

Lift on a Moving Sphere Near a Plane Wall in a Second Order Fluid

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Abstract: In this paper we examine the lift on a sphere moving very close to an infinite plane wall in a second-order fluid. The sphere is allowed to both translate and rotate along the plane. We focus on the limit when the sphere touches the wall. We found that due to the normal stress effect the flow gives rise to a positive elastic lift force on the sphere when gap between the sphere and the wall is small. For a moving particle in a shear flow, the ratio of the elastic lift to the buoyant weight of the particle is proportional to the particle radius, such that smaller particles will be easier to be suspended. We also found that the ratio of the inertial lift to the elastic lift is proportional to the square of the particle radius. Furthermore, the elastic lift force is singular when the minimum gap between the sphere and the wall approaches zero. Consequently, a moving particle in a viscoelastic fluid will be always suspended from a smooth surface.

Introduction.

The problem of the motion of an isolated sphere in bounded flows is of practical significance in a variety of applications, such as the cleaning of particles from surfaces, deposition of particles in filtration, fines mobilization in porous media and resuspension of particles in a packed bed. Often, in oil industry polymer solutions are used for the cleaning of drilling holes and for the transport of proppants. It was observed that some polymer solutions have a better capability to mobilize and transport particles. However, the mechanisms for the particle resuspension and the effects of fluid elasticity are still not

known. In this paper we examine the lift force on a sphere moving very close to an infinite plane wall in a second order fluid. The sphere is allowed to both translate and rotate along the plane. We focus on the effects of the normal stress and the limit when the sphere touches the wall.

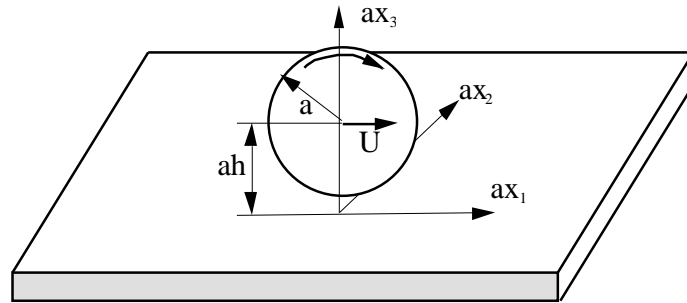


Figure 1. Sphere moving along a planar wall. The non-dimensional minimum gap between the sphere and the wall is $h - 1$.

A rigid sphere of radius a is moving in a direction parallel to a fixed infinite plane wall when the minimum clearance a between the sphere and the plane is much smaller than the radius of the sphere (in the limit 0 of no clearance), as shown in figure 1. The creeping flow solution for this problem can be obtained by superposing the solution for a sphere translating along the plane and the solution for a sphere rotating about a fixed axis parallel to the plane. The solutions of these two problems have been obtained by O'Neill & Stewartson [1967], Cooley & O'Neill [1968] respectively using matched asymptotic expansions. An 'inner' solution was constructed for the region in the neighborhood of the nearest points of the sphere and the plane wall where the velocity gradients and pressure are large; in this region the leading terms of the asymptotic expansion of the solution satisfies the equations of lubrication theory. A matching 'outer' solution was constructed which is valid for the remainder of the fluid where velocity gradients are moderate but it is possible to assume that $\tau = 0$. In summary, they found that the force acting on the sphere is expressible as $(F_x, 0, 0)$ with

$$\begin{aligned}
F_x = & -6 \mu Ua \left[\frac{8}{15} + \frac{64}{375} \log \frac{2}{\epsilon} + 0.58461 + O(\epsilon) \right] \\
& + 6 \mu a^2 \left[-\frac{2}{15} + \frac{86}{375} \log(\epsilon) - 0.25725 + O(\epsilon) \right]
\end{aligned} \tag{1.1}$$

and the couple acting on the sphere is expressible as $(0, G_y, 0)$ with

$$\begin{aligned}
G_y = & 8 \mu Ua^2 \left[\frac{1}{10} + \frac{43}{250} \log \frac{2}{\epsilon} - 0.26227 + O(\epsilon) \right] \\
& - 8 \mu a^3 \left[-\frac{2}{5} + \frac{66}{125} \log(\epsilon) + 0.37085 + O(\epsilon) \right]
\end{aligned} \tag{1.2}$$

where μ is the viscosity of the fluid, U and ϵ are the translational and angular velocity of the sphere, respectively, as indicated in figure 1.

