Correlation between R_e and R_G in Shear Thinning Fluid

by Jimmy Jing Wang

- Definition of shear Reynolds number R_e :

Use $V = \dot{\gamma}(y) \cdot d$, where $\dot{\gamma}(y)$ is the local shear rate, d is the diameter of particle.
From momentum equation,

$$\dot{\gamma}(y) = -\overline{p}\frac{w}{2\eta(y)}$$

where $\overline{p}$ is the pressure gradient, w is the width of the channel, $\eta(y)$ is local viscosity.
Shear Reynolds number:

$$R_e = \frac{\rho_f V d}{\eta(y)} = -\frac{\rho_f w d^2}{2\eta^2(y)}\overline{p}$$

using $\rho_f = 1, w = 12, d = 1$,

$$R_e = -6\frac{\overline{p}}{\eta^2(y)}$$

- Computation of R_G :

$$R_G = \frac{\rho_f(\rho_s - \rho_f)gd^3}{\eta^2(y)}$$

at equilibrium, lift force L satisfies:

$$L = (\rho_s - \rho_f)g\frac{\pi d^2}{4}$$

Therefore,

$$R_G = \frac{4\rho_f dL}{\pi\eta^2(y)} = \frac{4L}{\pi\eta^2(y)}$$

In Neelesh's paper, correlation was obtained between R_G and the critical R_e for lift-off. In our computation for shear thinning fluid, to simulate the case where equilibrium height equals 0.501d is very difficult. The computations often diverge. So we draw correlation between R_G and the R_e for different equilibrium height. For each equilibrium height, the plot of R_G vs. R_e is nearly straight line in log-log coordinates.

Table 1: R_G vs. R_e is for each equilibrium height for ***n=0.9***

he=0.75			he=1			he=2		
dpdx	Re	Rg	dpdx	Re	Rg	dpdx	Re	Rg
1.667	15.28255	28.98325	1.667	15.14131	14.84394	1.667	14.51274	9.579031
2.5	24.7573	53.58735	2.5	24.52856	22.81926	2.5	23.50846	24.38452
3.333	34.85989	81.97618	3.333	34.53864	29.82089	3.333	33.10481	45.5164
5	56.46626	143.0233	5	55.94861	48.79665	5	53.63664	105.0336
6.667	79.48371	204.4807	6.667	78.75819	85.92145	6.667	75.51687	189.3

he=3			he=4		
dpdx	Re	Rg	dpdx	Re	Rg
1.667	13.74554	5.053382	1.667	12.75368	0.79228
2.5	22.25613	12.7386	2.5	20.61341	3.50469
3.333	31.34088	23.96901	3.333	29.01513	7.971296

5	50.78768	57.68107	5	47.01692	22.24938
6.667	71.51979	106.7988	6.667	66.22145	44.00009

