Direct Simulation of the Motion of Solid Particles in Couette and Poiseuille Flows of Viscoelastic Fluids

by P. Y. Huang[1], J. Feng[2], H. H. Hu[3] and D. D. Joseph[1]

[1]Department of Aerospace Engineering and Mechanics and the Minnesota Supercomputer Institute, University of Minnesota, Minneapolis, MN 55455, USA
[2]Department of Chemical Engineering, University of California, Santa Barbara, CA 93106, USA
[3]Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia, PA 19104, USA

Abstract. This paper reports the results of direct numerical simulation of the motion of a two-dimensional circular cylinder in Couette flow and Poiseuille flow of an Oldroyd-B fluid. Both neutrally buoyant and non-neutrally buoyant cylinders are considered. The cylinder's motion and the mechanisms which cause the cylinders to migrate are studied. The stable equilibrium position of neutrally buoyant particles varies with inertia, elasticity, shear-thinning and the blockage ratio of the channel in both shear flows. Shear-thinning promotes the migration of the cylinder to the wall while inertia causes the cylinder to migrate away from the wall. The cylinder moves closer to the wall in a narrower channel. In a Poiseuille flow, the effect of elastic normal stresses is manifested by an attraction toward the nearby wall if the blockage is strong. If the blockage is weak, the normal stresses act through the curvature of the inflow velocity profile and generate a lateral force that points to the centerline. In both cases, the migration of particles is controlled by elastic normal stresses which in the limit of slow flow in two dimensions are compressive and proportional to the square of the shear rate on the body. A slightly buoyant cylinder in Couette flow migrates to an equilibrium position nearer the centerline of the channel in a viscoelastic fluid than in a Newtonian fluid; On the other hand, the same slightly buoyant cylinder in Poiseuille flow moves to a stable position farther away from the centerline of the channel in a viscoelastic fluid than in a Newtonian fluid. Marked effects of shear thinning are documented and discussed.

1. Introduction

We have developed a numerical scheme to simulate the motion of fluids and particles in two dimensions. The fully nonlinear nature of the problem is preserved and treated directly. This method was first applied to particle motion in Newtonian fluids (Hu, Joseph & Crochet 1992; Feng, Hu & Joseph 1994ab; Huang, Feng & Joseph 1994). Sedimentation in an Oldroyd-B fluid has been studied using the same scheme (Feng, Huang & Joseph 1996). The present paper deals with the motion of particles in Couette and Poiseuille flows of an Oldroyd-B fluid.

In a celebrated experiment, Segré and Silberberg (1961) discovered that neutrally buoyant spherical particles in a pipe flow of a Newtonian fluid migrate to a radial position roughly midway between the axis and the wall. The perturbation analysis of Ho and Leal (1974) demonstrated that inertia causes a sphere to assume an off-center position in a planar Poiseuille flow. Karnis and Mason (1966) studied the migration of a sphere in a viscoelastic liquid in a pipe flow when inertia is negligible. The sphere approaches the center of the pipe regardless of its initial position. Ho and Leal (1976) showed that normal stresses in a second-order fluid cause a sphere to migrate to the center of a planar Poiseuille flow. A few experiments on the motion of particles in Couette flows have been reported (Gauthier *et al*. 1971, Bartram *et al.* 1975). The purpose of these experiments was to study the migration of spheres in the slightly non-uniform shear flow in the gap between two concentric cylinders. No observation of particle motions in uniform shear flows of viscoelastic fluids has been reported.

Direct numerical simulations are not restricted to small non-linearity and the motion of particles may be followed in real time. Feng, Huang & Joseph (1996) simulated the sedimentation of particles in a vertical channel filled with an Oldroyd-B fluid and they showed that particles are forced toward or away from walls depending on the particle-wall separation and the width of the channel. The purpose of the present paper is to extend the same line of study to shear flows. We will investigate the effects of normal stresses, inertia and solid walls on the motion of a circular particle in a Couette or Poiseuille flow bounded by two parallel walls.

The motion of the incompressible viscoelastic fluid is governed by the equations of motion:

$$\begin{cases} \nabla \cdot \mathbf{u} = 0 \\ \rho_f \left(\dfrac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \nabla \cdot \mathbf{T} \end{cases} \tag{1}$$

where ρ_f is the fluid density. The constitutive equation for an Oldroyd-B fluid is:

$$\mathbf{T} + \lambda_1 \overset{\nabla}{\mathbf{T}} = 2\eta(\mathbf{D} + \lambda_2 \overset{\nabla}{\mathbf{D}}) \tag{2}$$

where $\mathbf{D} = (\nabla \mathbf{u} + \nabla \mathbf{u}^T)/2$ is the strain-rate tensor; λ_1 and λ_2 are constant relaxation and retardation times; η is the viscosity. The triangle denotes the upper-convected time-derivative:

$$\overset{\nabla}{\mathbf{T}} = \frac{\partial \mathbf{T}}{\partial \mathrm{t}} + \mathbf{u} \cdot \nabla \mathbf{T} - (\nabla \mathbf{u})^{\mathrm{T}} \cdot \mathbf{T} - \mathbf{T} \cdot (\nabla \mathbf{u}) \tag{3}$$

where $(\nabla \mathbf{u})_{\mathrm{ij}} = \partial u_{\mathrm{j}} / \partial x_{\mathrm{i}}$. Shear-thinning can be easily added to the Oldroyd-B model by using the Carreau-Bird viscosity law:

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \left[1 + \left(\lambda_3 \dot{\gamma}\right)^2\right]^{\frac{\mathrm{n}-1}{2}} \tag{4}$$

where $\dot{\gamma}$ is the strain-rate defined in terms of the second invariant in the usual way and $0<\mathrm{n}\leq 1$. In our computation, we fixed the fluid relaxation time ratio $\lambda_2/\lambda_1=1/8$ and viscosity ratio $\eta_\infty/\eta_0=0.1$.

Joseph and Feng (1996) did analysis of the forces that move particles in slow plane flows of a second-order fluid:

$$\mathbf{T} = -p\mathbf{I} + \eta \mathbf{A}_1 + \alpha_1 \mathbf{A}_2 + \alpha_2 \mathbf{A}_1^2 \tag{5}$$

where $\mathbf{A}_1$ and $\mathbf{A}_2$ are the first and second Rivlin-Ericksen tensors. Using the Giesekus-Tanner theorem they showed that

$$p = p_{\mathrm{N}} + \frac{\alpha_1}{\eta} \frac{Dp_{\mathrm{N}}}{Dt} + \left(\alpha_2 + \frac{3\alpha_1}{2}\right)\dot{\gamma}^2 \tag{6}$$

where p_N is the pressure of the Stokes flow for the prescribed values of velocity at the boundary of the flow domain. At the boundary of a solid body with outward normal $\mathbf{n}$, the normal component T_{nn} of $\mathbf{T}$ is given by

$$T_{nn} + p = (2\alpha_1 + \alpha_2)\dot{\gamma}^2 \tag{7}$$

and

$$T_{nn} = -p_N - \frac{\alpha_1}{\eta}\frac{Dp_N}{Dt} + \frac{\alpha_1}{2}\dot{\gamma}^2 \tag{8}$$

This shows that the normal component of the non-linear elastic part of the normal stress

$$\frac{\alpha_1}{2}\dot{\gamma}^2 = -\frac{\Psi_1(0)}{4}\dot{\gamma}^2 \tag{9}$$

where $\Psi_1(0)$ is the coefficient of the first normal stress, is always compressive. These compressive normal stresses are such as to turn long bodies into the stream and cause circular particles to aggregate and chain.

In the case of the Oldroyd-B fluid studied here, we find that $\alpha_1 = -\eta(\lambda_1 - \lambda_2)$ and $\alpha_2 = -2\alpha_1$ in the limit of slow flows, hence the coefficient of the second normal stress vanishes $(2\alpha_1 + \alpha_2) = 0$. Equation (7) shows that the entire contribution to T_{nn} comes from the pressure and none from the extra stress with

$$T_{nn} = -p = -p_N + (\lambda_1 - \lambda_2)\frac{Dp_N}{Dt} - \frac{1}{2}\eta(\lambda_1 - \lambda_2)\dot{\gamma}^2. \tag{10}$$

Of course, shear thinning does not appear at second-order in the asymptotic analysis leading to the second-order form of the Oldroyd-B model. We can stimulate some thoughts about the important role of shear thinning in the motion of particles by a heuristic argument due to Joseph suggested by writing the last term of (10) as

$$\kappa(\lambda_1 - \lambda_2)\dot{\gamma}^{n+1}. \tag{11}$$

Equation (11) shows that the normal stresses in the shear thinning form of the Oldroyd-B fluid still increase more rapidly than $\dot{\gamma}$ so that if the effect of shear thinning is to decrease the viscosity and increase the shear rate at places of high $\dot{\gamma}$ on the body, the overall effect would be to reinforce migration effects due to compressive normal stresses over and above what they would be without shear thinning. Another way to say this is that in a pipe flow with a prescribed pressure gradient, the pressure force balances the shear force at the wall so the shear stress $\tau_w = \eta(\dot{\gamma})\dot{\gamma}$ is the same for all viscosity functions. If the fluid thins in shear, the viscosity η goes down and the shear-rate $\dot{\gamma}$ up, keeping the product constant. Then $\eta(\dot{\gamma})\dot{\gamma}^2 = \tau_w\dot{\gamma}$ is larger than what it would be if the fluid did not shear thin because $\dot{\gamma}$ is larger.

Normal stresses are not the only forces which can cause a particle to migrate in a shear thinning fluid; a lateral imbalance of the shear stresses can also cause lateral migration.

In general the shear rates are large near the places where the velocities are large and the pressures due to inertia are small. On the other hand, where the stagnation pressures are high the compressive effects of the elastic normal stresses are small, and the places where particles migrate are worked out in this competition. This argument seems to be consistent with the results of the numerical experiments described below.