It is well known that the linearity of the creeping flow solutions in the absence of the inertia (at zero Reynolds number) requires the lift forces acting on the sphere moving along the wall to be identically zero. For a Newtonian fluid, the leading order contribution to the lift on a sphere is at $O(\text{Re})$. The inertial lift to this order may be calculated solely from the known creeping flow solutions using a reciprocal theorem.

The lift on a sphere in a Newtonian fluid and in the vicinity of a plane has been investigated in a number of studies. Cox and Brenner [1968] obtained general expressions for the lift force in terms of the Green's functions by assuming that the distance between the wall and the center of the sphere is large compared to the radius of the sphere. Cox and Hsu [1977] later used this formulation to evaluate the lift force on a sphere sedimenting near a flat wall in a stagnant fluid and neutrally buoyant and non-neutrally buoyant spheres in a fluid undergoing a planer quadratically varying flow. Drew [1988] applied perturbation techniques to evaluate the lift on a sphere translating in a shear field in the presence of a wall. The sphere was assumed to be very far from the wall and treated as a point force. The inertial lift on a sphere translating in a shear flow bounded by a single flat infinite wall was analyzed by McLaughlin [1993]. He derived an expression for the lift force by superposition of the disturbance flow created by the wall and migration velocity due to an unbounded shear field. The analysis is applicable when

the wall lies in the outer region or the inner region provided the distance between the wall and the sphere is large compared to the radius of the sphere. Later, Cherukat and McLaughlin [1994] considered the same problem when the distance between the sphere and the wall is comparable to the radius of the sphere. They obtained the inertial lift for separation distances down to 0.1 radius.

In the limit that the separation distance between the sphere and the plane vanishes, Leighton and Acrivos [1985] calculated the lift on a sphere in contact with a plane in a simple shear flow of a Newtonian fluid. They found that the lift points away from the wall and varies with the fourth power of the radius of the sphere and the square of the velocity gradient. However, they concluded that this inertial lift is far too small to be significance relative to the drag at Reynolds numbers of $O(10^{-2})$. They inferred that inertia plays only a minor role in bringing about the resuspension of settled particles in low Reynolds number shear fields. Recently, Krishnan and Leighton [1995] extended the work of Leighton and Acrivos [1985] to include the case where the sphere translates and rotates in the presence of shear in the limit that the separation distance between the sphere and the plane vanishes.

There were also a number of investigations on the motion of a sphere in a viscoelastic fluid and in the presence of a plane wall. A recent review was presented in Becker, McKinley & Stone [1996]. For flows with a small Deborah number that is the ratio of a characteristic relaxation time of the fluid to a convective time of the flow, one can use the “retarded-motion” expansion based on the asymptotic limit of a nearly-Newtonian fluid. The relevant constitutive model for viscoelastic materials is the Rivlin-Ericksen n th-order fluid which represents a generic limiting form for all constitutive equations for viscoelastic fluids. It was shown that the effects of viscoelasticity in these ordered fluids can be determined via a regular perturbation expansion in Deborah number (Leal [1980]).

Caswell [1972] examined the motion of various objects near plane and curves walls immersed in a non-Newtonian fluid used a reciprocal theorem. In his study, the flow was assumed to be weakly non-Newtonian up to third-order in the expansion, and the particle

was assumed to not close to the wall. It was found that a sedimenting sphere will be propelled away from the wall.