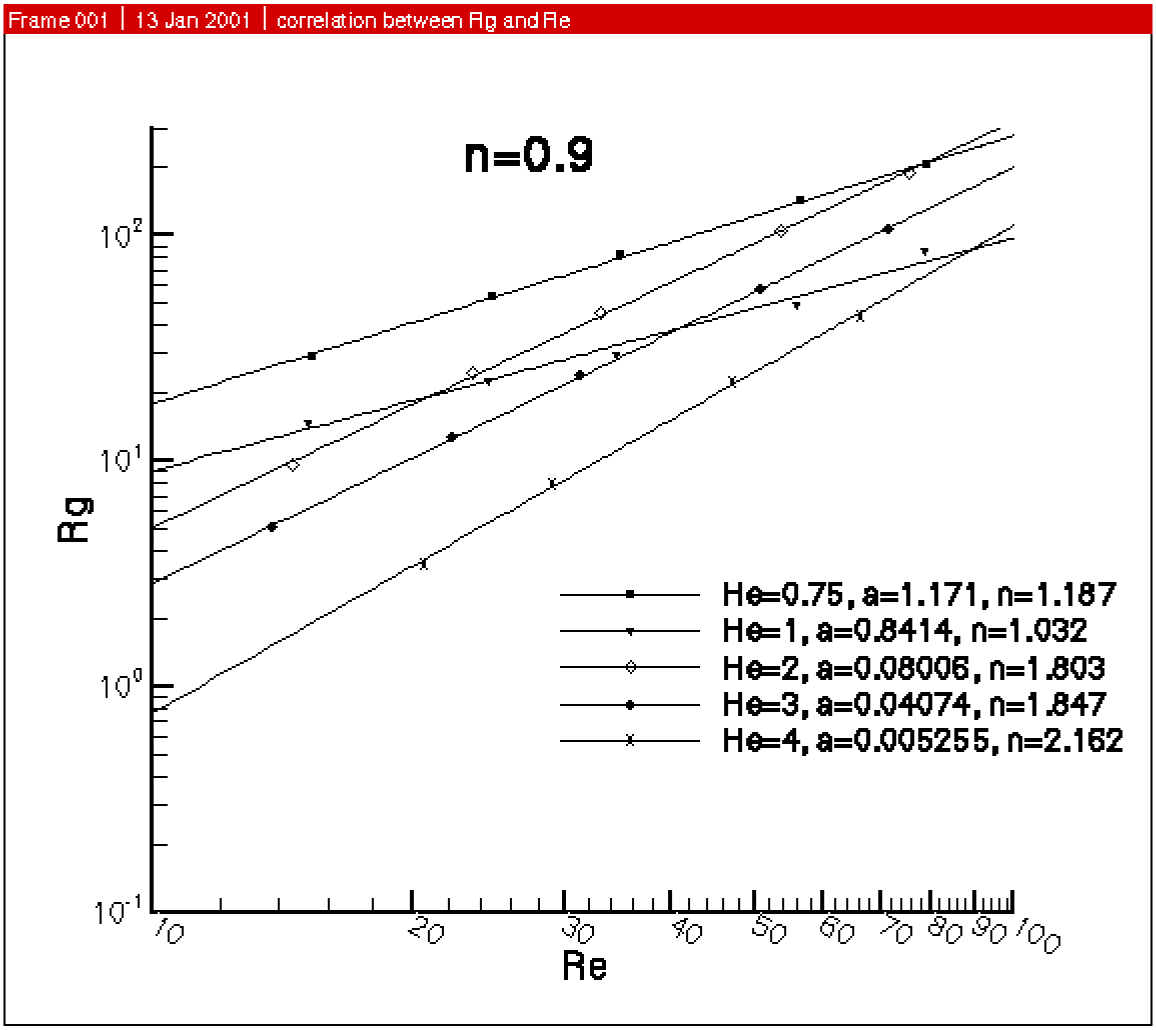

Figure 1: R_G vs. R_e is for each equilibrium height for ***n=0.9***

he=0.75			he=1			he=2		
dpdx	Re	Rg	dpdx	Re	Rg	dpdx	Re	Rg
1.667	25.06095	51.08183	1.667	24.57348	21.12577	1.667	22.45524	21.93816
2.5	44.18784	101.4774	2.5	43.34097	35.93177	2.5	39.65042	60.63187
3.333	65.97201	157.8694	3.333	64.7241	66.48622	3.333	59.27749	122.0733
5	115.7834	281.8425	5	113.6382	203.7458	5	104.2577	326.4864
6.667	172.21	458.7609	6.667	169.0701	411.0581	6.667	155.3217	649.6394
he=3			he=4					
dpdx	Re	Rg	dpdx	Re	Rg			
1.667	19.98467	9.751222	1.667	16.98205	1.556488			
2.5	35.3163	28.81428	2.5	29.96858	7.397405			
3.333	52.85983	60.6631	3.333	44.89156	18.30945			
5	93.16256	169.6834	5	79.30084	58.51825			
6.667	139.0199	347.2899	6.667	118.5798	127.6665			

