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List of symbols

English Symbols

1	Identity tensor
A	Area of cross section
A	Configuration tensor ($= \boldsymbol{\tau}_E + (\eta_E / \lambda_1) \mathbf{1}$)
a	Radius of the particles
b	Body force
D	Diameter of cylindrical tube
D	Drag force
d	Diameter of the particles
\bar{d}	Average diameter of the particles
$d_{i,j}$	Distance between the center of the i -th and j -th particles
\mathbf{e}_x	Unit vector in the x-direction
\mathbf{e}_y	Unit vector in the y-direction
F	Force
F'_i	Short-range repulsive force exerted on the i -th particle (Eq. VII.9)
$F_{i,j}^p$	Particle-particle repulsive force (Eq. VII.10)
$F_{i,j}^p$	Particle-wall repulsive force (Eq. VII.11)
g	Acceleration due to gravity
H	Indicator function
H	Height of the channel
H_e	Equilibrium height of the fluidized bed
h_v	Velocity mesh size
h_p	Pressure mesh size
I	Moment of inertia tensor
L	Channel length
L	Lift force
L'	Lift force per unit length
L_e	Elastic component of lift force
L_p	Pressure component lift force
L_s	Shear component lift force
M	Particle mass
N	Number of particles
n	Unit outward normal vector on the particle surface
n_ℓ	Growth rate of the wave amplitude
p	Pressure
\bar{p}	Average pressure gradient
p_a	Atmospheric pressure

r	Position vector of the point w.r.t. the center of mass of the particle
T	Torque
T	Dimensionless stress
T*	Stress tensor (Chap. 6)
T_f	Stress for the fluid phase (Chap. 6)
t	Time
t	Traction on the particle surface
U	Translational velocity of the particles
U_m	Mean velocity of the fluid (Chap. 9)
U_s	Slip velocity (= $U_f - U_p$)
u	Velocity of the fluid
V_c	Composite velocity (= $\epsilon \mathbf{V}_f + \phi \mathbf{V}_s$) (Chap. 6)
V_f	Average velocity of the fluid phase (Chap. 6)
V_m	Mass averaged velocity (Chap. 6)
V_p	Volume per unit length of the particles (Chaps. 9 and 11)
V_s	Average velocity of the solid phase (Chap. 6)
W	Width of domain
X	Coordinate of the center of mass of the particle

Greek Symbols

ϵ	Volume fraction of the fluid phase
ϵ_p and ϵ_w	Stiffness parameter for the collision scheme
Φ_e	Elastic lift fraction
Φ_p	Pressure lift fraction
Φ_s	Shear lift fraction
ϕ	Volume fraction of the solid phase
Γ	Circulation
γ	Surface tension
$\dot{\gamma}$	Shear rate
$\dot{\gamma}_w$	Shear rate at the wall
η	Viscosity of the fluid
η_2	Effective viscosity of the composite (Eq. VIII.8)
η_E	Elastic viscosity
η_m	Effective viscosity of the composite (Chap. 14)
λ	Lagrange multiplier field
λ	Wavelength (Chap. 8)
λ_1	Relaxation time
λ_2	Retardation time
ν	Kinematic viscosity
Π	Pressure in a periodic domain
π	3.141592
θ	Angular position of the particle

ρ_f	Density of the fluid
ρ_2	Effective density of the composite (Chap. 8)
σ	Stress tensor
τ	Shear stress
τ_E	Elastic stress
v	composite velocity
v_m	Migration velocity (Chap. 10)
Ω	Computational domain, angular velocity
Ω_f	Fluid domain
Ω_s	Particle domain
Ω_f	Angular velocity of the fluid
Ω_p	Angular velocity of the particle
Ω_s	Slip angular velocity ($=\Omega_f - \Omega_p$)
ω	Angular velocity of the particle

Dimensionless Groups

C_D	Drag coefficient
De	Deborah number
E	Elasticity number De/R
G	Gravity parameter
R	Shear Reynolds number
R_G	Gravity Reynolds number
R_s	Slip Reynolds number

Subscript

2D	Two dimensional
3D	Three dimensional
0	Initial Value $t = 0$ (Chap. 14)
c	Composite
E	Elastic
f	Fluid
i and j	i -th and j -th particle
p	particle
s	solid
sed	Sedimentation
T	Total
∞	$t \rightarrow \infty$

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