Experiments at the University of Minnesota (draft 2)

September 17, 2001

Studies of migration and lift and of the orientation of particles in shear flows

Experiments to determine positions of spherical and long particles and the orientation of long particles can be carried out in a stationary frame by fluidizing particle in a balance of the buoyant weight of the heavier than liquid particle with

(1) drag, which will be realized in a vertical tube apparatus

(2) hydrodynamic lift (perpendicular to flow, as in a lifting aircraft) which will be realized in the rotating cylinder apparatus described in the sequel.

(1) Migration and orientation of particles in a vertical tube under a balance of buoyant weight and drag.

We propose to study problems of lift, migration, positioning and orientation of long particles in a vertical tube in which fluid is driven up by a pressure gradient reaching an equilibrium height under a balance of weight and drag. If the diameter/length ratio and fluids are suitably chosen, a developed Poiseuille flow can be achieved and the position and orientation of particles and their orbital motions in a shear flow can be conveniently studied in a stationary window.

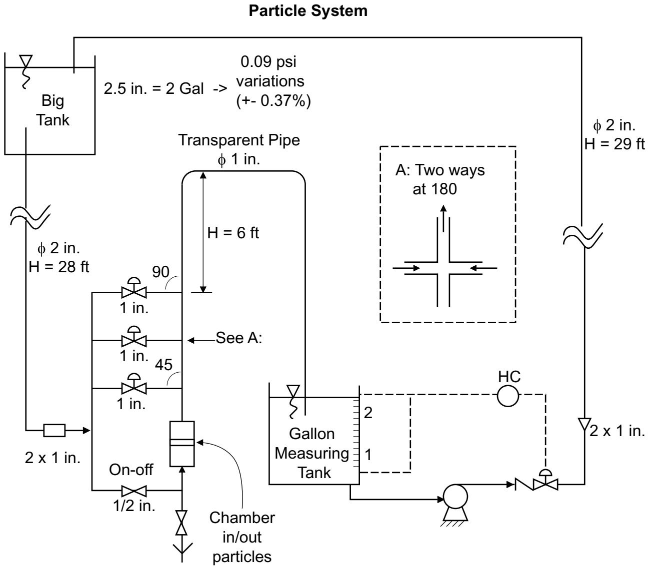


Figure 1.

We have built a flow loop for these studies (Figure 1). This loop is very simple but very well adapted to studies of the effects of weight, size and shape of particles and the associated changes of Reynolds number required to attain a balance of buoyant weight and drag. We propose also to study the behavior of particles suspended in fluids with different rheological behavior.

There are a small number of theoretical studies that have been published of low Reynolds number sedimentation of single particles perturbing Stokes flow in tubes and channels. The problem of fluidization particles of different shape, heavy spheres and many spheres at moderate and high Reynolds numbers apparently has not been studied. Extended discussion of the subtle problems arising in low Reynolds number analysis of lift have been presented by Renner 1966, Cox and Mason 1971, Leal 1980, Fauillebois 1989, Cherakut and McLaughlin 1994 and Asmolov 1999.

The problem of positioning fluidized particles in round tubes seems not to have been considered before. It is always true that the realized positions are compatible with the balance of buoyant weight, which does not change with speed, and drag. It is possible that under certain conditions the particle moves in such a way as to maintain the same drag at different speeds. The "bed" expansion of a single line of particles could be applied to the problem of placing sensors downhole in producing wells.