2. Numerical method

The equations of motion and the constitutive equation are solved at each time step using an EVSS (Elastic-Viscous-Split-Stress) formulation. The main solver was adapted from that of the popular code POLYFLOW. The solid-fluid coupling is treated by an Arbitrary Lagrangian-Eulerian method with mesh velocities. The numerical method has been explained before (Huang and Feng 1995; Feng, Huang & Joseph 1996) and the details will not be discussed here. Because of the lower-order scheme used, our computations are limited to low Deborah numbers; above a critical *De* value, convergence is lost. This critical *De* depends on the Reynolds number *Re*, the degree of shear-thinning in the fluid and the blockage ratio of the channel β; higher *De* can be reached for smaller *Re*, smaller β and milder shear-thinning. As an example, we have obtained convergent results up to *De*=3 at *Re*=5, β =0.25 and n=1 in a Poiseuille flow.

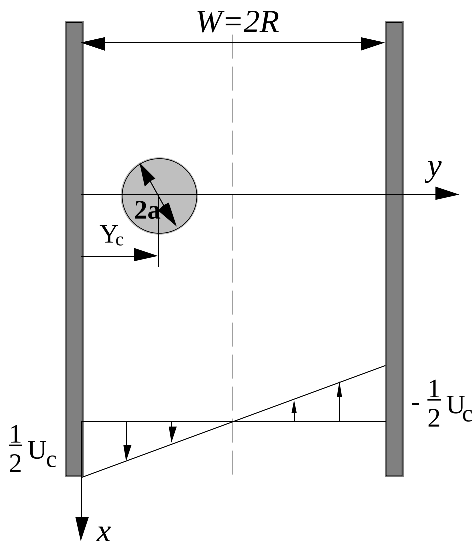

Figure 1. Lateral migration of a solid particle in Couette flow between two walls moving in opposite directions. *W* is the width of the channel. Y_c is the distance of the cylinder center from the wall. We call the final equilibrium value of Y_c/W the standoff distance.

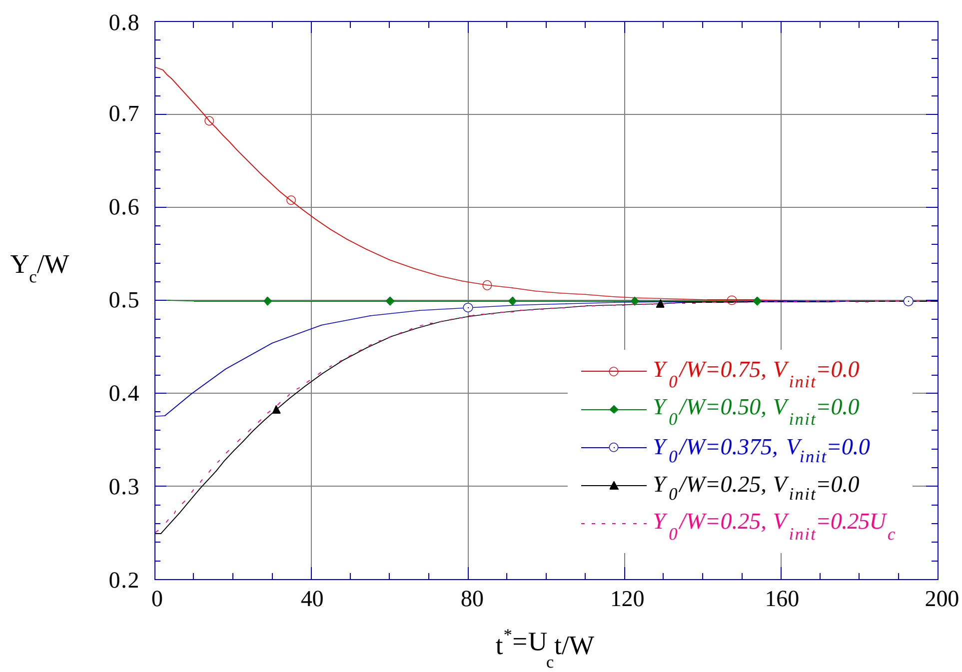

Figure 2. Lateral migration of a neutrally buoyant particle released from different initial positions in Couette flow of an Oldroyd-B fluid (β=0.25, *Re*=5, *De*=1.0). Y_0 indicates the initial position of the particle. V_{init} is the initial velocity of the particle.

In the following sections, numerical results for the motion of a two-dimensional circular cylinder in a Couette flow and a Poiseuille flow will be presented. Both neutrally buoyant and non-neutrally buoyant particles are considered.

3. Particle migration in a Couette flow

The lateral migration of a circular particle in a two-dimensional Couette shear flow is studied. In our setup, the walls are moving in opposite directions with speed $\pm\frac{1}{2}U_c$ (Figure 1). The motion of a solid particle in the channel is determined by the following dimensionless parameters: Reynolds number $Re=\rho_f U_c a/\eta$, Deborah number $De=U_c\lambda_1/a$, blockage ratio of the channel $\beta=a/R$ and ratio of solid particle to fluid density ρ_s/ρ_f . Here a is the radius of the particle, R is the semi-width of the channel.

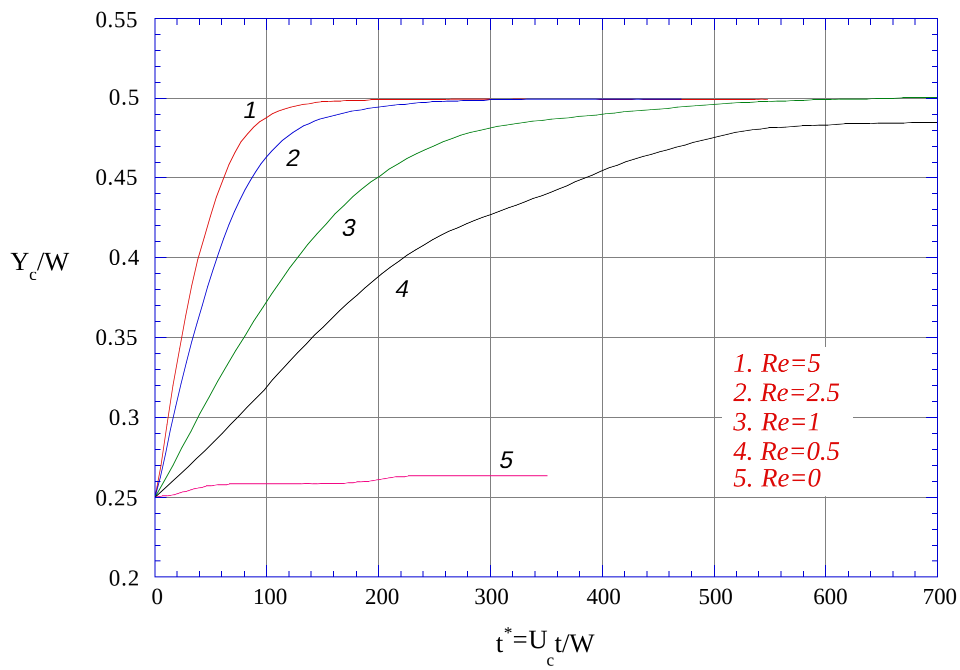

Figure 3. The effect of the Reynolds number on the migration of a neutrally buoyant particle in Couette flow of an Oldroyd-B fluid (β=0.25, De=1.0). The centerline of the channel is no longer a stable equilibrium when the Reynolds number is small.

3.1. Neutrally buoyant particles

A neutrally buoyant circular cylinder is released at different initial positions in a channel with β=0.25, *Re*=5 and *De*=1.0, as shown in figure 2. The particle reaches an equilibrium position at the centerline of the channel, regardless of its initial position and velocity. This is the same as in a Newtonian fluid (Feng, Hu & Joseph 1994b).

The numerical solution of Feng, Hu & Joseph (1994b) and the perturbation solutions of Ho & Leal (1974) and Vasseur & Cox (1976) showed that the centerline of a channel is the stable equilibrium position for a neutrally-buoyant particle in a Couette flow of a Newtonian fluid regardless of the Reynolds number. For a particle sedimenting in an Oldroyd-B fluid, Feng, Huang & Joseph (1996) found that the particle is pushed further away from the side wall as the Reynolds number is increased. To study the effect of Reynolds number on the motion of a solid particle in a Couette flow of an Oldroyd-B fluid, we fixed the Deborah number *De*=1.0 and changed the Reynolds number *Re* by changing both the Couette inflow velocity U_c and relaxation time λ_1. Figure 3 shows that the particle moves toward the centerline very fast when inertia is strong (*Re*=5). As *Re* is decreased the centerline of the channel is apparently no longer a global attractor of trajectories of the neutrally buoyant particle; normal stress effects become dominant as *Re* is decreased. If we set Reynolds number *Re* to zero by removing the inertia term $\mathbf{u} \cdot \nabla \mathbf{u}$ in equation (10), the particle no longer moves to the centerline but stays very close to the wall (see figure 3). Particle migrations in viscoelastic Couette flows and sedimentation are such as to have normal stresses generated on the side of the particle away from the wall, forcing it to the wall and the migration is opposed by lubrication forces associated with slow flow in smaller gap near the wall (see Feng, Huang and Joseph, 1996).