Table 2: R_G vs. R_e is for each equilibrium height for ***n=0.8***

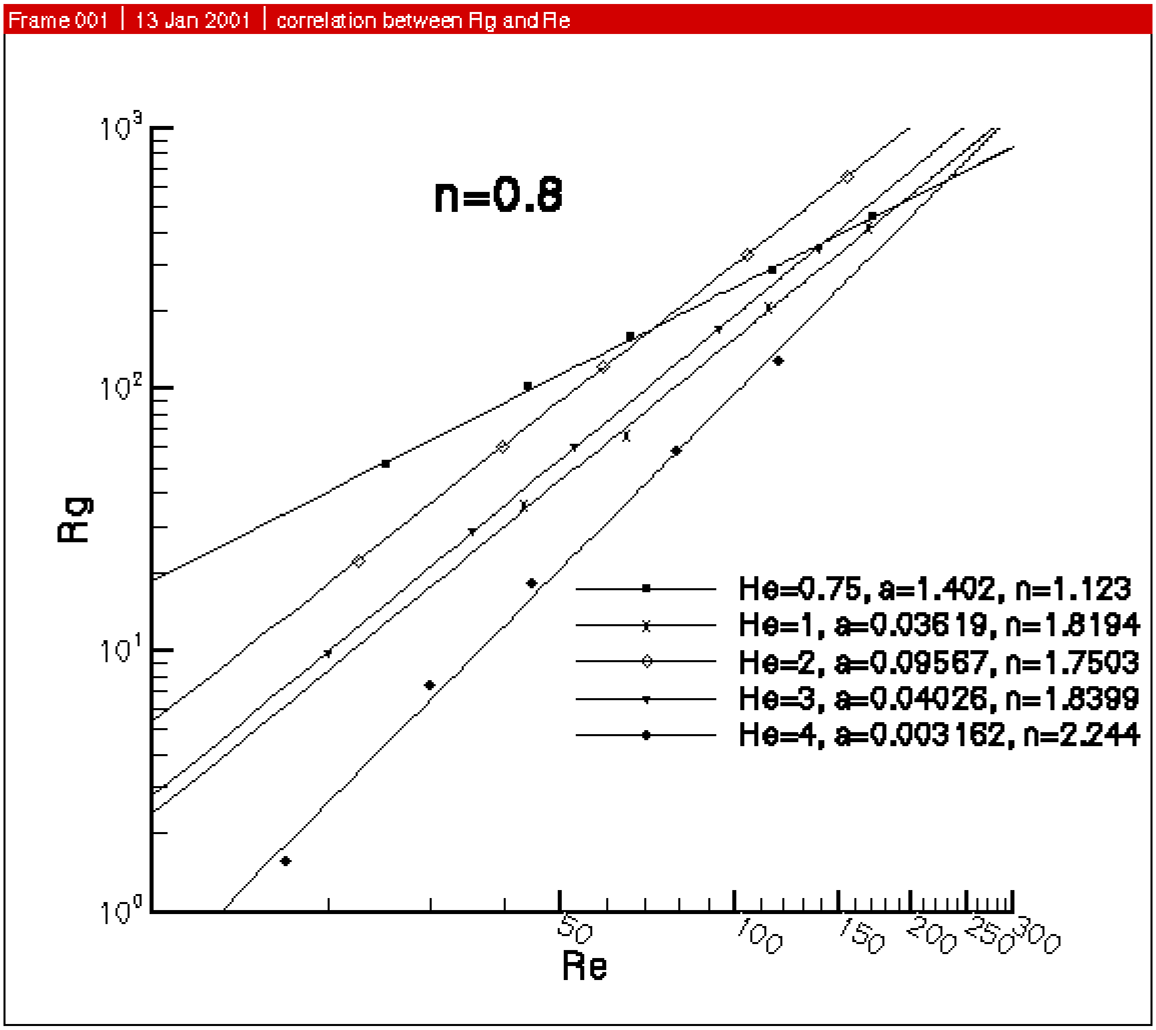

Figure 2: R_G vs. R_e is for each equilibrium height for ***n=0.8***

From figure 1 and 2, we can see that:

1. The correlation between R_G and R_e for different equilibrium height in shear thinning fluid can be represented by $R_G = aR_e^n$. For different shear thinning fluids (parameter n=0.8 or 0.9), *a* and *n* are constant at each equilibrium height. These parameters are listed in figure 1 and 2.
2. Larger R_e is required to lift a heavier particle to a certain equilibrium height.
3. Corresponding to the lift force profile where there's a minimum lift force at about He=1, in figure 1 and 2 we can see the same thing. At He=0.75, the lift force has a high value; at He=1, the lift force decreases to a low value; at He=2, the lift force return to a rather high value; then the lift force decreases monotonically with the He increasing.
4. The plots for different equilibrium heights cross each other. The intersection points indicate that for a particle with certain density, at a given R_e, it may have multiple equilibrium positions.

5. For the two shear thinning fluids (n=0.8 and n=0.9), the slopes of the R_G vs. R_e curves are similar.

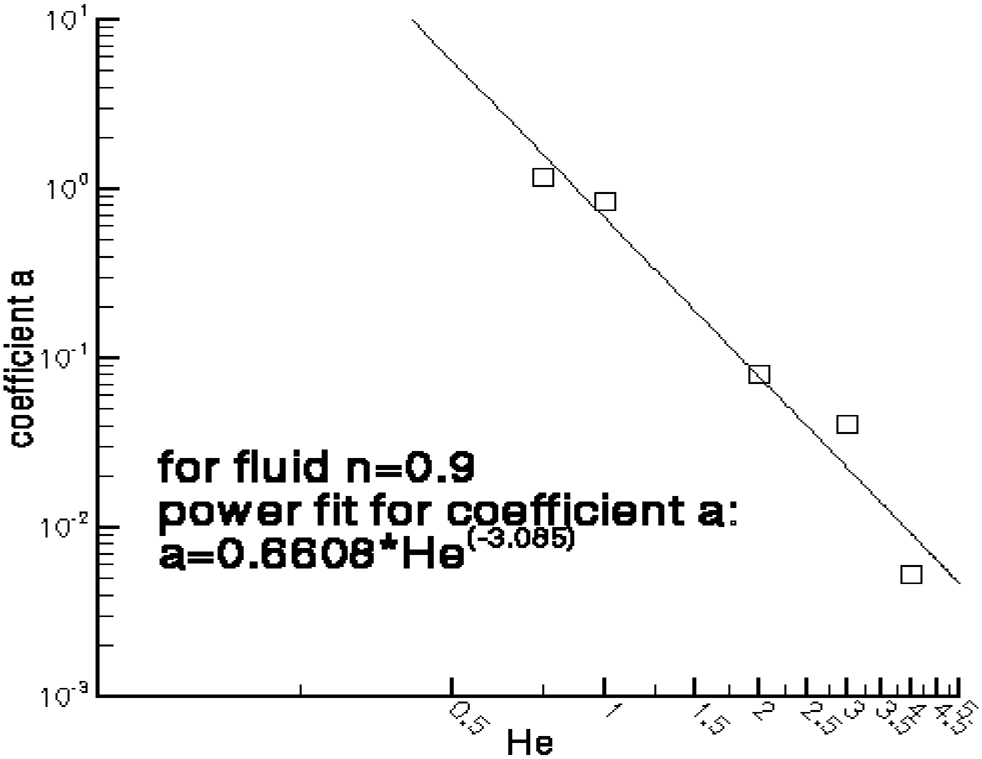

Figure 3: power fit for coefficient ***a*** for fluid ***n=0.9***
Extrapolate to **He=0.501**, **a=5.573**