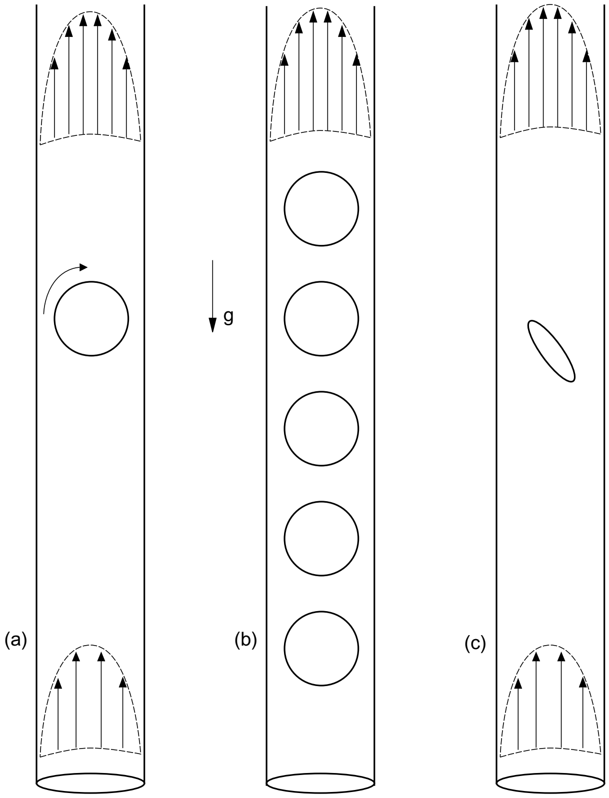


Figure 2. Fluidization by drag of particles in a vertical tube. (a) "Off center" fluidization of single spheres of different size and weight if the particle is stationary in a balance of weight and drag. It will flow out of the bed if the fluidizing velocity is too great, and fall out if it is too small. (b) "Bed" expansion of a row of particles. As in a fluidized bed, hindered settling effects should allow us to expand the row of particles without blowing them out. (c) Orientation of a long particle in a Poiseuille flow.

(2) Migration and orientation of particles in a rotating cylinder apparatus under a balance of buoyant weight and hydrodynamic lift

Review of past experimental studies

Several authors have already reported experimental data on hydrodynamic lift. Eichhorn and Small (1964) suspended a small sphere in Poiseuille flow through an inclined tube and determined the lift and drag coefficients on the particle. By suspending the sphere in the flow, the lift and drag forces are calculated from the tangential and normal components of the buoyancy force with the flow. The results are not extensive, but at the time did provide new information about the drag and lift on spheres in such flows. These experiments showed that the lift coefficient increases with Reynolds number. However, the accuracy of the measurements was limited and the variables such as rotation speed and radial position are related by the operating characteristics of the apparatus and could not be varied independently.

Bagnold (1974) measured the lift and drag forces on spheres and cylinders in the gravity flow of a liquid in an open channel (the upper, free boundary of the liquid is frictionless). The objects are placed near the lower, solid boundary of the device and are allowed to translate down the channel in the shear flow. Bagnold overcame the problem of the limited channel length by creating a “stationary flow” where the lower boundary is replaced with an endless belt that translates in the direction opposite the free stream velocity. By producing the proper flow kinematics to balance the inertial motion of the sphere down the channel, the particle can be suspended in the flow and the equilibrium height can be measured. In separate measurements, a linkage assembly is used to measure the drag and lift forces on bodies fixed in the flow field. In general, Bagnold observed that the lift force decreases rapidly with increasing distance from the solid boundary and disappears when the clearance exceeds on particle diameter. Unfortunately, the author admits, “the experiments must be regarded as exploratory only” because of the accuracy of the device and the limited scope of the experiments.

Cherukat et al. (1994) used a homogenous shear flow apparatus (HFA) to measure the shear-induced inertial lift on a rigid sphere. The HFA device creates a uniform linear shear flow between two timing belts moving in opposite directions. Spheres were injected onto the mid-plane between the two belts will translate between the belts, migrating laterally towards one of the walls. A system of cameras recorded the motion of the particle, which was then used to calculate the dimensionless lift force on the sphere. The lift force increases with the ratio of $\varepsilon = \sqrt{\mathrm{Re}}/\mathrm{Re}_s$, where Re is the Reynolds number of the flow and Re_s is the Reynolds number of the sphere based on the slip velocity. These results, within experimental error, validated the theory of McLaughlin (1991) and they showed that Saffman’s expression overpredicts the lift if $\varepsilon >> 1$ is not satisfied. However, these experiments are not able to measure the equilibrium height from the wall, and the rotation velocity of the sphere is not presented.

Experiments by Mollinger et al. (1995) examined the lift forces on particles within the viscous sublayer of highly turbulent flows ($\mathrm{Re} \sim 10^6$). Their experimental device measures the lift force on a small particle permanently affixed to cantilever beam at the surface over which air flows in a wind tunnel. Optical methods allow for measuring the lift force to an accuracy of 10^{-9} N. Their results show a substantial difference between experiments and theory for this regime of Reynolds numbers. However, because the particle is fixed onto the surface, measurements of important parameters such as equilibrium height, slip velocity, and rotation rate are unobtainable.

