

## **XIV Fluidization by lift of 300 circular particles in plane Poiseuille flow by DNS**

In fluidized beds and sedimentation columns in which particles are in a balance of buoyant weight and drag, the cooperative effects of other nearby particles enter strongly into the dynamics. These effects are described by theories of hindered motion. There is a very definite relation between the fluidization by drag of one single isolated particle and the fluidization of a particle in a swarm of other particles; for example, the Richardson-Zaki correlation 1954. Analogous ideas must come into play in problems of slurries which are fluidized by lift rather than by drag; hindered motion effects involving the effective viscosity and the effective density of a suspension and other cooperative effects surely enter here but are not well understood.

Choi and Joseph 2001 carried out studies of fluidization by lift of 300 particles in the same plane Poiseuille flow used to study the lift to equilibrium of a single particle. The computation is carried out in a long periodic domain in which the volume fraction of solid circles ranges roughly between 78 % and 31%. The study is framed as an initial value problem in which a closely spaced cubic array of particles resting on the bottom of the channel are lifted into suspension.

The following picture emerges from this study. At early times the top of the array is only slightly disturbed; since the cubic crystal array is not tightly packed the top layers move forward relative to the bottom. The lifting of particles out of suspension is accomplished by a pressure mechanism clearly revealed by the simulation; liquid is driven into the bed by high pressure at the front and low pressure at the back of each circle in the top row. The particles are dislodged by this pressure mechanism. At higher Reynolds numbers single particles are thrown out of the bed in a manner resembling saltation. Typically isolated particles will fall back into the bed because the drag on an isolated particle is less than when it is among many. The permanent lifting of more particles of the bed takes shape in the formation of waves, which resemble water waves. The wave amplitude grows as the pressure gradient and flow speed increase; particles are levitated out of the bed and the levitated particles form a fluidized suspension over a basically fixed bed. This can be described as bed erosion. It is possible to erode the whole bed and fluidize all of the particles by lift if the pressure gradient is high enough and the bed depth small enough. The wave amplitude decreases when the particles are fully fluidized.

The evolution to full fluidization is associated with a transition from a basically vertical stratification of dynamic pressure to a basically horizontal stratification of dynamic pressure. The final state of full fluidization is not steady. Internal pressure waves which propagate horizontally are associated with the propagation of particle depleted regions which could be described as an internal wave of the volume fraction.

A single heavier than liquid particle will not lift off at low pressure gradients; the particle slides and rolls on the wall; lift-off to equilibrium occurs at critical values. In the fluidized suspension we can track the evolution of the rise of the mass center of the particles. The final state of full fluidization can be determined as the leveling off of the rise of the mass center curve. The mass center did not rise even after a long computation when the pressure gradient was below the one critical for the rise of a single particle.

When, as in a slurry, there are many particles the number  $N$  and places of  $N$  boundaries enter into the problem description. In the present case, 300 is the number and the places of these 300

particles evolve as part of the solution. We have similarity then for all evolution problems for which  $N$ ,  $d/W$ ,  $\rho_p/\rho_f$ ,  $R$ ,  $G$  are identical. In fact, the density ratio  $\rho_p/\rho_f$  enters only as the coefficient of the acceleration terms, hence does not enter for steady flow. Though steady flows of a single particle do occur, steady flow of many particles apparently do not occur after lift off; it is possible that the accelerations are small, however after the bed has fully expanded.

Here we study the problem of lift of 300 solid circles in plane Poiseuille flow (figure IX.2) when  $\rho_p = 1.01 \rho_f$ ,  $\rho_f = 1$  gm/cc,  $W = 12d$ ,  $d = 1$  cm for viscosities  $\eta = 1, 0.2$  and  $0.01$  poise (from light oil to water). The calculations are carried out in a periodic domain  $L = 63d$  which is long enough to contain four or more of the waves of pressure which characterize fully fluidized beds of 300 particles. Initially the circles are arranged in a cubic crystal array in which the particles do not touch; they are separated by  $0.05$  cm when  $\eta = 1$  and  $0.2$  poise, and by  $0.1$  cm when  $\eta = 0.01$  poise. The height of the array is  $5.25$  cm and  $5.35$  cm.

