Heated Reservoir - 2D Model

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The purpose of this model is to study blocking. We look for the size of the heated zone outside the fracture. The fracture is heated by microwaves on nanoparticles in the proppant. The heat source

$$\tilde{Q}(x,y) = \begin{cases} \hat{Q} & \text{constant in the fracture} \\ 0 & \text{in the reservoir} \end{cases} \tag{1}$$

is spatially uniform in each region. *A summary of the equations to be solved is given by* (32) *through* (36).

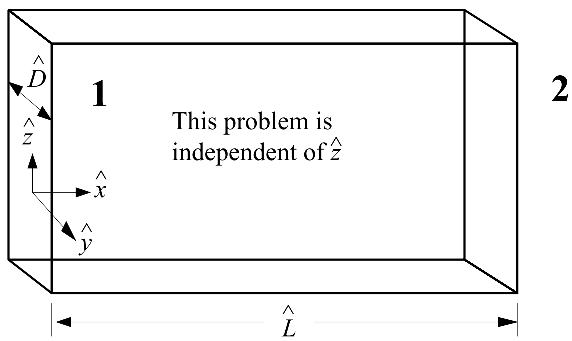

Figure 1: *Geometry of the fracture*

The fracture is infinitely long; conditions do not vary with $\hat{z}$.

$$0 \le \hat{x} \le \hat{L}, \quad |y| \le \frac{\hat{D}}{2} \text{ in the fracture} \tag{2}$$

The region outside the fracture is the reservoir **2**.

Nonmenclature

$$\left.\begin{array}{rl} \hat{P}(\hat{x},\hat{y}) & \text{pressure} \\ \hat{P}_R & \text{reservoir pressure} \\ \hat{p} = \hat{P}(\hat{x},\hat{y}) - \hat{P}_R & \text{pressure difference} \end{array}\right\} \tag{3}$$

$$\left.\begin{array}{rl} \hat{T}(\hat{x},\hat{y}) & \text{temperature} \\ \hat{T}_R & \text{reservoir temperature} \\ \hat{\theta} = \hat{T}(\hat{x},\hat{y}) - \hat{T}_R & \text{pressure difference} \end{array}\right\} \tag{4}$$

$$\left.\begin{array}{rl} \mu(\hat{\theta}) & \text{viscosity} \\ \mu_0 = \mu(0) & \text{viscosity of oil at } \hat{T}_R \\ \rho & \text{oil density, assumed constant} \end{array}\right\} \tag{5}$$

$$\left.\begin{array}{rl} k_1, k_2 & \text{permeability in fracture } \mathbf{1} \text{ and reservoir } \mathbf{2} \\ \alpha_1, \alpha_2 & \text{porousity} \\ \tilde{\kappa}_1, \tilde{\kappa}_2 & \text{thermal conductivity} \\ c_1, c_2 & \text{specific heats} \end{array}\right\} \tag{6}$$

$$\hat{\kappa}_1 = \frac{\tilde{\kappa}_1}{\rho c_1}, \qquad \hat{\kappa} = \frac{\tilde{\kappa}_2}{\rho c_2} \tag{7}$$

$$\hat{\mathbf{u}} \quad = (\hat{u}, \hat{v}) \quad \text{velocity components} \tag{8}$$

$$\left.\begin{array}{rl} \mu(\hat{\theta}) & = \mu_0 \mathrm{e}^{-\lambda\hat{\theta}} \text{viscosity flow} \\ \lambda & = \text{rate constant} \end{array}\right\} \tag{9}$$

Problem set up. We will need boundary conditions for $\hat{p}$ and $\hat{\theta}$.
Assume that reservoir conditions prevail for

$$\sqrt{\hat{x}^2 + \hat{y}^2} > \hat{L} \tag{10}$$

Then

$$\hat{p}(\hat{x},\hat{y}) = 0 \left\{ \begin{array}{rl} \hat{x} > & \hat{L} \\ |\hat{y}| > & \hat{L} \end{array}\right. \tag{11}$$

$$\hat{\theta}(\hat{x},\hat{y}) = 0 \left\{ \begin{array}{r} \hat{x} > L \\ |\hat{y}| > L \end{array}\right. \tag{12}$$

At $\hat{x} = 0$ we have two choices

$$\hat{p}(0, \hat{y}) = \hat{p}_0 \text{ prescribed pump pressure} \tag{13a}$$

$$\left.\begin{aligned} p(0,\hat{y}) &= \hat{p}_0 \text{ for } |\hat{y}| \leq \tfrac{\hat{D}}{2} \\ \frac{\partial \hat{p}}{\partial \hat{x}} &= 0 \text{ for } |\hat{y}| \geq \tfrac{\hat{D}}{2} \end{aligned}\right\} \tag{13b}$$

The choice (13b) says that there is no production outside the fracture.