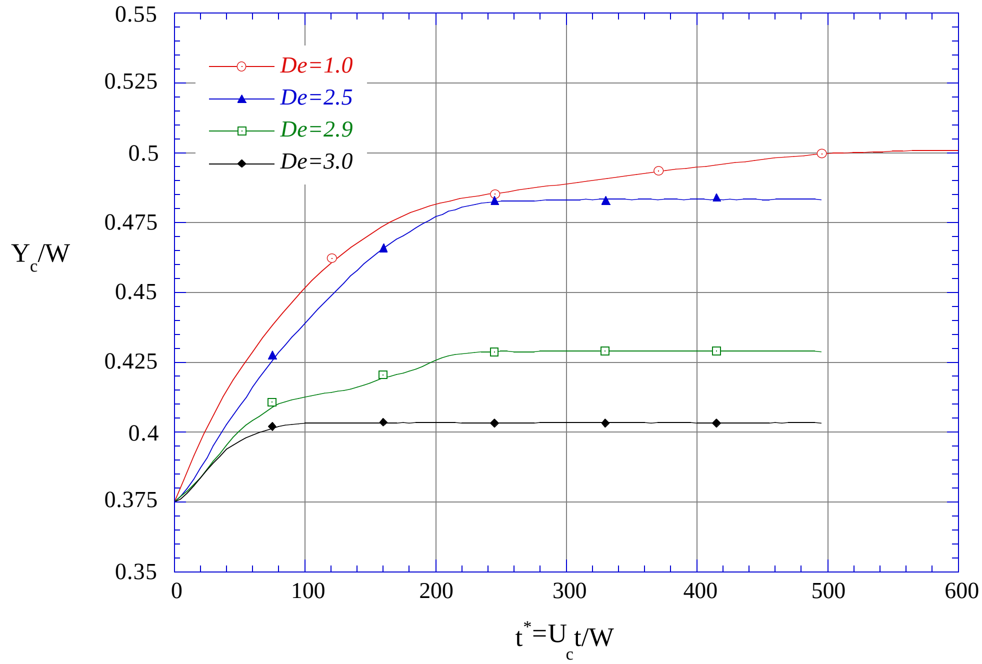

Figure 4. The effect of Deborah number on the migration of a neutrally buoyant particle in Couette flow of an Oldroyd-B fluid (β=0.25, *Re*=1.0).

In figure 4, we study the effects of *De* with a fixed Reynolds number *Re*=1.0. When *De*=1.0, the particle moves to the centerline of the channel. As the Deborah number is increased, the particle migrates toward the side wall. Perhaps there is a critical *De* for any Reynolds number such that the center attracts for *De* smaller than critical and a position nearer the wall attracts when *De* is greater than critical.

The effect of normal stresses is strengthened by the effect of the blockage. To emphasize this, we suppressed inertia by setting *Re*=0. For a wide channel with β=0.25 or smaller, the particle goes to the centerline eventually regardless of its initial position. When the gap is smaller than critical, the centerline of the channel is no longer a stable equilibrium position as can be seen in figure 5. The particle is pushed toward the side wall. For β=0.5, the particle migrates until it almost touches the wall. At this point our numerical computation fails. Figures 6 and 7 give the contours of streamlines as seen in a coordinate system fixed on the particle. As shown in figure 6 with β=0.25, the particle at the time t^* has drifted to its equilibrium position which is determined by a balance of compressive normal stresses on the side near the centerline where the streamlines are crowded and lubrication forces on the other side. This feature is even clearer in figure 7 with β=0.5.

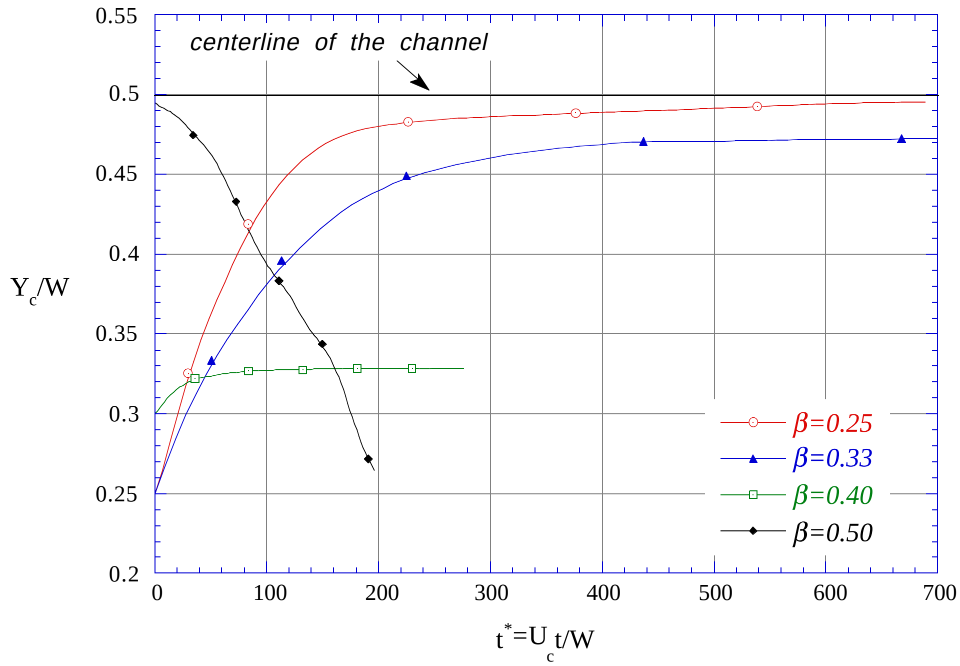

Figure 5. The effect of the blockage ratio β=a/R on the migration of a neutrally buoyant particle in Couette flow of an Oldroyd-B fluid (*De*=1.0, *Re*=0). The cylinder will touch the wall when the center of the cylinder is at $Y_c/W=0.5\beta$ in the channel.

In the sedimentation problem studied by Feng, Huang & Joseph (1996), the final equilibrium position of the particle is also closer to the side wall when the channel is narrower but the particle never approaches the wall so closely that the computation fails even when *Re*=0.

Figure 6. Streamlines for the Couette flow around a cylinder under conditions specified in figure 5, β=0.25 and t^*=689. At this t^* the particle has drifted to its equilibrium position.

The position of equilibrium is determined by a balance of the normal compressive stresses where the streamlines are crowded and the lubrication forces which arise in the gap between the cylinder and the wall. Lubrication forces are associated with the change from high to low pressures, linear in velocity, when the fluid is forced slowly through the gap between the cylinder and the wall. For faster flow through the gap, stagnation pressures proportion to the square of the velocity are induced by inertia, hence lubrication forces are at work.

Figure 7. Streamlines for the Couette flow around a cylinder under conditions specified in figure 5, β=0.50 and t^*=196.5. The particle is still migrating to the wall. The computation fails for larger t^*.

The effect of shear thinning is also to drive the particle closer to the wall. This effect has already been noted by Huang & Feng (1995). In figure 8, we used a Carreau-Bird viscosity law with λ_3/λ_1=10; the power index *n*=1.0 corresponds to no shear-thinning, and smaller *n* indicates more shear-thinning. The inertia effect was removed in these computations. Shear-thinning enhances the migration of the cylinder to the wall. The results are consistent with the argument we gave in the first section; shear thinning promotes the migration toward the wall because of the action of the compressive normal stresses on the side of the particle away from the wall, as shown in equation (11). For *n*=0.8 and *n*=0.6, the particle approaches the side wall rapidly and so closely that our computation fails.

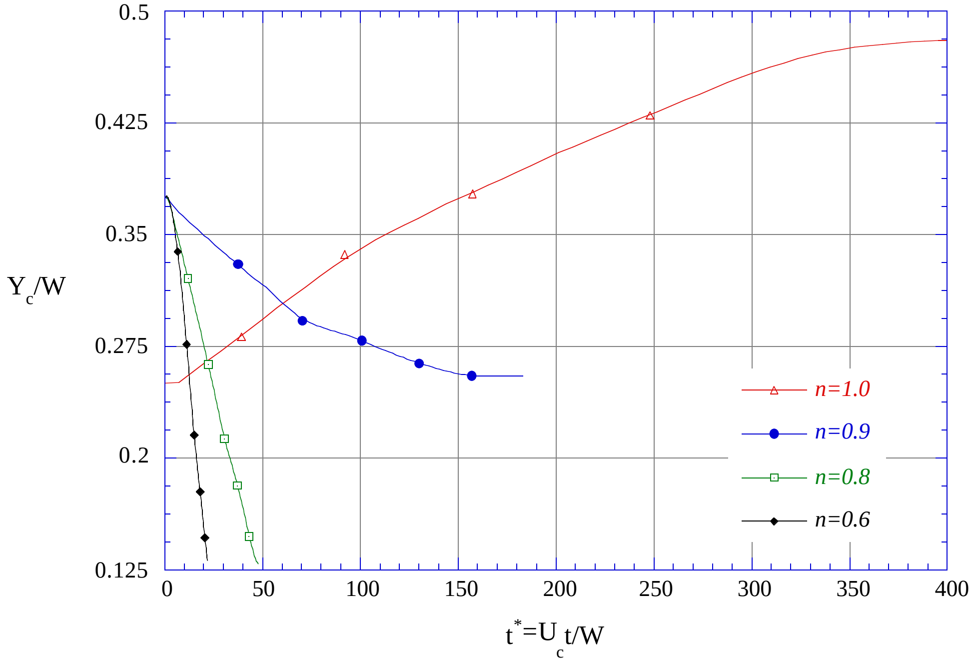

Figure 8. The effect of shear-thinning on a neutrally buoyant particle in Couette flow of an Oldroyd-B fluid (β=0.25, *Re*=0, *De*=2.8). The cylinder will touch the wall when the center of the cylinder is at Y_c/W=0.125 in the β=0.25 channel.

3.2. Non-neutrally buoyant particles

Now we focus our attention on the motion of non-neutrally buoyant particles. In the following simulations, we keep these parameters unchanged: β=0.25, *Re*=5 and *De*=1.0 and release particles with different densities.

Figure 9 shows that the particle reaches a stable position close to either of the walls when it is only slightly buoyant (ρ_s/ρ_f=0.998 and ρ_s/ρ_f=1.002). A light particle is always at the side of the channel in which the flow is down and a heavy particle is at the other side. As the density difference increases, the slip velocity of the particle increases, enhancing the effects of inertia which push particles away from the wall. As in a Newtonian fluid (Feng *et al.*, 1994b), this effect is not quite symmetric with respect to the sign of the density difference because the solid inertia varies (see figure 10).