We recall that the results of calculations may be generalized by computing the values of the shear Reynolds number  $R = \dot{\gamma}_w d^2 / \nu$  and the gravity number  $G = d(\rho_p - \rho_f)g / \dot{\gamma}_w \eta$  defined in (IX.13). The value  $R_G = RG = \rho_f d^3 (\rho_p - \rho_f)g / \eta^2 = 9.81 / \eta^2$  is independent of  $\dot{\gamma}_w$ . The running index in our calculation is the pressure gradient  $\bar{p}$ ; given  $\eta$  this determines  $R = \rho d^2 W \bar{p} / 2\eta^2 = 6\bar{p} / \eta^2$ .

#### ▪ Case 1: $\eta = 1$ poise, $R_G = 9.81$

Figure XIV.1 shows the height of the center of gravity of the 300 particles as a function of  $R = 6\bar{p}$ . We say that the bed height has attained its final fully fluidized value when the rise curve levels off. Average values of the bed height  $\bar{H}$ , the average velocity  $\bar{U}$  cm/sec and angular velocity  $\bar{\Omega}$  sec<sup>-1</sup> at full inflation are given in Table XIV.1. The time taken for the center of gravity to reach its fully fluidized value increases as the Reynolds number  $R$  decreases. Figure XIV.2, XIV.3, XIV.4 and XIV.5 show snapshots of the evolution of the bed to full fluidization at values of  $R$ ,  $G$ , given in the caption to figure XIV.1.

Animations for these snapshots can be found at our URL <http://www.aem.umn.edu/Solid-Liquid-Flows>. The snapshots are decorated by shades of gray coded to reveal the distribution of dynamic pressure ( $\bar{p}$  in equation (IX.6)); dark means low pressure. Figures XIV.6 and XIV.7 give graphs of the pressure at different cross-sections of the channel for  $R = 120$  at an early time  $t = 0.9$  sec and when the suspension is fully fluidized at  $t = 27$  sec. At early times the pressure is stratified vertically, but not horizontally; at the later time the pressure is stratified horizontally and not vertically.