Becker, Mckinley and Stone [1996] study the motion of a sphere sedimenting near a single vertical plane wall. They included the full wall effects, and non-Newtonian effects up to second order in Deborah number using a general third-order fluid expansion. They explored the first effects of normal stress differences, shear thinning and inertia on the motion of a sphere. Their theoretical calculations indicate that a sphere settling near a wall the first effects of elasticity result in a drift velocity of the sphere away from the wall; a drag decrease beyond the value expected in the unbounded case with shear thinning further enhances the drag reduction; and no tendency for a sphere to exhibit anomalous rotation, as observed by Liu, Nelson, Feng and Joseph [1992], near a single plane wall unless shear thinning in the viscosity is also incorporated. Their results were valid in the limit that the wall is much closer to the sphere than the Oseen distance Re^{-1} , i.e. $\ll Re^{-1}$ for the gap. Results for the distance between the sphere and the wall down to 0.1 radius of the sphere were presented.

In this paper we examine the lift force on a sphere translating and rotating very close to a single plane wall with the limit of separation distance $\rightarrow 0$. The flow is assumed to be slow enough to be approximated by a second order fluid expansion. Since the flow is antisymmetric with respect to the front and back of the sphere, the second order fluid does not generate correction to the drag (1.1) and torque (1.2) on the sphere at the leading order; while it gives rise to the first order correction to the lift force, as noted in Becker et. al [1996]. The justification of using a second-order fluid expansion or the range of validity of its application will be discussed later in the paper.

2. Formulation of the Problem

Consider an incompressible second-order fluid with density ρ and viscosity μ , and second order stress coefficients α_1 and α_2 . The steady flow around a moving sphere in this fluid, as shown in figure 1, can be defined as,

$$\begin{aligned}
&= \text{Re}(\mathbf{u} \cdot \nabla) \mathbf{u} + \text{De} \left(\mathbf{B} + \frac{-2}{1} \mathbf{A}^2 \right) \\
&\mathbf{u} = 0 \\
&\text{where } \tau = -p\mathbf{I} + \mathbf{A}[\mathbf{u}]
\end{aligned} \tag{2.1}$$

The equations are made dimensionless with the particle radius a as the length scale, and some arbitrary V as the velocity scale. The pressure is scaled with $\mu V/a$. $\text{Re} = Va/\mu$ is the Reynolds number. $\text{De} = (-1)V/(\mu a)$ is the Deborah number representing elastic effects of the fluid, where $-2/1 = 1$ is the coefficient of the first normal stress difference which is usually positive.

If the coordinate system is located on the wall and centered at the point of closest contact between the sphere and the plane wall, and moves with the translational velocity of the sphere, the boundary conditions for this problem are given by

$$\begin{aligned}
\mathbf{u} &= -U/V \mathbf{e}_1 && \text{on plane wall} \\
\mathbf{u} &= a/V \mathbf{e}_2 \times (\mathbf{x} - h\mathbf{e}_3) && \text{on sphere surface } A \\
\mathbf{u} &= -U/V \mathbf{e}_1 && \text{for } |x| \gg a
\end{aligned} \tag{2.2}$$

Expanding the velocity field in powers of the small parameters, Re and De , we have

$$\mathbf{u} = \mathbf{u}^{(0)} + \text{Re} \mathbf{u}^{(1)} + \text{De} \mathbf{u}^{(2)} + o(\text{Re}, \text{De}) \tag{2.3}$$

and similarly for the pressure p and the total stress tensor τ .

The creeping flow equations at the leading order reduces to

$$\nabla^2 \mathbf{u}^{(0)} = 0 \quad \text{and} \quad \nabla^2 p^{(0)} = 0 \tag{2.4}$$

with the same boundary conditions given in (2.2).

The dimensionless lift force acting in the x_3 direction is given by integrating the stress over the surface of the sphere,

$$L = \int_A \mathbf{e}_3 \cdot (\mathbf{n}) dS = \int_A n_j n_{i3} dS \quad (2.5)$$

or

$$\begin{aligned} L &= \text{Re} \int_A n_j^{(1)} n_{i3} dS + \text{De} \int_A n_j^{(2)} n_{i3} dS + o(\text{Re}, \text{De}) \\ &= \text{Re} L^{(1)} + \text{De} L^{(2)} + o(\text{Re}, \text{De}) \end{aligned} \quad (2.6)$$

since the contribution from the zeroth order creeping flow solution vanishes. It can be shown that the second order contribution in De^2 term for a third-order fluid expansion is also zero since the flow is antisymmetric with respect to the front and back of the sphere (Becker et. al [1996]). Using a reciprocal theorem, it can be shown that the first term in the (2.6) due to the inertia reduces to

$$L^{(1)} = \int_V \mathbf{v} \cdot \left\{ \left(\mathbf{u}^{(0)} \right) \mathbf{u}^{(0)} \right\} dV \quad (2.7)$$

where \mathbf{v} is the creeping flow solution for a sphere approaching a plane with unit velocity in a quiescent fluid. The integration is over the entire space occupied by the fluid.