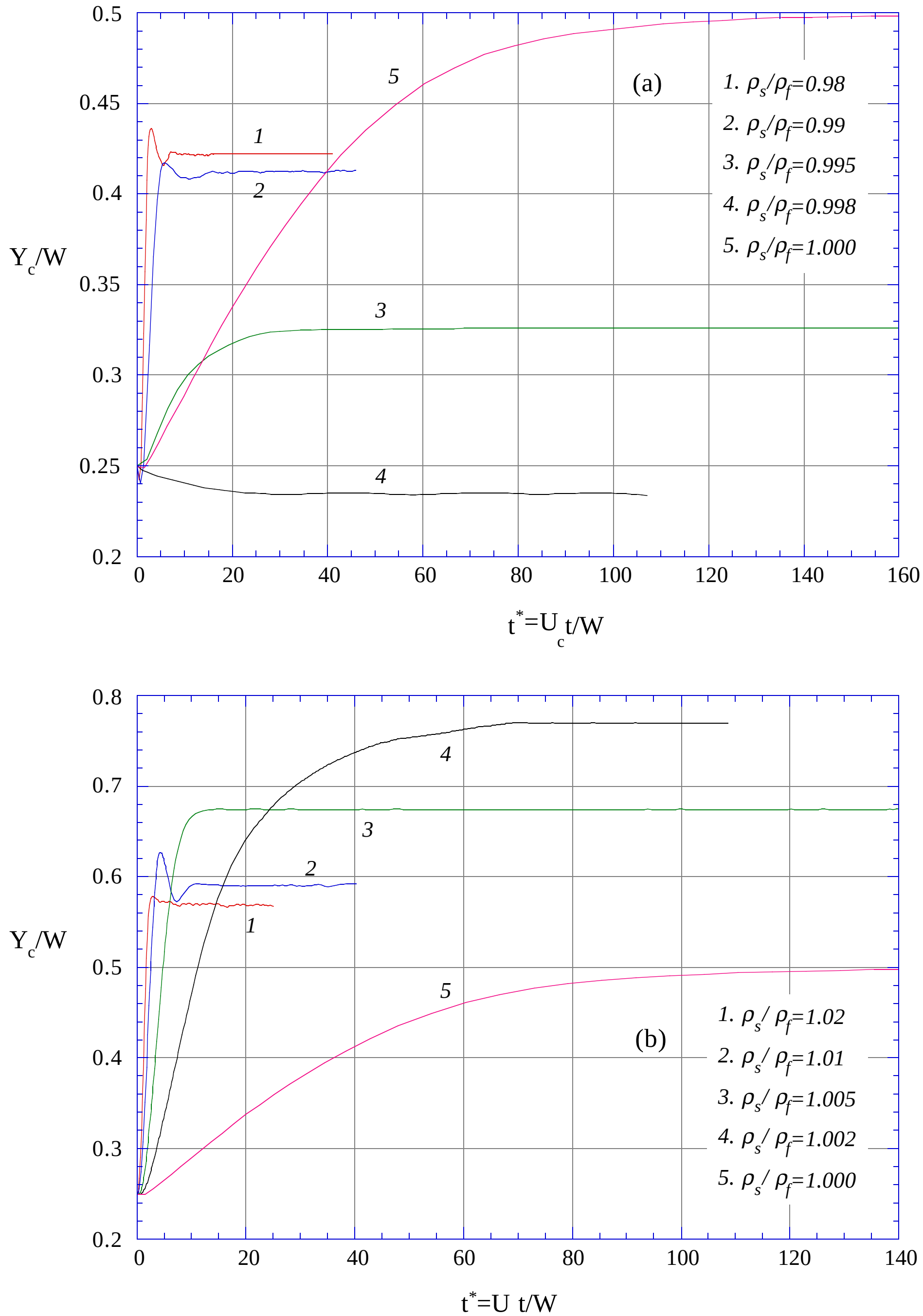

Figure 9. Trajectories of non-neutrally buoyant particles in Couette flow of an Oldroyd-B fluid (β=0.25, *Re*=5, *De*=1.0). (a) Particles with density ρ_s smaller than the fluid ρ_f; (b) particles with density ρ_s larger than the fluid ρ_f.

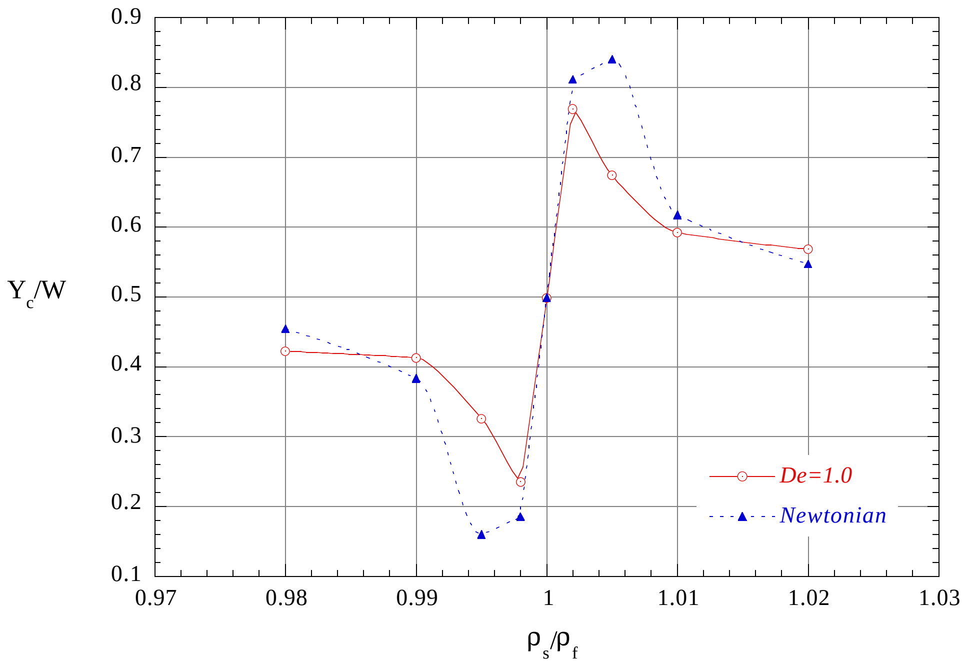

Figure 10. Equilibrium position of a non-neutrally buoyant particle in Couette flow of an Oldroyd-B fluid with *De*=1.0 and a Newtonian fluid (β=0.25, *Re*=5).

A slightly buoyant ($0.99<\rho_s/\rho_f<1.01$) particle in an Oldroyd-B fluid moves closer to the centerline of the channel than in a Newtonian fluid whereas a more buoyant particle moves closer to the wall. This is similar to the shift of the equilibrium position in sedimentation studied by Feng, Huang and Joseph (1996).

4. Particle migration in a Poiseuille flow

Now let us consider the lateral migration of a circular particle in a two-dimensional Poiseuille flow of an Oldroyd-B fluid (Figure 11). The geometry of the channel is the same as that in a Couette flow. All the dimensionless parameters can be defined as in Couette flow, except that the velocity U_c is replaced by the maximum velocity of the Poiseuille inflow U_0.

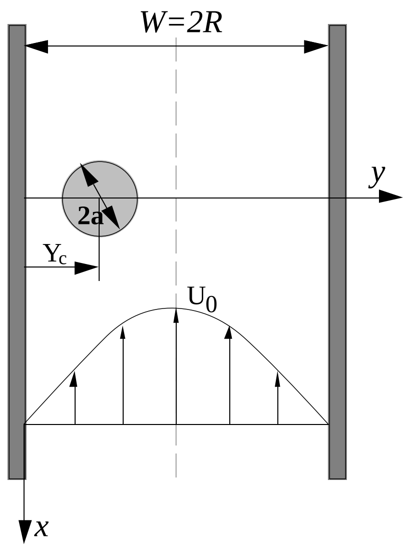

Figure 11. Lateral migration of a solid particle in a plane Poiseuille flow. U_0 is the maximum velocity at the centerline. (X_c, Y_c) is the center of the cylinder.

4.1. Neutrally buoyant particles

The motion of a neutrally buoyant particle in a viscoelastic fluid varies with the Reynolds number *Re*, the Deborah number *De*, the blockage ratio β and the parameters of shear-thinning; to study these effects, we change one of the parameters while keeping the others fixed.

At first a neutrally buoyant circular cylinder is released from different initial positions in a channel with β=0.25, *Re*=5 and *De*=0.2. The behavior is similar to that in a Newtonian fluid; all the particles reach the same equilibrium position roughly half-way between the wall and the center, as shown in figure 12. The final position of a particle is independent of the initial conditions.

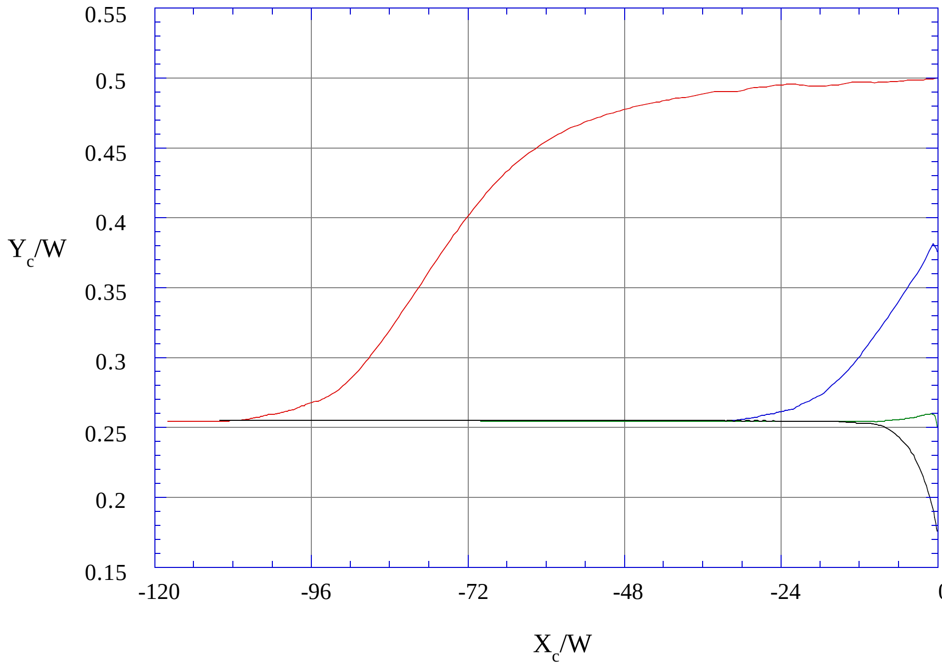

Figure 12. Trajectories of a neutrally buoyant particle released at different initial positions in Poiseuille flow of an Oldroyd-B fluid (β=0.25, *Re*=5, *De*=0.2).