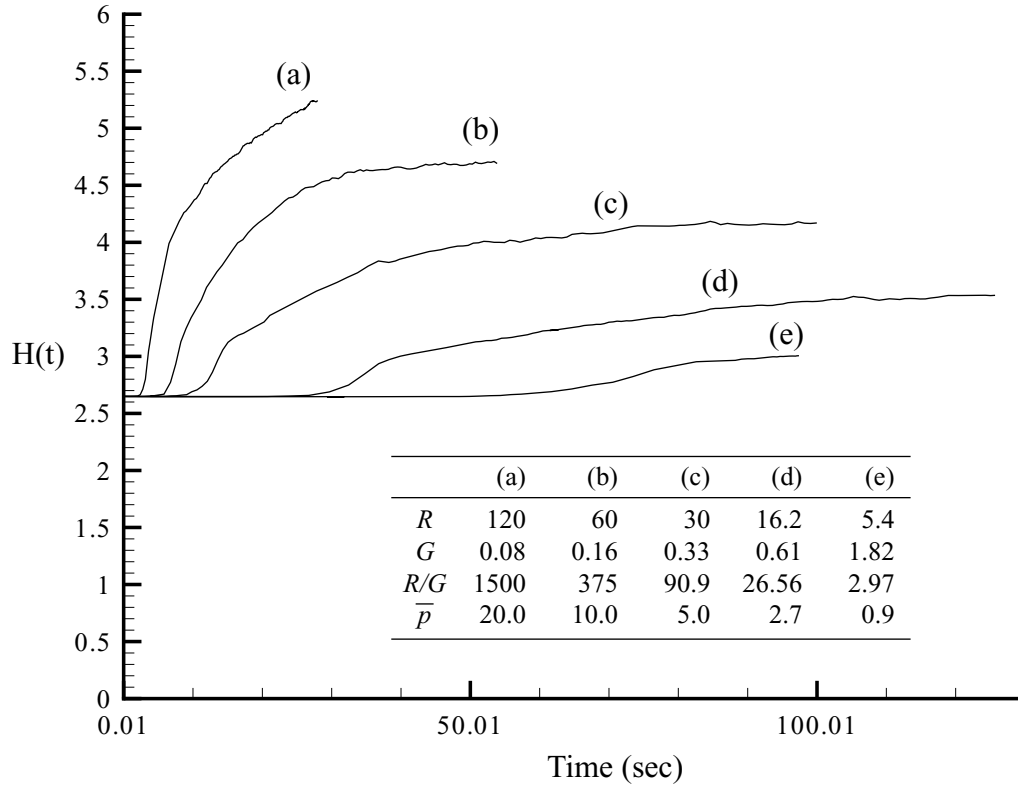


Figure XIV.1. Rise curves for the center of gravity of 300 circular particles fluidized by lift (fluid viscosity = 1.0 poise,  $G = 9.81/R$ ). The time scale for the slow rise at  $\bar{p} = 0.9$  dynes/cm<sup>2</sup> has been compressed by 2; the real time corresponding say, to 50.01 is 100.02 sec. The bed is said to be fully fluidized when the rise curves level off. The time to full fluidization is longer when the Reynolds number is smaller.

Table XIV.1 Data for the forward motion of a fluidized suspension of 300 particles after the bed has fully inflated and the average height  $\bar{H}$  of all particles has stopped increasing ( $\eta = 1.0$ ).  $\bar{H} = \bar{H}_o = 2.65d$  at  $t = 0$ .  $\bar{U}$  and  $\bar{\Omega}$  are the average velocity and angular velocity of the particles.

$R$	$G$	$\bar{p}$	$\bar{H}$	$\bar{U}$	$\bar{\Omega}$
5.4	1.82	0.9	3.01	2.09	0.28
16.2	0.61	2.7	3.54	9.55	1.026
24	0.41	4.0	4.00	18.62	1.75
30	0.33	5.0	4.17	25.63	2.25
60	0.16	10.0	4.69	55.60	3.85
120	0.08	20.0	5.24	119.76	4.90