The second term in (2.6) is due to the normal forces in the viscoelastic fluid. By using the same reciprocal theorem and applying the Giesekus theorem for a second order fluid with $\alpha_1 = -\alpha_2$, we have

$$L^{(2)} = \int_V \mathbf{v} \cdot \left\{ \left(\mathbf{B}[\mathbf{u}^{(0)}] - \mathbf{A}^2[\mathbf{u}^{(0)}] \right) \right\} dV = \int_V \mathbf{v} \cdot \left[\mathbf{n}_3 \right] dV = - \int_A \left[n_3 \right] dS \quad (2.8)$$

where A is the surface of the sphere, and

$$= \mathbf{u}^{(0)} \cdot \mathbf{p}^{(0)} + \frac{1}{4} \mathbf{A}[\mathbf{u}^{(0)}] : \mathbf{A}[\mathbf{u}^{(0)}] \quad (2.9)$$

Substituting (2.9) into (2.8), we have

$$L^{(2)} = - \int_A \left[\mathbf{u}^{(0)} \cdot \mathbf{p}^{(0)} \right] n_3 dS - \frac{1}{4} \int_A \mathbf{A}[\mathbf{u}^{(0)}] : \mathbf{A}[\mathbf{u}^{(0)}] n_3 dS \quad (2.10)$$

Taking advantage of the linearity of the creeping flow equations, the zeroth order solution $(\mathbf{u}^{(0)}, p^{(0)})$ induced by the motion of a sphere moving very close to a plane may be obtained from the superposition of two different creeping flow solutions: (a) (\mathbf{u}^t, p^t) , for a non-rotating, translating sphere in a quiescent fluid; (b) (\mathbf{u}^r, p^r) , for a sphere rotating in a quiescent fluid about a fixed axis parallel to a plane. Solutions to the two problems have been provided by O'Neill & Stewartson [1967] and Cooley & O'Neill [1968]. It is important to note that in order to keep the flow steady in the translational flow case, the sphere should be kept fixed, while the plane wall and fluid are moving with a constant velocity, as indicated in the boundary conditions (2.2). Thus we write for the zeroth order dimensional velocity field as

$$\mathbf{u}^{(0)} = (U)\mathbf{u}^t + (a)\mathbf{u}^r \quad (2.11)$$

where the flow velocity for the two cases are scaled differently. Using (2.11) to write down the dimensional lift force, we get

$$\begin{aligned} \mathbf{L} &= \mathbf{L}^{(1)} + \mathbf{L}^{(2)} \\ &= a^2 \left\{ \begin{matrix} (1) \\ 1 \end{matrix} (U^2) + \begin{matrix} (1) \\ 2 \end{matrix} (a^2) + \begin{matrix} (1) \\ 3 \end{matrix} (U a) \right\} \\ &\quad + (-1) \left\{ \begin{matrix} (2) \\ 1 \end{matrix} (U^2) + \begin{matrix} (2) \\ 2 \end{matrix} (a^2) + \begin{matrix} (2) \\ 3 \end{matrix} (U a) \right\} \end{aligned} \quad (2.12)$$

where the inertial terms are evaluated by the Krishnan and Leighton [1995], $\begin{matrix} (1) \\ 1 \end{matrix} = 1.755$, $\begin{matrix} (1) \\ 2 \end{matrix} = 0.546$, $\begin{matrix} (1) \\ 3 \end{matrix} = -2.038$; while the elastic lift coefficients are defined as