Various choices for $\hat{\theta}(\hat{x}, \hat{y})$ at $\hat{x} = 0$ are possible. For blocking we could say that

$$\hat{\Theta}(0, \hat{y}) = 0 \tag{14}$$

Quasi Steady State: It is probable that $\partial\hat{\theta}/\partial t$ is very small. Then the parameters α_1, α_2 will not enter this problem.

Equations in the fracture

$$\left.\begin{aligned} \hat{u}_1 &= \frac{-k_1}{\mu_0 \mathrm{e}^{-\lambda\hat{\theta}}} \frac{\partial \hat{p}}{\partial \hat{x}} \\ \hat{v}_1 &= \frac{-k_1}{\mu_0 \mathrm{e}^{-\lambda\hat{\theta}}} \frac{\partial \hat{p}}{\partial \hat{y}} \end{aligned}\right\} \tag{15}$$

$$\begin{aligned} \frac{\partial}{\partial \hat{x}} \left(\mathrm{e}^{\lambda\hat{\theta}} \frac{\partial \hat{p}}{\partial \hat{x}} \right) + \frac{\partial}{\partial \hat{y}} \left(\mathrm{e}^{\lambda\hat{\theta}} \frac{\partial \hat{p}}{\partial \hat{y}} \right) &= 0 \\ \hat{u}_1 \frac{\partial \hat{\theta}}{\partial \hat{x}} + \hat{v}_1 \frac{\partial \hat{\theta}}{\partial \hat{y}} &= \hat{\kappa}_1 \left(\frac{\partial^2 \hat{\theta}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{\theta}}{\partial \hat{y}^2} \right) \frac{\hat{Q}}{\rho c_1} \end{aligned} \tag{16}$$

Equations in the reservoir

$$\begin{aligned} \hat{u}_2 &= \frac{-\kappa_2}{\mu_0 \mathrm{e}^{-\lambda\hat{\theta}}} \frac{\partial p}{\partial \hat{x}} \\ \hat{v}_2 &= \frac{-\kappa_2}{\mu_0 \mathrm{e}^{-\lambda\theta}} \frac{\partial p}{\partial \hat{y}} \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{\partial}{\partial \hat{x}} \left(\mathrm{e}^{\lambda\hat{\theta}} \frac{\partial \hat{p}}{\partial \hat{x}} \right) + \frac{\partial}{\partial \hat{y}} \left(\mathrm{e}^{\lambda\hat{\theta}} \frac{\partial \hat{p}}{\partial \hat{y}} \right) &= 0 \\ \hat{u}_2 \frac{\partial \hat{\theta}}{\partial \hat{x}} + \hat{v}_2 \frac{\partial \hat{\theta}}{\partial \hat{y}} &= \hat{\kappa}_2 \left(\frac{\partial^2 \hat{\theta}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{\theta}}{\partial \hat{y}^2} \right) \end{aligned} \tag{18}$$

Dimensionless variables

$$\left.\begin{array}{rl} \Delta P & = P_R - P_0 = -\hat{p}_0 \text{ reservoir minus pump pressure} \\ p & = \hat{p}/\Delta P \end{array}\right\} \tag{19}$$

$$(x, y) = (\hat{x}, \hat{y})/\hat{D} \tag{20}$$

$$\theta = \hat{\theta}\lambda \tag{21}$$

$$(u, v) = (\hat{u}, \hat{v}) \Big/ \frac{\Delta P k_2}{\mu_0 \hat{D}} \tag{22}$$

$$\left.\begin{array}{rll} a & = \tilde{\kappa}_1/\tilde{\kappa}_2 & \text{conductivity ratio} \\ d & = c_2/c_1 & \text{specific heat ratio} \\ ad & = \tilde{\kappa}_1/\tilde{\kappa}_2 & \\ b & = k_1/k_2 & \text{permeability ratio} \end{array}\right\} \tag{23}$$

Coupling conditions. At the fracture surface, the normal component of velocity, the pressure, the temperature and heat flux are continuous, but the tangential component of velocity is discontinuous.