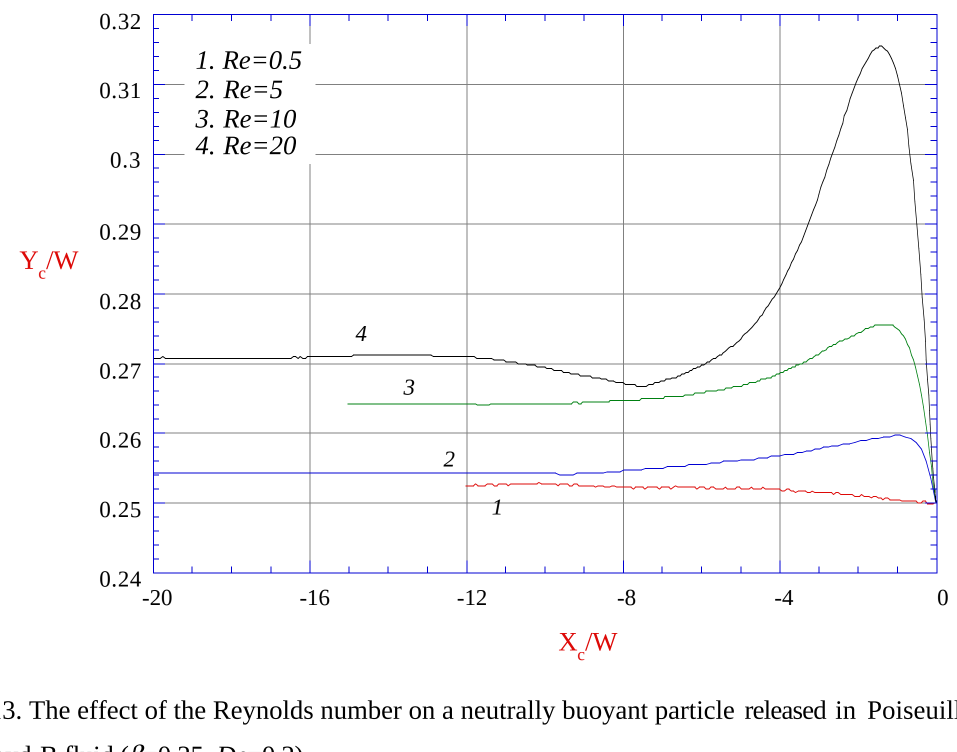

Figure 13. The effect of the Reynolds number on a neutrally buoyant particle released in Poiseuille flow of an Oldroyd-B fluid (β=0.25, *De*=0.2).

Figure 13 shows that the effects of inertia, at higher *Re*, are such as to push the particle away from the wall after an initial overshoot as in the Couette flow of an Oldroyd-B fluid (cf. figure 3). In Poiseuille flow of a Newtonian fluid, the effect of inertia, at higher *Re*, are such as to push the particle toward the wall (Feng *et al.*, 1994b).

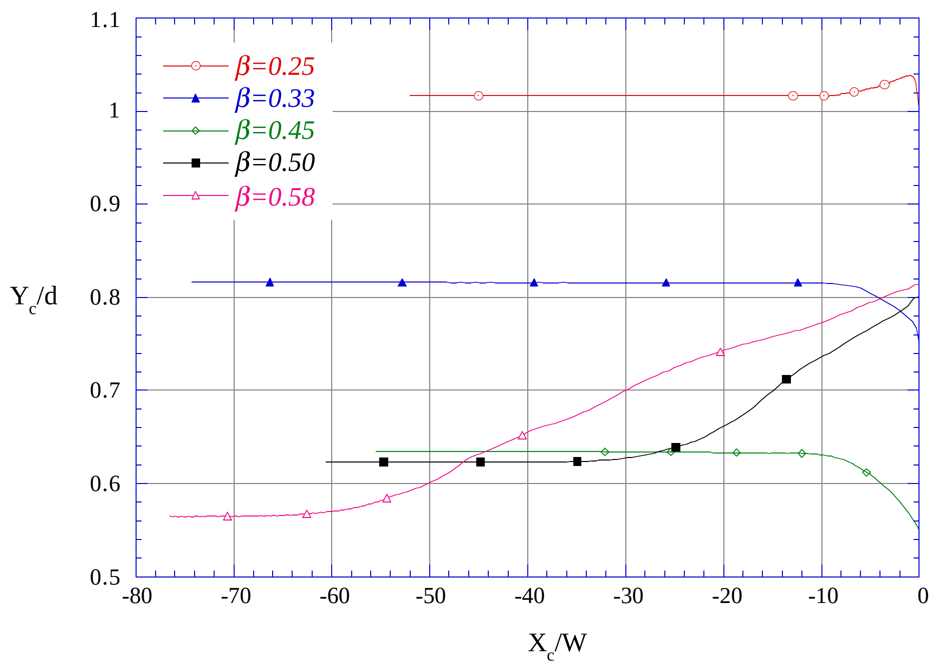

Figure 14. The effect of the blockage ratio $\beta = a/R$ on the motion of a neutrally buoyant particle in Poiseuille flow of an Oldroyd-B fluid (*Re*=5, *De*=0.2). The cylinder will touch the wall when the center of the cylinder is at Y_c/d=0.5 where *d* is the diameter of the particle.

The width of the channel plays a very important role in the migration of the solid particles. Karnis & Mason (1966) observed the migration of a sphere in a viscoelastic fluid in a pipeline flow when inertia is negligible. They found that the sphere moves to the center of the pipe regardless of its initial position. The perturbation results of Ho and Leal (1976) showed that normal stresses of a second-order fluid cause the neutrally buoyant particle to migrate to the center of the channel. On the other hand, Dhahir & Walters (1989) measured the lateral force acting on a circular cylinder fixed in a slow channel flow of a viscoelastic fluid. This force pulls the cylinder toward the nearby wall. Their results are later verified by the numerical results of Carew & Townsend (1991) and Feng, Huang & Joseph (1996).

The principal difference between the two groups of results is the blockage ratio. Dhahir & Walters (1989) and Carew & Townsend (1991) used a very large cylinder in their planar Poiseuille flow: β=7/12. In the perturbation analysis of Ho and Leal (1976), they restricted the solid particle to be very small as $\beta = a/R \ll 1$. The effect of the wall is merely to generate the undistributed flow field and has no direct contribution to the lateral migration of the particle. The experiment of Karnis & Mason (1966) also used very small particles, though their results might have been affected by the density mismatch as discussed in the next subsection. It appears that the effects of viscoelastic stresses are modulated by the wall blockage; a particle experiences an attraction toward the nearby wall at large β, but feels a lateral force toward the center of the Poiseuille flow at small β.

The effects of the blockage ratio β are illustrated in figure 14. As the width of the channel is decreased (larger β), the particle stabilizes closer to the wall because the streamlines in a narrower channel crowd in such a way as to intensify the shear rates in the more open portions of the channel away from the wall (see figures 15 and 16), intensifying compressive normal stresses there.

Figure 15. Streamlines for the Poiseuille flow around a cylinder when *Re*=5, *De*=0.2 and β=0.25. The particle has reached its equilibrium position which is determined by a balance of compressive normal stresses on the open side near the centerline where the streamlines are crowded and lubrication forces on the other side.

To explore the limit of small β, we have done a series of computations with β=0.025 (figure 17). The inertia term is also turned off so we can focus our attention on the effects of the normal stresses induced by the curvature of the inflow velocity. Now the particle approaches the center of the channel where the shear rate of the undisturbed flow is zero. Without wall effects, the migration of the particle is controlled by the normal stresses through the curvature of the velocity profile. The larger the Deborah number *De* is, the faster the particle goes toward the center of the channel. The results agree with the perturbation solution of Ho & Leal (1976). Our simulation is in two dimensions where the wall effect can be even stronger than in three dimensions.

Figure 16. Streamlines for the Poiseuille flow around a cylinder when *Re*=5, *De*=0.2 and β=0.50. The particle has drifted to an equilibrium position very close to the wall. Streamlines on the open side near the centerline are crowded.

The effects of Deborah number *De* at relatively large β are shown in figure 18. The inertia is again turned off (*Re*=0). The particle moves all the way to the wall and this migration is faster when *De* is larger. Without inertia effects, the strong effects of the wall blockage overwhelm the effects of the curvature of the velocity profile and increase the effects of the compressive normal stresses acting on the particle surface so as to drive the particle toward the wall. This is consistent with the experimental results of Dhahir & Walters (1989) and the numerical results of Carew & Townsend (1991).