$$\begin{matrix} (2) \\ 1 \end{matrix} = -\frac{1}{4} \int_A \mathbf{A}[\mathbf{u}^t] : \mathbf{A}[\mathbf{u}^t] n_3 dS \quad (2.13a)$$

$$\begin{matrix} (2) \\ 2 \end{matrix} = - \int_A \begin{bmatrix} \mathbf{u}^r & p^r \end{bmatrix} n_3 dS - \frac{1}{4} \int_A \mathbf{A}[\mathbf{u}^r] : \mathbf{A}[\mathbf{u}^r] n_3 dS \quad (2.13b)$$

$$\begin{matrix} (2) \\ 3 \end{matrix} = - \int_A \begin{bmatrix} \mathbf{u}^r & p^t \end{bmatrix} n_3 dS - \frac{1}{2} \int_A \mathbf{A}[\mathbf{u}^r] : \mathbf{A}[\mathbf{u}^t] n_3 dS \quad (2.13c)$$

since $\mathbf{u}^t = 0$ on the surface of the sphere.

We can write the integrals in (2.13) in the polar-cylindrical coordinate system (r, θ, z) where z coincides with x_3 . The velocity components and pressure of the solution for the two creeping flows around a sphere (described above) can be written in a special form of

$$\{u_r, u_\theta, u_z, p\} = \{U \cos \theta, V \sin \theta, W \cos \theta, P \cos \theta\} \quad (2.14)$$

where U, V, W and P are functions of r and z only.

Using the velocity boundary conditions on the sphere surface, the first surface integration in (2.13) related to the pressure reduces to,

$$\int_A [\mathbf{u}^r \cdot \mathbf{n}_3] p dS = \int_0^2 (z-1) \left[\frac{P}{r} + \frac{P}{r} - r \frac{P}{z} (z-1) \right] dz \quad (2.15)$$

and the second integral becomes

$$\begin{aligned} & \frac{1}{2} \int_A (\mathbf{A}[\mathbf{u}] : \mathbf{A}[\mathbf{u}]) n_3 dS \\ &= \int_0^2 4 \left[\frac{U^2}{r} + \frac{V^2}{r} + \frac{W}{r} + \frac{U^2}{z} + \frac{V}{z} - \frac{W}{r} \right] (z-1) dz \end{aligned} \quad (2.16)$$

1. Contribution of the Inner Solution to the Lift

The creeping flow solutions for a sphere moving along a wall when the minimum clearance a between the sphere and the plane is very much smaller than the radius of the sphere can be obtained using a matched asymptotic expansions technique (O'Neill & Stewartson [1967], Cooley & O'Neill [1968]). An 'inner' solution can be constructed for the region in the neighborhood of the nearest points of the sphere and the plane where the velocity gradients and pressure are large. A matching 'outer' solution can be constructed which is valid for the remainder of the fluid where velocity gradients are moderate but it is possible to assume that $\epsilon = 0$. In the limit of $\epsilon = 0$, the 'outer' solution obviously provides the $O(1)$ contribution to the lift force. We are going to show that the 'inner' solution leads to $O(1/\epsilon)$ contribution to the lift. Therefore, to the leading order we only need to calculate the contribution to the lift force from the flow in the 'inner' region, using the 'inner' solution for a translating sphere along a plane wall.

The inner solution for a translating sphere along a plane wall is given by O'Neill and Stewartson [1967],

$$\begin{aligned}
 P(r, z) &= -\mathcal{O}(\epsilon^2)P_0(R, Z) + \mathcal{O}(\epsilon^4)P_1(R, Z) + \dots \\
 U(r, z) &= U_0(R, Z) + \mathcal{O}(\epsilon^2)U_1(R, Z) + \dots \\
 V(r, z) &= V_0(R, Z) + \mathcal{O}(\epsilon^2)V_1(R, Z) + \dots \\
 W(r, z) &= \mathcal{O}(\epsilon^2)W_0(R, Z) + \mathcal{O}(\epsilon^4)W_1(R, Z) + \dots
 \end{aligned} \tag{3.1}$$