More recently, King and Leighton (1997) used a rotating parallel-plate device to measure translation velocity and the angular velocity of particles in contact with a wall. The velocities were determined by timing the particle as it travels through an arc length. A wide range of Reynolds numbers could be sampled because the local shear rate between the parallel plates is proportional to the radial distance. Unfortunately, secondary currents drive the particle inwards at larger Reynolds numbers. This study showed that the particle undergoes three different modes of motion as the flow Reynolds number is increased: (i) solid body rotation along the wall, (ii) particle rotation with translational slip along the wall, and (iii) translation and rotation in the free stream following lift-off. Careful attention was paid to the surface roughness of the sphere and the frictional coefficient of the wall that pays an important role in the motion when the sphere is in contact with the wall. The transition from (i) to (ii) was observed to scale with $\mathrm{Re}/\mathrm{Re}_s \sim 0.1$; while lift-off occurred when $\mathrm{Re}^2/\mathrm{Re}_s \sim 4$. These experiments were in good agreement with the rough sphere model proposed by Krishnan and Leighton (1995) for a range of reasonable surface roughness values. No values of the equilibrium height were presented.

Proposed Experiments

In proposed experiments, we will quantify lifting dynamics of a spherical particle and the lifting and orientation dynamics of long particles in contact with a solid boundary. For the spherical particle we seek to measure the magnitude of the drag (D) and lift (L) forces exerted on the body, the translational (U_s) and the rotational (Ω_s) slip velocities of the particle, and the equilibrium suspension height (h_e) away from a wall that the particle achieves. We want to measure the orientation and possible orbital motions of long particles. The behavior of such particles in shear flows like that shown in figure 3 has not been studied before. We will confine the description to the case of fluidized spheres, which are better understood. The same measurement technique will be used for long particles.

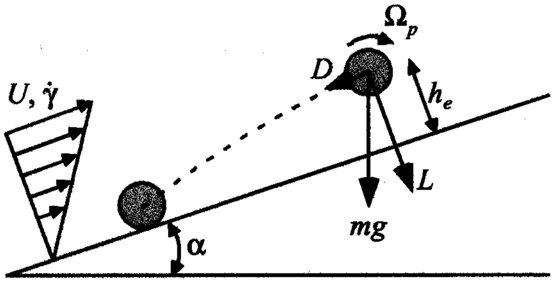


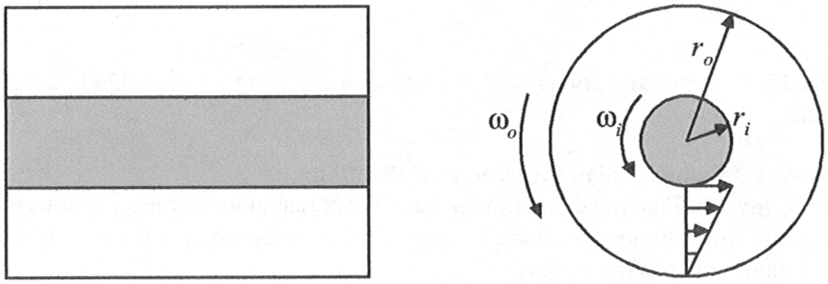
Figure 3. Schematic diagram illustrating the principle behind the proposed experiments.

The inspiration for the proposed experiments comes from the work of Eichhorn and Small (1964). In their experiments, they are able to measure all the desired variables (D, L, U_s, Ω_s) by creating a flow field in which the lift and drag forces are balanced exactly by gravitational body forces. Consider the illustration in Figure 3. A sphere starts out at rest in contact with an inclined surface. For a given sphere, radius a and sphere density ρ_p, there exists a flow state where the particle will experience a lift force great enough to lift it off the surface of the and into the flow field. The lift and drag forces exerted on the sphere will be aligned perpendicular to and parallel to the direction of flow. Adjusting the flow kinematics (U and $\dot{\gamma}$) will vary the forces exerted on the sphere, so that a state can be achieved where the sphere remains stationary within the flow, i.e., the particle velocity $U_p = 0$. In this state, the drag and lift forces will be balanced by the parallel and perpendicular components of the gravitational body force,