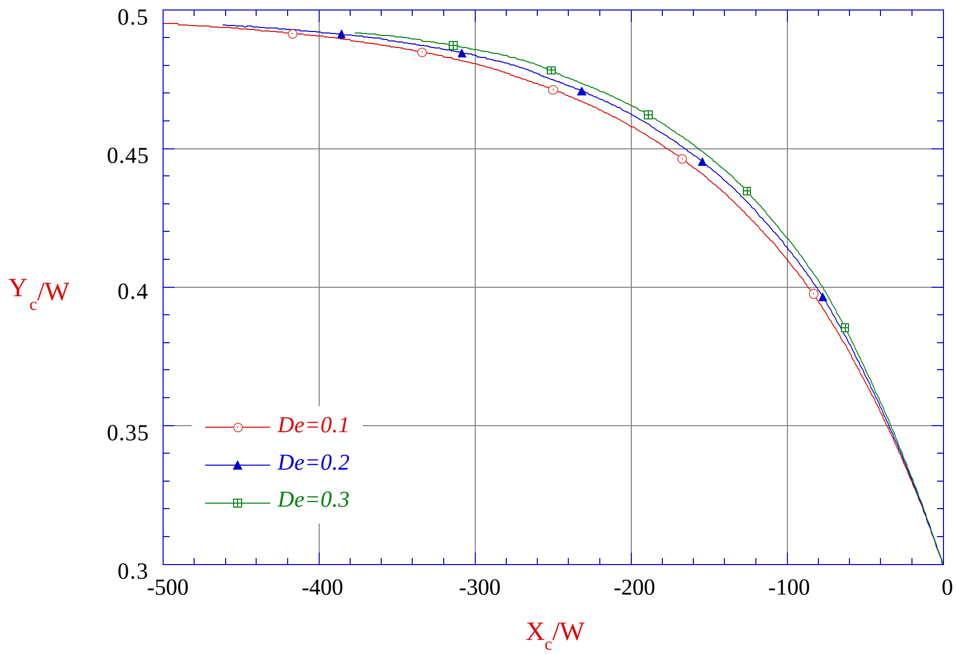

Figure 17. The effect of normal stresses on a neutrally buoyant particle released in Poiseuille flow of an Oldroyd-B fluid without wall effects (β=0.025, *Re*=0). The cylinder drifts toward the center of the channel.

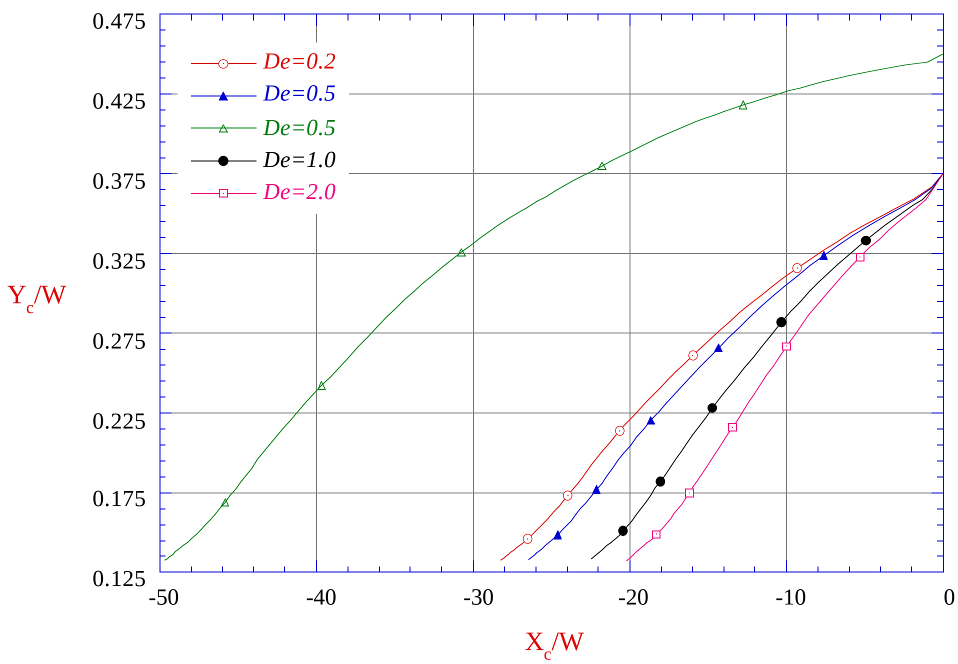

Figure 18. The effect of normal stresses on a neutrally buoyant particle in Poiseuille flow of an Oldroyd-B fluid in the limiting case of no inertia (β=0.25, *Re*=0).

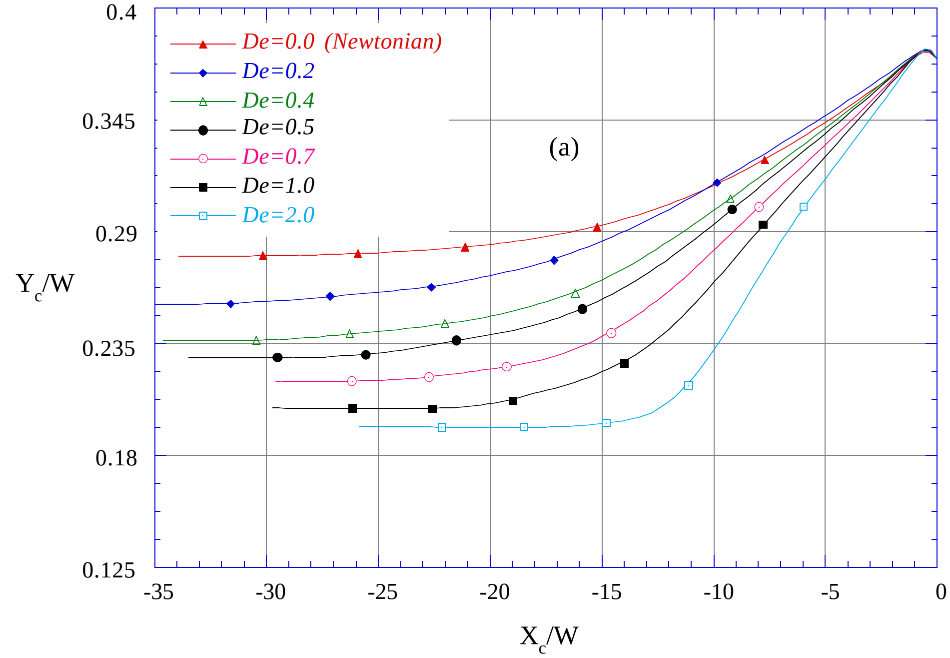

Figure 19. The effect of normal stresses on a neutrally buoyant particle released in Poiseuille flow of an Oldroyd-B fluid (β=0.25, *Re*=5). The cylinder will touch the wall when the center of the cylinder is at Y_c/W=0.125 in this β=0.25 channel. (a) The particle trajectories; (b) the standoff distances.

The effect of inertia on the interplay between β and *De* is such that the particle never touches the wall. Figure 19 shows a series of competition. At β=0.25, the elastic stresses push the particle toward the wall. But unlike in figure 18, the particle will stop at a standoff distance from the wall. This standoff distance Y_c/W decreases as *De* increases, and is expected to increase with *Re*.

From figures 17, 18 and 19 we can draw the conclusion that inertia always pushes the particle away from the wall. The normal stresses push the particle toward the wall at strong blockage. At weak blockage, the wall attraction becomes minimal and the normal stresses generate a lift force, through the curvature of the velocity profile, that leads the particle to the center of the channel where the shear rate is zero. The final equilibrium position of the particle is determined by a competition between inertia and normal stresses. In the sedimentation problem studied by Feng *et al.* (1996), the particles assume a finite standoff distance close to the wall even for *Re*=0.

The effect of shear-thinning always promotes the particle's motion toward the wall. A heuristic argument, associated with equation (11), was given in section 1 whose main thrust is that compressive normal stresses are intensified by shear thinning because of the increase of the shear rate at walls where the resistance to flow is weakened. The shear stress τ_w may not change much at a particle surface, but $\tau_w\dot{\gamma}$ will definitely increase there. This idea is consistent with the numerical results displayed in figure 20 where it is shown that the final standoff distance of the particle comes closer to the wall as the power law index *n* is decreased. More shear thinning also gives rise to a larger transient overshoot.

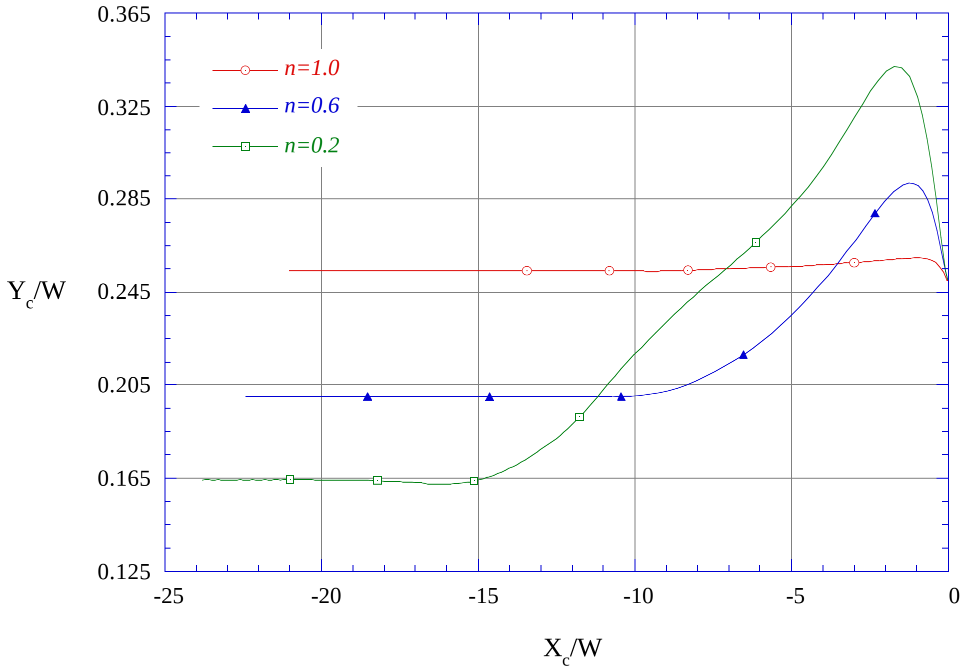

Figure 20. The effect of shear-thinning on the migration of a neutrally buoyant particle in Poiseuille flow of an Oldroyd-B fluid (β=0.25, *Re*=5, *De*=0.2, λ_3/λ_1=100). The cylinder will touch the wall when the center of the cylinder is at Y_c/W=0.125 in the β=0.25 channel.