where the inner variables are defined as

$$R = r/\sqrt{\epsilon} \quad \text{and} \quad Z = z/\epsilon. \tag{3.2}$$

The gap between the sphere and the plane, expressed as a function of r , is given by

$$= 1 + \mathcal{O}(\epsilon^2) - (1 - r^2)^{1/2} = H + \frac{1}{8}\epsilon^2 R^4 + \dots \tag{3.3}$$

where

$$H = 1 + \frac{1}{2}\epsilon R^2. \tag{3.4}$$

The leading order solution to a translating sphere is

$$\begin{aligned}
 P_0' &= \frac{6R}{5H^2} \\
 U_0' &= \frac{6 - 9R^2}{10H^3} Z^2 + \frac{2 + 7R^2}{5H^2} Z - 1 \\
 V_0' &= -\frac{3}{5H^2} Z^2 - \frac{2}{5H} Z + 1 \\
 W_0' &= \frac{8R - 2R^3}{5H^4} Z^3 + \frac{2R^3 - 8R}{5H^3} Z^2
 \end{aligned} \tag{3.5}$$

Similarly, leading order solution to a rotating sphere is

$$\begin{aligned}
P_0^r &= \frac{6R}{5H^2} \\
U_0^r &= \frac{6-9R^2}{10H^3} Z^2 - \frac{8-2R^2}{5H^2} Z \\
V_0^r &= -\frac{3}{5H^2} Z^2 + \frac{8}{5H} Z \\
W_0^r &= \frac{8R-2R^3}{5H^4} Z^3 - \frac{R^3+26R}{10H^3} Z^2
\end{aligned} \tag{3.6}$$

The first lift coefficient in (2.13) are calculated, at the leading order, as

$$C_{l1}^{(2)} = \frac{R_0}{2} \left[\frac{U_0^r}{Z} \right]^2 + \frac{V_0^r}{Z} + O(\epsilon) \quad \text{at } Z=H \tag{3.7}$$

Substituting the ‘inner’ solutions (3.5) into (3.7), we found

$$C_{l1}^{(2)} = \frac{1}{2} \frac{16 R_0^2 (32 + 20R_0^2 + 5R_0^4)}{25 (2 + R_0^2)^3} + O(\epsilon) \tag{3.8}$$

Evaluating (3.8) at large values of the inner variable R_0 , we have

$$C_{l1}^{(2)} = \frac{8}{5} \frac{1}{R_0} + O(1) \tag{3.9}$$

To the leading order, the second lift coefficient in (2.13) is calculated as

$$C_{l2}^{(2)} = \frac{R_0}{2} \left[-2 \frac{P_0^r}{R} + \frac{P_0^r}{R} + \frac{U_0^r}{Z} \right]^2 + \frac{V_0^r}{Z} + O(\epsilon) \quad \text{at } Z=H \tag{3.10}$$

Substituting the inner solutions (3.6) into (3.10), we found

$$C_{l2}^{(2)} = \frac{1}{2} \frac{16 R_0^2 (-28 - 10R_0^2 + 5R_0^4)}{25 (2 + R_0^2)^3} + O(\epsilon) \tag{3.11}$$

or

$$\mathbf{z}_2^{(2)} = \frac{8}{5} \frac{1}{\delta} + O(\delta). \quad (3.12)$$

with the inner variable R_0 , which is the same as $\frac{1}{\delta}$.

Similarly, the third lift coefficient in (2.13) is evaluated as

$$\begin{aligned} \mathbf{z}_3^{(2)} &= \frac{1}{2} \frac{R_0}{\delta} - 2 \frac{P_0^r}{R} + \frac{P_0^r}{R} + \frac{U_0^r}{Z} - \frac{U_0^r}{Z} + \frac{V_0^r}{Z} - \frac{V_0^r}{Z} + O(\delta) \quad \text{at } z=H \\ &= \frac{1}{2} \frac{1}{\delta} - \frac{4}{25} \frac{R_0^2 (92 + 80R_0^2 + 5R_0^4)}{(2 + R_0^2)^3} + O(\delta) \end{aligned} \quad (3.13)$$

or
$$\mathbf{z}_3^{(2)} = -\frac{4}{5} \frac{1}{\delta} + O(\delta), \quad (3.14)$$

with the inner variable R_0 .