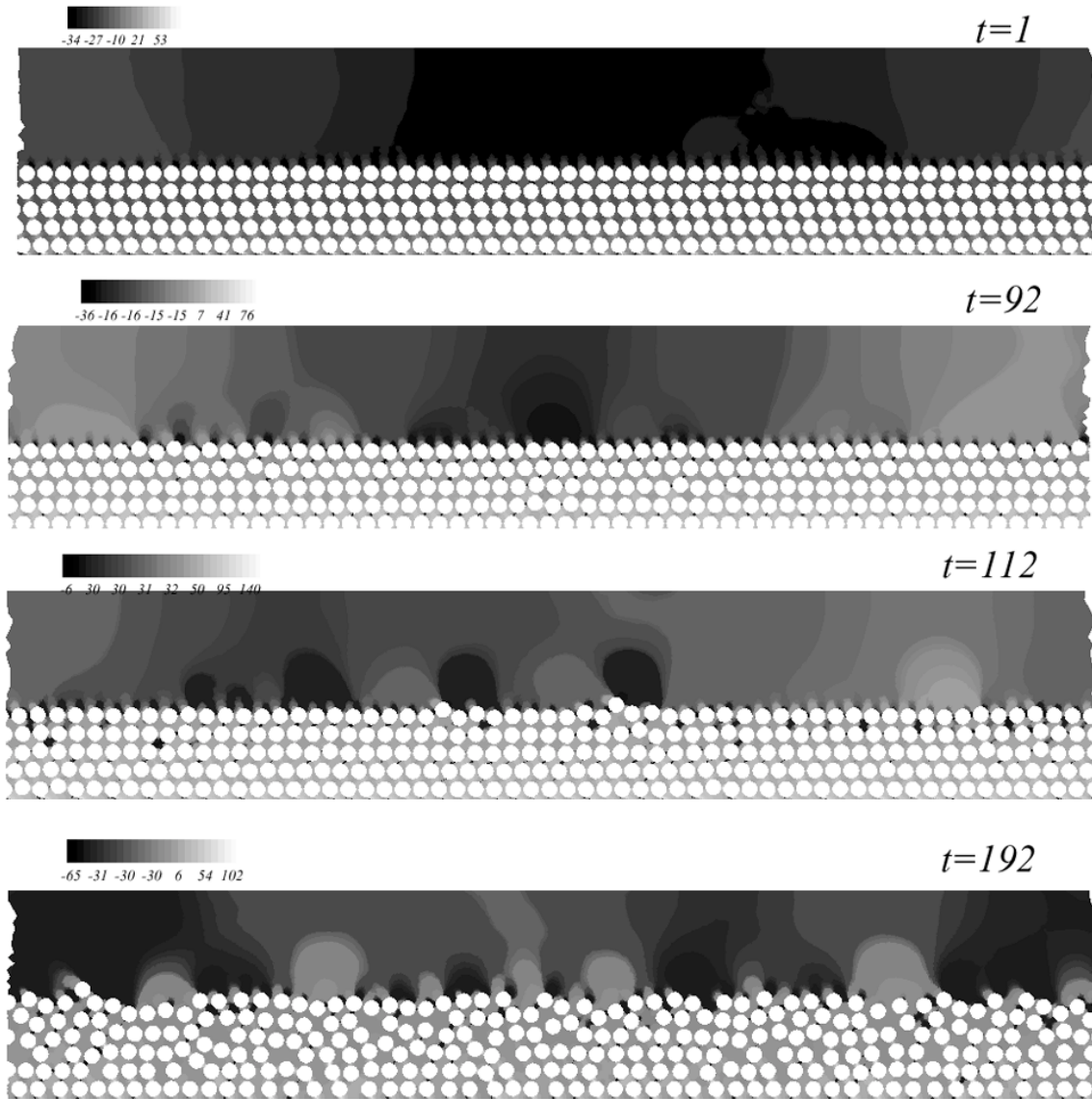


Figure XIV.2. Snapshots of the fluidization of lift of 300 circular particles  $\rho_p = 1.01 \text{ g/cm}^3$  when  $\eta = 1$  poise ( $R = 5.4$ ,  $G = 1.82$ ). The flow is from left to right. The gray scale gives the pressure intensity and dark is for low pressure. At early times particles are wedged out of the top layer by high pressure at the front and low pressure at the back of each and every circle in the top row. The vertical stratification of pressure at early times develops into a "periodic" horizontal stratification, a propagating pressure wave. The final inflated bed has eroded, rather tightly packed at the bottom with fluidized particles at the top.