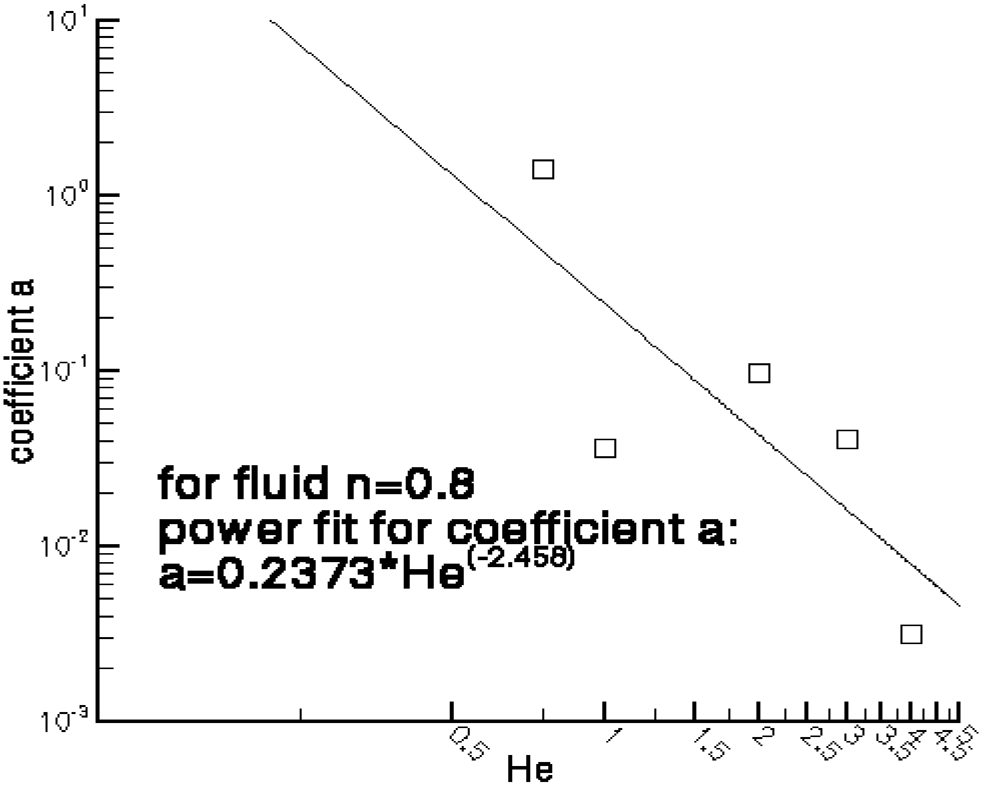

Figure 4: power fit for coefficient ***a*** for fluid ***n=0.8***
Extrapolate to **He=0.501**, **a=1.297**