It is necessary to explain why particles in Couette flow are pushed all the way to the wall in situations where particles in Poiseuille flow do not touch. It is true, as seen in figure 17, that particles in Poiseuille flow will drift all the way to the wall without the intervention of inertia (*Re*=0). For *Re*>0, the compressive normal stresses pushing the particle from the open channel side are balanced by inertia forces in the gap. Obviously inertia forces and normal stresses act all around the particle and not only on one or the other side. We can understand the difference between Couette and Poiseuille flow by noting that the shear rates at the particle are due to the main flow and a perturbation and the main flow shear is uniform across the particle in Couette flow and greater in the gap and weaker in the open part of the channel in Poiseuille flow (compare the streamlines in figure 16 to figure 7). So there is a greater action of compressive normal stresses on the gap side of the particle in Poiseuille flow and this helps inertia to keep the particle off the wall.

4.2. Non-neutrally buoyant particles

Particles with densities larger or smaller than the fluid are released midway between the wall and the centerline of the channel with β=0.25. The trajectories are shown in figure 21 for *Re*=5, *De*=0.2.

If the density difference is large, the particle moves to the center of the channel no matter whether the particle is lighter or heavier than the fluid but lighter particles with ρ_s/ρ_f close to one (ρ_s/ρ_f=0.998 in figure 21) move to the wall. In many cases there is a transient in which the particle first drifts away from its equilibrium position in narrow channels (figure 21) and wider ones (figure 22). The effects of density differences are greater in wider channels. Particles only slightly lighter than the fluid (ρ_s/ρ_f=0.998) move closer to the wall than neutrally buoyant particles. The equilibrium positions of heavier particles are closer to the centerline.

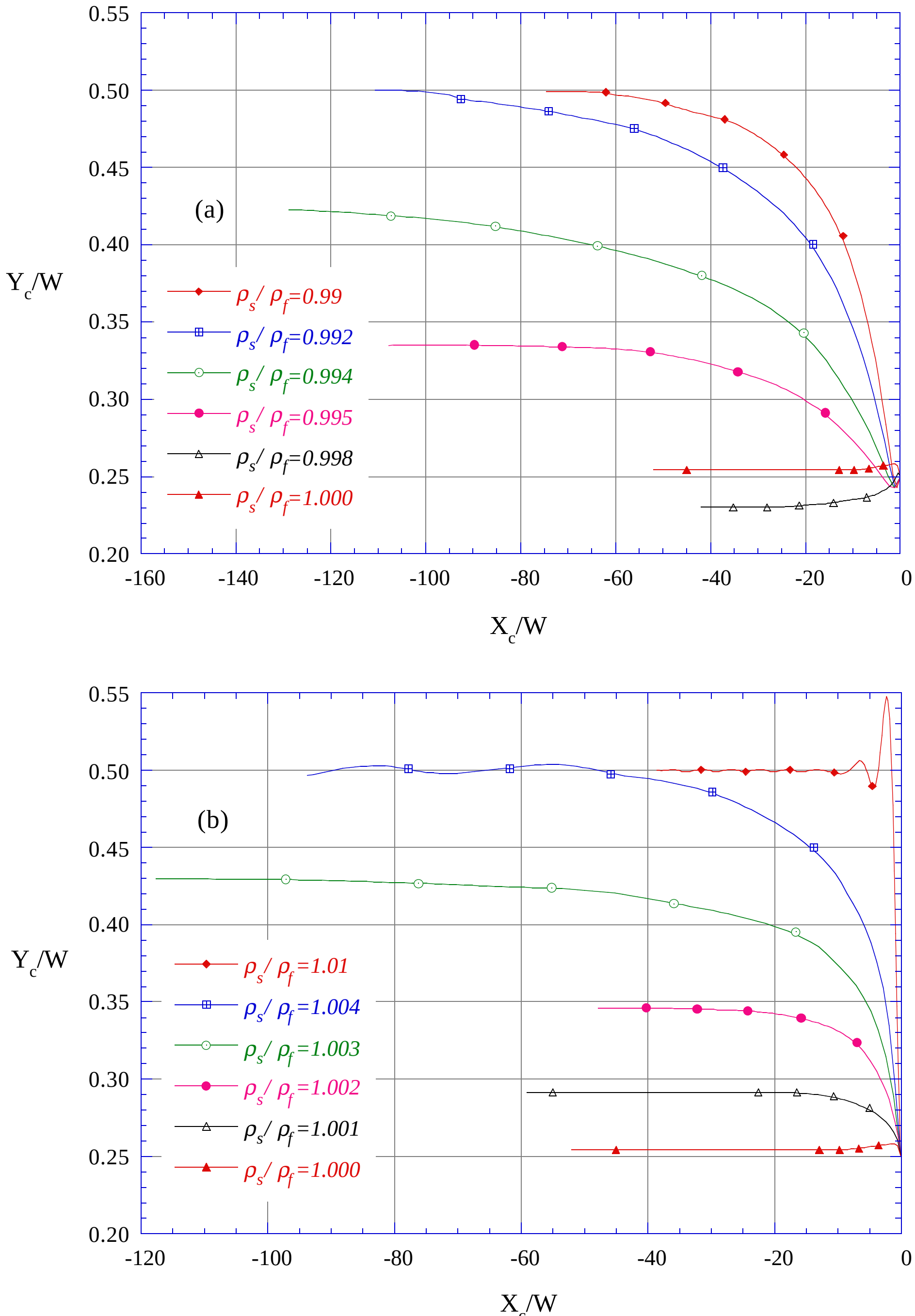

Figure 21. Trajectories of non-neutrally buoyant particles in Poiseuille flow of an Oldroyd-B fluid (β=0.25, *Re*=5, *De*=0.2) when (a) the density of the particle ρ_s is smaller than the density of the fluid ρ_f and (b) the density of the particle ρ_s is larger than the density of the fluid ρ_f.

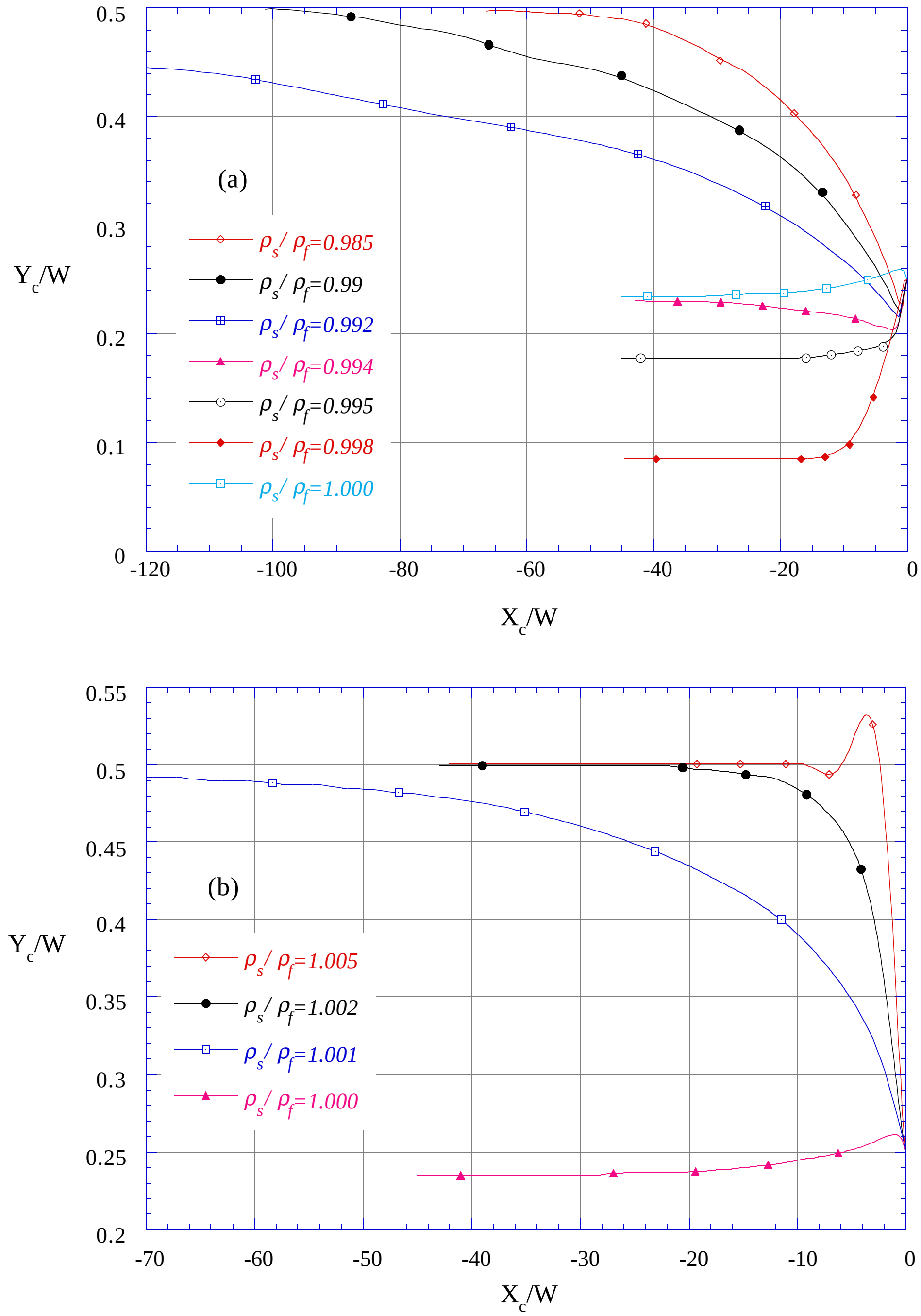

Figure 22. Trajectories of non-neutrally buoyant particles in Poiseuille flow of an Oldroyd-B fluid (β=0.10, *Re*=5, *De*=0.2) when (a) the density of the particle ρ_s is smaller than the density of the fluid ρ_f and (b) the density of the particle ρ_s is larger than the density of the fluid ρ_f.