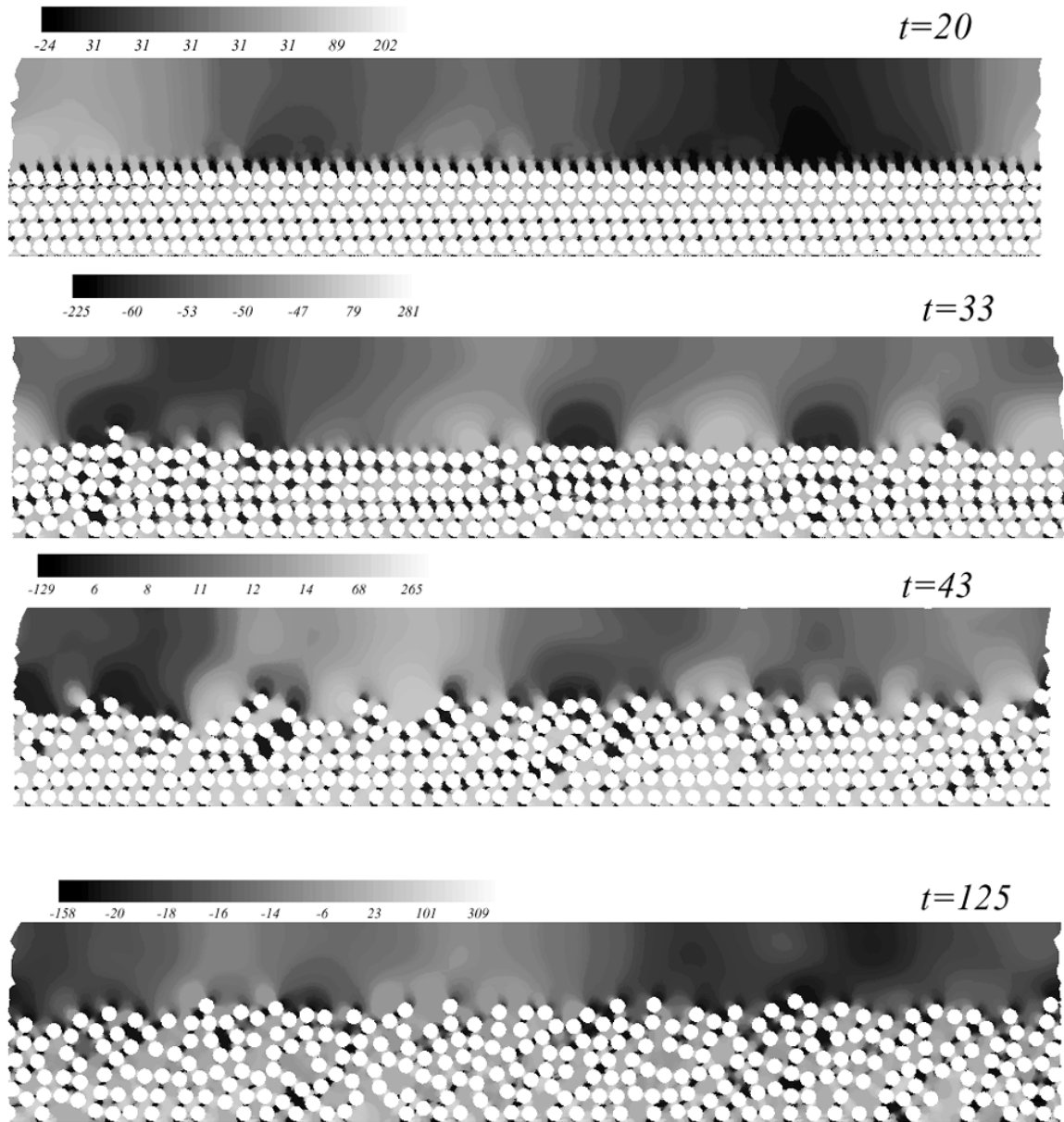


Figure XIV.3. Fluidization of 300 particles ( $R = 16.2$ ,  $G = 0.61$ ). The conditions are the same as in figure XIV.2 except that the lift forces are greater leading to a more complete and faster fluidization.