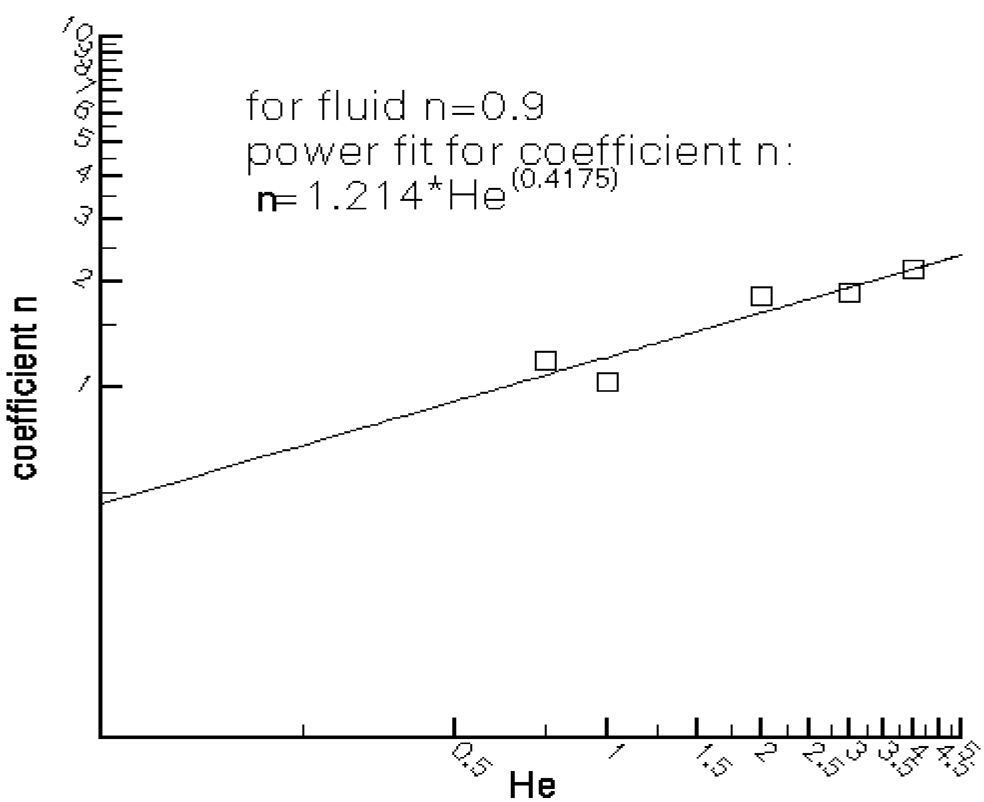

Figure 5: power fit for coefficient ***n*** for fluid ***n=0.9***

Extrapolate to **He=0.501**, **n=0.9097**

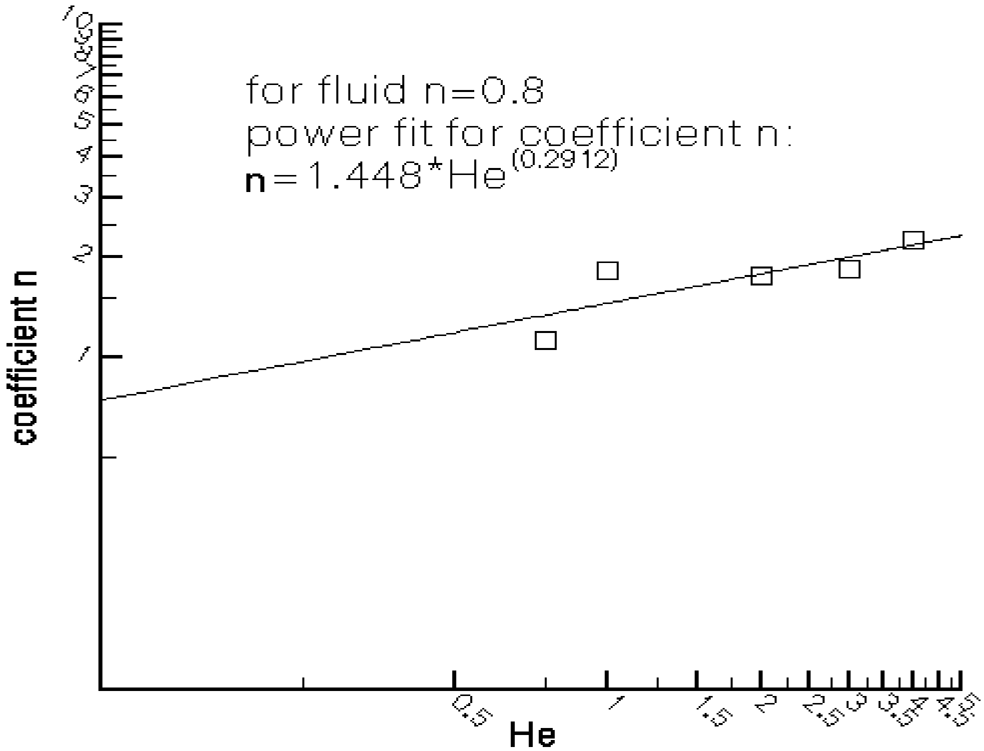

Figure 6: power fit for coefficient ***n*** for fluid ***n=0.8***
Extrapolate to **He=0.501**, **n=1.184**