Therefore, for an arbitrary moving (both translating and rotating) sphere, we obtain the dimensional lift due to the normal stress

$$\mathbf{F}^{(2)} = \left(-\frac{1}{\delta} \right) \left(U^2 + \frac{1}{2} a^2 - \frac{1}{2} U a \right) \frac{8}{5} \frac{1}{\delta} + O(\delta), \quad (3.15)$$

where U and a are the translational and angular velocity of the sphere, respectively. When the sphere is in perfect rolling along the plane $U = a$, we have a non-zero lift force

$$\mathbf{F}_{\text{rolling}}^{(2)} = \left(-\frac{1}{\delta} \right) \frac{1}{2} a^2 \frac{12}{5} \frac{1}{\delta} + O(\delta). \quad (3.16)$$

4. Discussion and Conclusions

The linear dependence of the lift on the inverse of the minimum gap size in (3.15) and (3.16) can be easily explained. In addition to the Newtonian pressure, the pressure in a second-order fluid is modified by a term which is proportional to the square of the shear rate (see Joseph and Liao [1994]). Within the gap between the sphere and the wall and to the leading order, this term scales as $1/\delta^2$, since the shear rate there is proportional to

$1/\lambda$. The circular area of the gap region between the sphere and wall shrinks as the gap size reduces and scales as λ^{-2} , as indicated by the inner variable R in (3.2). Therefore, the lift generated by the additional term for the second-order fluid varies linearly with $1/\lambda$.

From (3.15), we conclude that the flow due to a moving sphere along a plane wall gives rise to a positive elastic lift force on the sphere when gap between the sphere and the wall is small, since in flows of viscoelastic fluids the first normal stress difference which is proportional to $(-\tau_{11})$ is positive.

Equation (2.12) indicates that the ratio of the inertial lift to the elastic lift is proportional to the square of the particle radius. Therefore, for fine particles moving in a viscoelastic solution, the elastic lift could be much bigger than the inertial lift.

For a sphere moving in a shear flow of constant shear rate far away from the sphere, the expression (3.15) is still valid to the leading order, since the addition of shear flow around a stationary sphere does not induce any flow in the ‘inner region’ between the sphere and the wall (O’Neill, [1968]). In this situation, the shear flow around the sphere is driving the sphere forward. For a fixed relative gap size between the particle and the wall, the translational velocity of the sphere is proportional to its radius. Thus the elastic lift in (3.15) will be proportional to the square of the sphere radius. When the lift force on a heavy particle moving along a wall exceeds the effective weight of the particle, the particle will be lifted from the wall and suspended in the fluid. Therefore, the ratio of the elastic lift to the effective weight of the particle is proportional to the particle radius. Namely, smaller particles will be easier to be suspended by the elastic lift due to the normal stress in the fluid. However, under the same condition, the inertial lift that is proportional to the fourth power of the particle radius as discussed above does not suspend small particles.

Furthermore, the elastic lift force is singular when the minimum gap between the sphere and the wall approaches zero. A direct consequence of this result is that in a viscoelastic fluid a moving particle will be always suspended from a smooth surface. However, as the gap size approaches zero, the application of the second-order fluid

expansion is no longer valid since the shear rate in the gap becomes increasingly large. In order for a retarded motion expansion to be valid, the second-order terms are normally required to be small corrections to the Newtonian terms. We may compare the magnitude of the nondimensional pressures in the gap due to the creeping flow, $O(\epsilon^{-3/2})$, with that of the correction in a second-order fluid expansion, $O(De^{-2})$. For valid use of the second-order fluid expansion, one needs that

$$De \ll O(\epsilon^{1/2}) \text{ or } De \sim O(\epsilon) . \quad (4.1)$$

Certainly this is not a rigorous proof for the validity of the expansion. However, in the perturbation sense, there is always a valid range of Deborah number for the second-order fluid expansion, however small the range may be. Expression (3.15) will be always valid for this limited range of De .

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