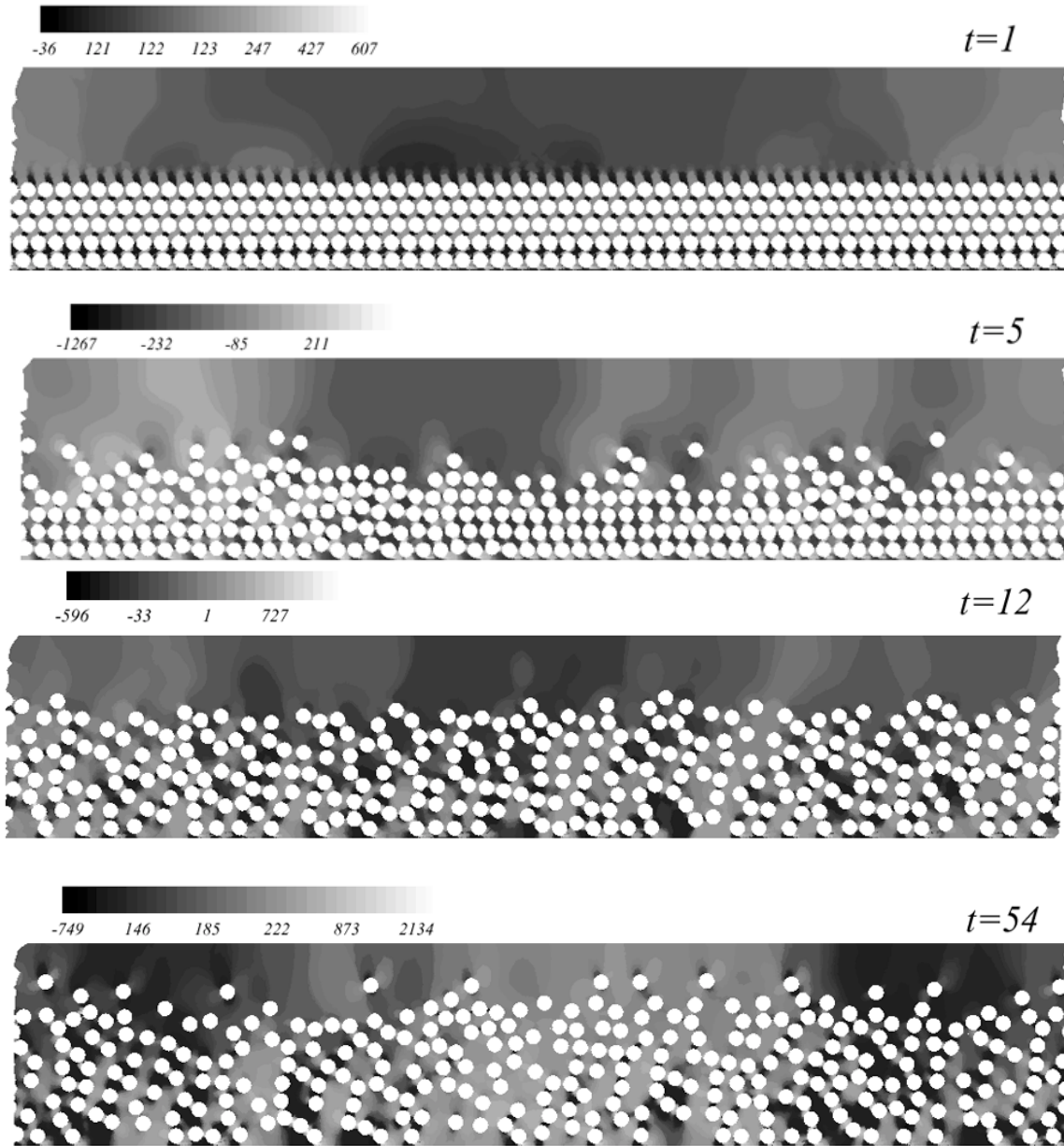


Figure XIV.4. Fluidization of 300 particles ( $R = 60$ ,  $G = 0.16$ ). The conditions are as in figure XIV.2. The ratio  $R/G = \dot{\gamma}_w^2 d / (g\Delta\rho / \rho)$  measures the ratio of lift to buoyant weight. Here the ratio is very large leading to fast and complete fluidization; the entire bed has eroded.