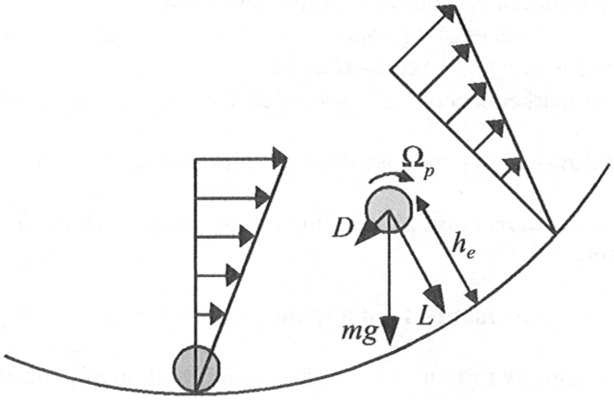
$$D = mg \sin \alpha \tag{1}$$

$$L = mg \cos \alpha \tag{2}$$

where $mg = 4\pi a^3 g(\rho_p - \rho_f)/3$. With the sphere at a stationary position, the equilibrium height (h_e), rotation velocity (Ω_p), and the detailed flow kinematics can be easily measured using optical methods. The benefit of this method is that the sphere need not be constrained in any way to obtain these measurements, and by adjusting the plane angle (α), the drag and lift forces can be varied over a wide range.



(a) Side view (b) End view



(c) Detailed zoom

Figure 4. Schematic diagram of the experimental device.

Eichhorn and Small used this method to suspend small spheres of varying density in a Poiseuille flow in an inclined tube. Our proposed experiments are based on the same principle, but will use a Couette flow device to generate a shear flow between two rotating concentric cylinders as illustrated in Figure 4. A sphere placed between the two cylinders and resting on one of the surfaces will experience lift and drag forces as the cylinders are rotated to generate a the desired shear flow. Under the proper flow kinematics, the sphere will experience a lift force great enough to allow it separate from the wall. The sphere is carried along by the flow field experiencing lift and drag forces that are in the radial and tangential directions, respectively. The sphere will reach a location at which the lift and drag forces are balanced by the radial and tangential components of the gravitational body force. At this location, the sphere will be suspended ($U_p = 0$) and the lift and drag forces can be determined by a simple force balance. A high precision video imaging system will allow us to follow the position of the sphere and its measure its distance from the wall. We will also be able to observe the rotation rate of the sphere by placing markers on the sphere. A detailed study of the velocity field can be obtained using point-wise laser-Doppler velocimetry (LDV) to determine the slip velocity ($U_s = v_r(r_e)$) and the angular slip velocity ($\Omega_s = \dot{\gamma}/2 - \Omega_p$).

The benefit of this system is that for a prescribed flow field, the sphere will naturally achieve an equilibrium position without adjustments because it will migrate to a location where the lift and drag forces balance the gravitational body force. However, the device does introduce some experimental difficulties. First, we no longer have a plane surface, but rather the surface is curved. The effects of the curvature can be reduced using small spheres. Second, for large Reynolds numbers, there will be a centripetal acceleration force directed radially inwards which can easily be incorporated into the lift force calculation. Finally, and most importantly, this arrangement of rotating concentric cylinders is known to become unstable. It is unknown what effect, if any, that these instabilities will have on the motion of the sphere and the lift forces.

The device as designed is a closed system, so the fluid can easily be changed quite easily. This allows us to examine the effects of rheology on the lift force. We will examine a Newtonian fluid, a constant viscosity elastic fluid (a dilute polymer solution), and a shear-thinning, viscoelastic polymer solution. The Newtonian fluid will be a mixture of glycerin and water that can be easily diluted to vary the viscosity and hence the Reynolds numbers. Using the polymer solutions, we can examine the effects of fluid elasticity and shear-thinning rheology on the lift forces. Although the constant viscosity elastic fluid may lack real-world practicality, it is useful for isolating the elastic effects and for comparisons made with theoretical and numerical models that use constant viscosity constitutive equations such as Maxwell and Oldroyd-B. Measurements of lift forces in viscoelastic solutions have never be made. Recent simulations by Hu and Joseph show small particles can be easier to suspend in an elastic fluid due to the normal stresses in the fluid. We will attempt to collaborate these observations with experimental evidence of that fact.

The proposed experiments are intended to provide a complete set of experimental data for the lift force exerted on a sphere in a shear flow and on the orientation and orbital dynamics of long particles in such a flow. We intend to compare these experimental results with direct numerical simulations and theoretical models.

References

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