravity. For two identical spheres or circular cylinders settling side by side (figure 6b), strong shears occur on the outside and the resulting compressive stresses push the particles together; they then act like a long body and are turned into the stream by torques like those in figure 6(a). Two particles settling in tandem experience imbalanced compressive normal stresses at the bottom of the leading particle and the top of the trailing particle, causing them to chain as in figure 6(c). The lateral attraction of a particle to a nearby wall can be explained by a similar mechanism (figure 6b). Experimental evidence of particle-particle and particle-wall interactions has been documented in⁴.

The compressive stresses which are generated by the motion of particles in plane flow of a second-order fluid produce aggregation rather than dispersion; they align long bodies with the stream and produce chains of particles aligned with the stream.

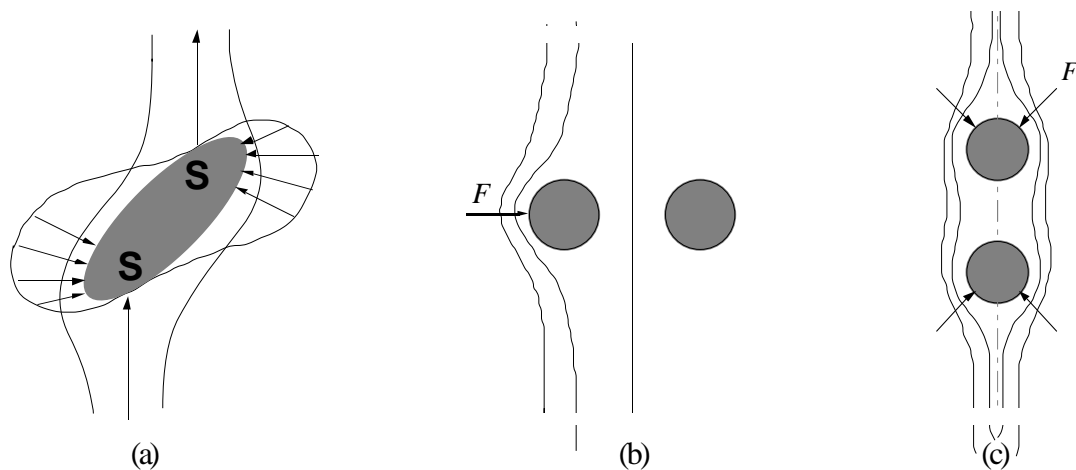


Figure 6. Cartoons of streamlines around bodies settling in Stokes flow. The normal stresses are negative and proportional to $\dot{\gamma}^2$; they are large and compressive where the streamlines are crowded, basically where the flow is fast. Inertial pressures are large at stagnation point $\dot{\gamma} = 0$, where normal stresses vanish. (a) Normal stresses turn the major axis of the ellipse into the stream. For slow flows inertial forces are smaller than normal stresses. (b) The normal stresses force side by side particles together and they urge particles to the wall. (c) Compressive forces cause particles in tandem to chain.

INERTIA DOMINATES

When the body is moving at a much faster velocity than the stream inertia will dominate whether or not the fluid is viscoelastic. A good measure of this is the stable orientation of a long body. This was studied in a number of experiments^{2,3}; it was found that long bodies which fall with their long sides parallel to gravity abruptly turn their long sides perpendicular to gravity when the fall speed is large. Huang et al.¹⁵ showed that this abrupt change can be framed in terms of stability; there are only two possibilities, falling parallel or perpendicular to gravity. In viscoelastic fluids the stable orientation is parallel to gravity when the fall velocity is slower than critical and perpendicular when faster. Inertia dominates when the fall velocity is greater than critical; that is, greater than the speed v/d of diffusion and greater than the speed $c(= \sqrt{\eta/\lambda\rho})$ of shear waves. Under these conditions, signals cannot reach the fluid before the falling body and the body feels the pressures of potential flow at its front side. Such pressures turn the body broadside-on. We may frame this criterion for the dominance of inertia in the following way: let U be the relative speed of the particle, then $U > v/d$ means that the Reynolds number $Re = Uv/d > 1$ and $U > c$ means that the viscoelastic Mach number $M = U/c$ is also greater than one.

When the relative velocity of particles is high enough the microstructural properties of fluidized suspensions will always be dominated by inertia, no matter whether the fluid is a viscoelastic or Newtonian liquid or a turbulent gas; in all these cases particle microstructure is controlled by wakes and turning couples on long bodies. The basic rearrangement sequence here has been described as drafting, kissing and tumbling which can be seen in particles settling in liquids and in ping pong balls fluidized in turbulent air.

CONCLUSIONS

- Fluidized suspensions have an *anisotropic structure* which is determined by the *microstructure*.
- The microstructure depends on the dynamics of *pair particle interactions*.

- Pair particle interactions are dominated by *wakes* and *turning couples* on long and broad bodies.
- Pair particle interactions in Newtonian and viscoelastic fluids are *maximally different*.
- Stagnation pressures due to inertia turn long bodies across the stream. Spherical particles draft, kiss and tumble into cross stream arrays. Inertial forces force particles to separate; they are dispersive.
- Normal stresses are compressive and in plane flows are proportional to the square of the shear rate. Normal stresses vanish at stagnation points and are large where the flow is fast and inertia is small. Spherical particles draft, kiss and chain, linked along rather than across the stream. Normal stresses force particles together; they are aggregative rather than dispersive.
- Normal stress effects dominate flows for which the Reynolds number and viscoelastic Mach numbers are less than one.

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