The condition that

$$v_1 = v_2 \text{ at the fracture}$$

can be written as

$$\left.\begin{array}{rcl} b\dfrac{\partial \hat{p}}{\partial \hat{y}|_1} & = & \dfrac{\partial \hat{p}}{\partial \hat{y}|_2}\,, \\ \hat{p}_1 & = & \hat{p}_2 \\ \hat{\theta}_1 & = & \hat{\theta}_2 \\ a\dfrac{\partial \hat{\theta}}{\partial \hat{y}_1} & = & \dfrac{\partial \hat{\theta}}{\partial \hat{y}|_2} \end{array}\right\} \tag{24}$$

Dimensionless form of coupling condition

$$\left.\begin{array}{rcl} p_1 & = & p_2, \\ \theta_1 & = & \theta_2, \\ a\dfrac{\partial \theta_1}{\partial y_1} & = & \dfrac{\partial_2 \theta}{\partial y} \\ b\dfrac{\partial p_1}{\partial y} & = & \dfrac{\partial p_2}{\partial y} \end{array}\right\} \tag{25}$$

Equation of motion in dimensionless form. In Region **2**, the reservoir

$$(u, \upsilon) = -\mathrm{e}^{\theta}\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}\right) \tag{26}$$

$$\left.\begin{aligned} \frac{\partial}{\partial x}\left(\mathrm{e}^{\theta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mathrm{e}^{\theta}\frac{\partial p}{\partial y}\right) &= 0 \\ \frac{\partial p}{\partial x}\frac{\partial \theta}{\partial x} + \frac{\partial p}{\partial y}\frac{\partial \theta}{\partial y} &= \beta\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) \end{aligned}\right\} \tag{27}$$

$$\beta = \frac{\hat{\kappa}_2\mu_0}{\Delta P k_2 C_2} \tag{28}$$

Region **1**, heated fracture

$$(u, \upsilon) = -b\mathrm{e}^{\theta}\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}\right) \tag{29}$$

$$\left.\begin{aligned} \frac{\partial}{\partial x}\left(\mathrm{e}^{\theta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mathrm{e}^{\theta}\frac{\partial p}{\partial y}\right) &= 0 \\ b\left(\frac{\partial p}{\partial x}\frac{\partial \theta}{\partial x} + \frac{\partial p}{\partial y}\frac{\partial \theta}{\partial y}\right) &= \beta a d\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + Q \end{aligned}\right\} \tag{30}$$

$$Q = \hat{Q}\lambda\mu_0 \Big/ \rho\mathrm{e}_1\Delta P k_2 \tag{31}$$

This problem is determined by 5 dimensionless numbers $a,\ b,\ d,\ \beta$ and Q.
Summary. Our problem may be framed in terms of p and θ alone; later we get u and υ.
Region **1 fracture** $0 \le x \le \hat{L}/D, |y| \le 1/2$.

$$\left.\begin{aligned} \frac{\partial}{\partial x}\left(\mathrm{e}^{\theta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mathrm{e}^{\theta}\frac{\partial p}{\partial y}\right) = 0 \\ b\left(\frac{\partial p}{\partial x}\frac{\partial \theta}{\partial x} + \frac{\partial p}{\partial y}\frac{\partial \theta}{\partial y}\right) = \beta a d\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + Q \end{aligned}\right\} \tag{32}$$

$$\left.\begin{aligned} p_0 &= p(0,y) \text{ is the prescribed pump pressure difference} \\ p(\left(\frac{\hat{L}}{\hat{D}}, y\right) &= 0 \\ \theta(0,y) &= \Theta\left(\frac{\hat{L}}{\hat{D}}, y\right) = 0 \end{aligned}\right\} \tag{33}$$

Region **2 reservoir** $\hat{x} > \hat{L}/\hat{D}$ all $\hat{y}$ (say $< \hat{L}/\hat{D}, x < \hat{L}/D, y > 1/2$.

$$\left.\begin{aligned} \frac{\partial}{\partial x}\left(\mathrm{e}^{\theta}\frac{\partial b}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mathrm{e}^{\theta}\frac{\partial b}{\partial y}\right) &= 0 \\ \left(\frac{\partial p}{\partial x}\frac{\partial \theta}{\partial x} + \frac{\partial p}{\partial y}\frac{\partial \theta}{\partial y}\right) &= \beta\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) \end{aligned}\right\} \tag{34}$$

$$\left.\begin{aligned} p(x,y) &= 0 \quad \forall x > \hat{L}/D \\ p(x,y) &= 0 \quad \forall y > \hat{L}/D \\ \frac{\partial p}{\partial x}(0,y) &\leq 0 \quad \text{no flow} \\ \theta(0,y) &= 0 \\ \theta(x,y) &= 0 \quad x \geq \hat{L}/D,\ y \geq \hat{L}/D \end{aligned}\right\} \tag{35}$$

Coupling condition on the fracture

$$p_1 = p_2, \quad \theta_1 = \theta_2, \quad a\frac{\partial \theta_1}{\partial y} = \frac{\partial \theta_2}{\partial y}, \quad b\frac{\partial p_1}{\partial y} = \frac{\partial p_2}{\partial y} \tag{36}$$