Figure 23 shows how the equilibrium positions of particles vary with density. Heavy particles drift closer to the center in a wider channel while light particles drift further from the center in a wider channel. It is interesting to note the difference between the migration of the particle in an Oldroyd-B fluid and that in a Newtonian fluid. A particle with slight density difference (between ρ_s/ρ_f=0.993 and ρ_s/ρ_f=1.001) moves a bit closer to the wall in a viscoelastic fluid but a particle with larger density difference stabilizes closer to the center of the channel. This trend is opposite to that in a Couette flow (Figure 10).

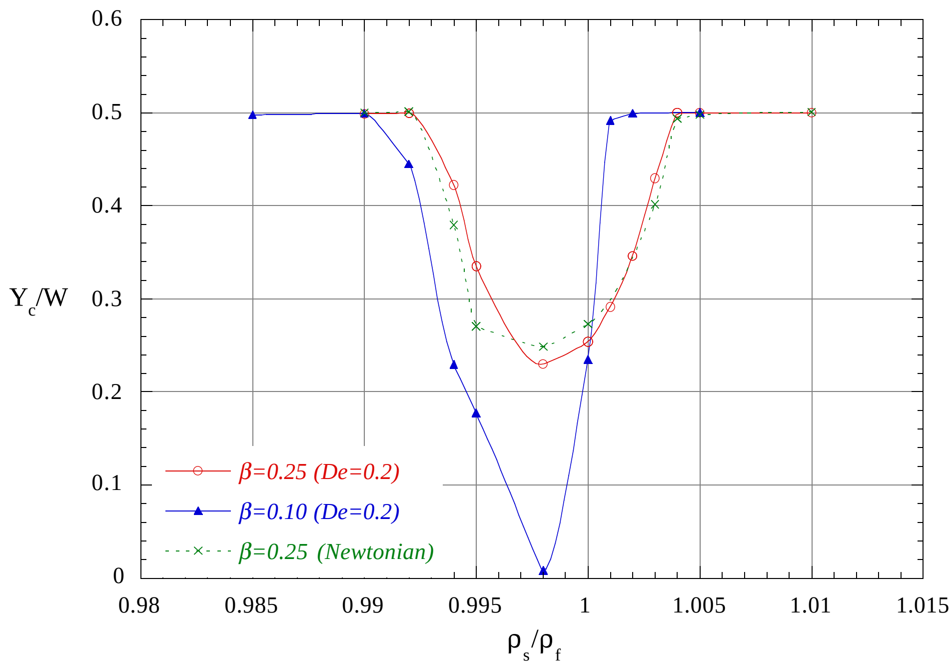

Figure 23. The stable position of a non-neutrally buoyant particle in Poiseuille flow of an Oldroyd-B fluid. *Re*=5. A Newtonian curve for β=0.25 is also shown for comparison.

Karnis & Mason (1966) matched the density of the fluid and particles in their experiments on Couette flow but they did not report procedures for density matching in their experiments on Poiseuille flow even though the experiments were for different concentrations of polymer using the same particles. They observed that particles move toward the center of the channel in Poiseuille flow but did not say that these particles were neutrally buoyant. It seems to us that the particles used in their experiments on Poiseuille flow could not all be neutrally buoyant, though the mismatch of density might be small. Our results in figure 23 shows that the particle will approach the center of the channel if the density of the particle ρ_s is 0.5% larger or 1% smaller than the density of the carrying fluid ρ_f.

5. The effects of shear thinning in sedimentation

In this section we present some results on the effects of shear thinning in the sedimentation of the particles in an Oldroyd-B fluid which was not given in the previous study of Feng, Huang & Joseph (1996). All the dimensionless parameters are defined in the same way as in Couette and Poiseuille flows except the characteristic velocity is replaced by the particle velocity U_p.

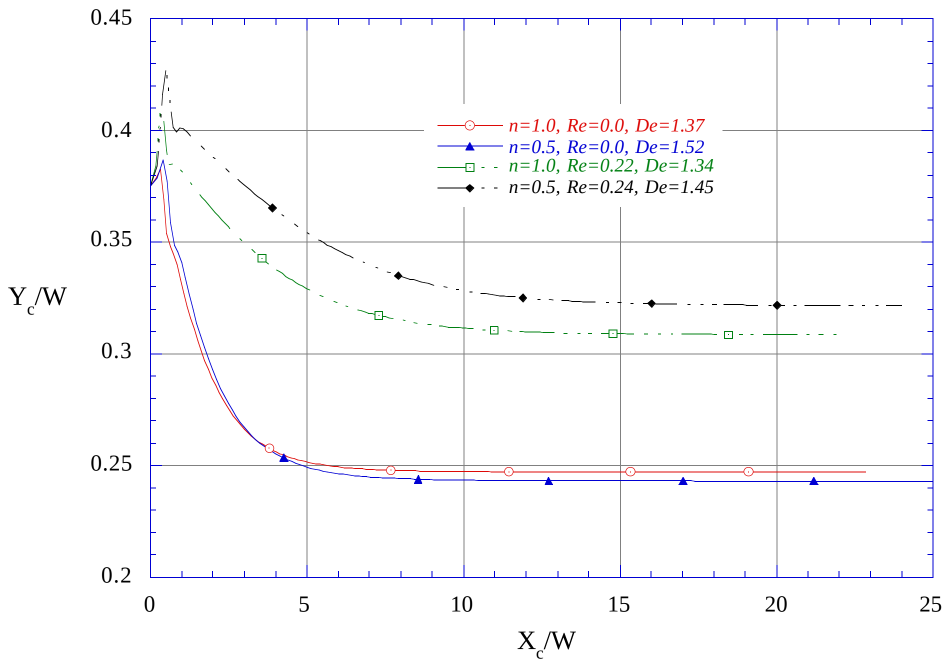

Figure 24. The effects of shear thinning on the sedimentation of particle in an Oldroyd-B fluid. β=0.25.

Shear thinning leads to larger sedimentation velocity of the particle. This means larger inertial effect on one hand, which drives the particle away from the wall, and larger effect of the compressive normal stresses on the other hand, which pushes it toward the wall. These features are clearly seen in figure 24 where the particle is released in a β=0.25 channel. The dash curves show that the effects of inertia are greater than the effects of normal stresses due to the increasing velocity induced by the shear thinning so the particle moves to an equilibrium position closer to the center. To emphasize the effects of the normal stresses, we remove the inertial term so as Re=0. The solid curves show that the compressive normal stresses push the particle closer to the wall as a result of shear thinning.

6. Conclusions

In this paper we studied the unsteady motion of both neutrally buoyant and non-neutrally buoyant particles in a Couette flow and a Poiseuille flow. Using direct numerical simulation, we can follow the particle's motion and verify our concepts of the mechanisms which move the solid particles in the viscoelastic fluids.

For a neutrally buoyant particle in a viscoelastic fluid, the equilibrium position varies as a result of inertia, elasticity, the blockage ratio and shear-thinning.

(a) Effects of inertia. Lower Reynolds number leads to stronger attraction to the wall on a particle in a viscoelastic fluid so the particle migrates closer to the wall in both a Couette flow and a Poiseuille flow. This is similar to the attraction toward the wall on a particle sedimenting in a viscoelastic fluid but is different from the migration in a Newtonian fluid. In a Newtonian fluid, a particle will move to the centerline of the channel in a Couette flow regardless of Reynolds number and move closer to the centerline in a Poiseuille flow as Reynolds number is decreased.

(b) Effects of the blockage ratio. A particle moves closer to the wall as the width of the channel becomes smaller, as in the sedimentation of a particle. The streamlines in a narrower channel crowd in such a way as to intensify the shear rates in the more open portions of the channel away from the wall so as to intensify the compressive normal stresses.

(c) Effects of shear-thinning. Shear-thinning strengthens the attraction to the wall on the particle in both Couette and Poiseuille flows. In a Couette flow, a particle migrates and finally touches the nearby wall in the absence of inertia. This is because the compressive normal stresses at places of high shear-rate on the body increase as a result of shear thinning. In sedimentation, the settling velocity of the particle increases because of shear thinning, increasing both the effect of the inertia which drives the particle away from the wall and the effect of the compressive normal stresses which pushes it toward the wall.

(d) Effects of elasticity. Elasticity gives rise to the compressive normal stresses on the particle and attracts it to the wall in Couette flow and in sedimentation. If the blockage ratio is large enough (β=0.25), the effect of the curvature of the inflow velocity profile is overwhelmed by the effect of the blockage ratio, and the particle in Poiseuille flow moves toward the side wall until it touches the wall if the fluid inertia is put to zero. The compressive normal stresses on the surface of the particle where the streamlines are crowded cause the particle to move closer to a side wall. It is different from sedimentation where a particle goes closer to the wall with larger elasticity but never touches the wall since the elasticity generates a repulsion from the wall in the region next to the wall. If the blockage ratio is very small (β=0.025), the particle in Poiseuille flow moves to the center of the channel where the shear rate of the undisturbed flow is zero. In this case, the migration of the particle is controlled by the normal stresses induced by the curvature of the inflow velocity profile. Larger Deborah number *De* will drive the particle toward the center of the channel faster.

In general, a particle's motion is controlled by the competition between the effect of inertia and the effect of the compressive normal stresses. The effect of the normal stresses comes from the local shear rate on the particle surface induced by the motion of the particle when the effect of the blockage ratio is strong and by the curvature of the inflow velocity profile when the effect of the blockage ratio is weak.

All non-neutrally buoyant particles migrate toward the centerline of the channel when they are 1% lighter or 0.5% heavier than the carrying fluid. This is the same as in a Newtonian fluid.

(e) A slightly buoyant particle in a Couette flow migrates closer to the centerline of the channel in a viscoelastic fluid than it does in a Newtonian fluid. A more buoyant particle in a Couette flow moves closer to the side walls than it does in a Newtonian fluid.

(f) In a Poiseuille flow, a slightly buoyant particle moves closer to the side wall and a very buoyant particle moves closer to the centerline of the channel in a viscoelastic fluid than it does in a Newtonian fluid.

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