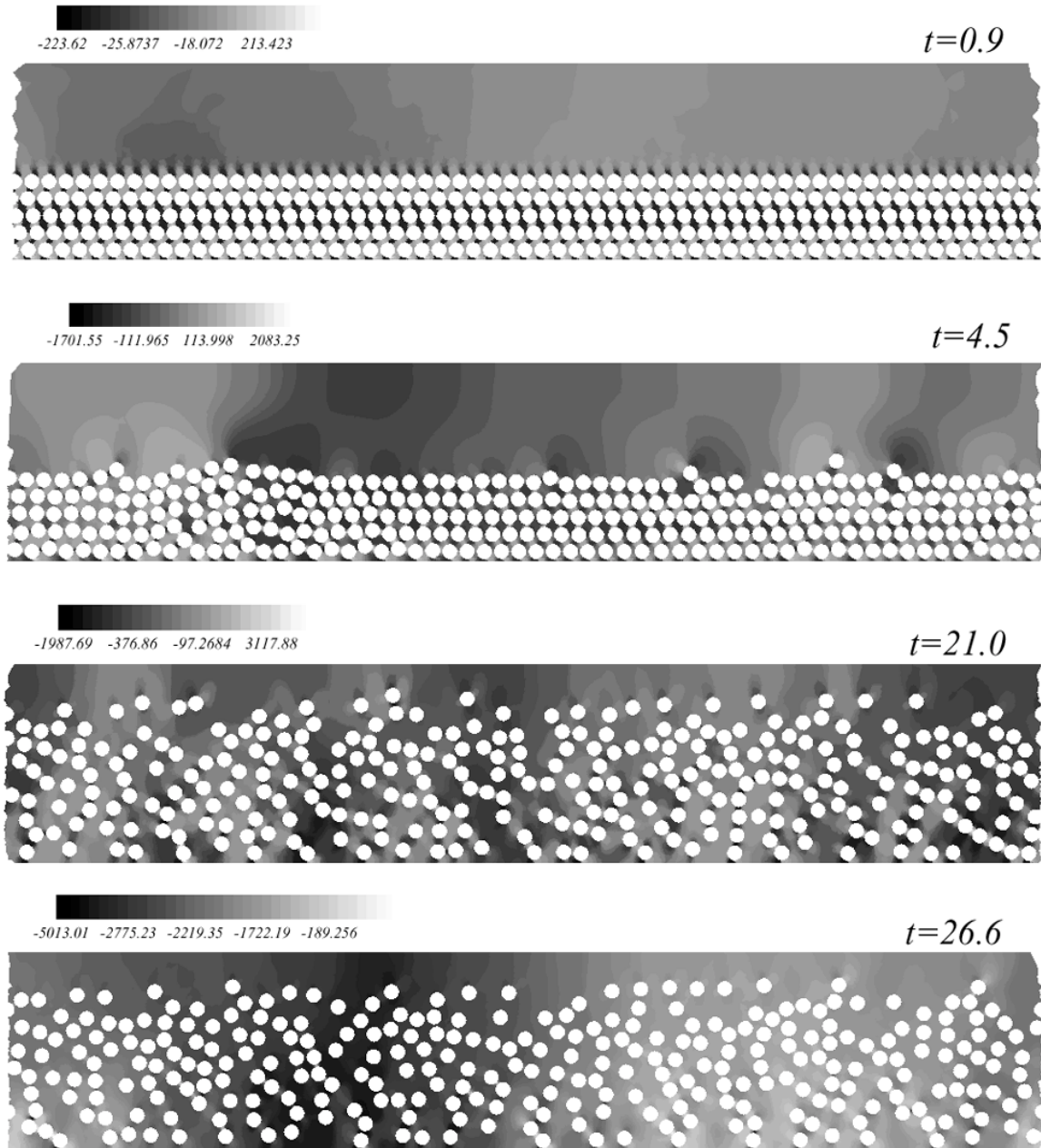


Figure XIV.5. Fluidization of 300 particles ( $R = 120$ ,  $G = 0.08$ ). The conditions are the same as in figure XIV.2 and figure XIV.4 but the ratio of lift to buoyant weight is greater and the fluidization is faster and the particle mass center rises higher than in the previous figures.

In figures XIV.6 and XIV.7 we plotted the distribution of pressures at an early and late time when  $R = 120$ ,  $G = 0.08$ . The figures show that the vertical stratification of pressure at an early time evolves to waves of pressure which are associated with propagating number density or voidage waves.

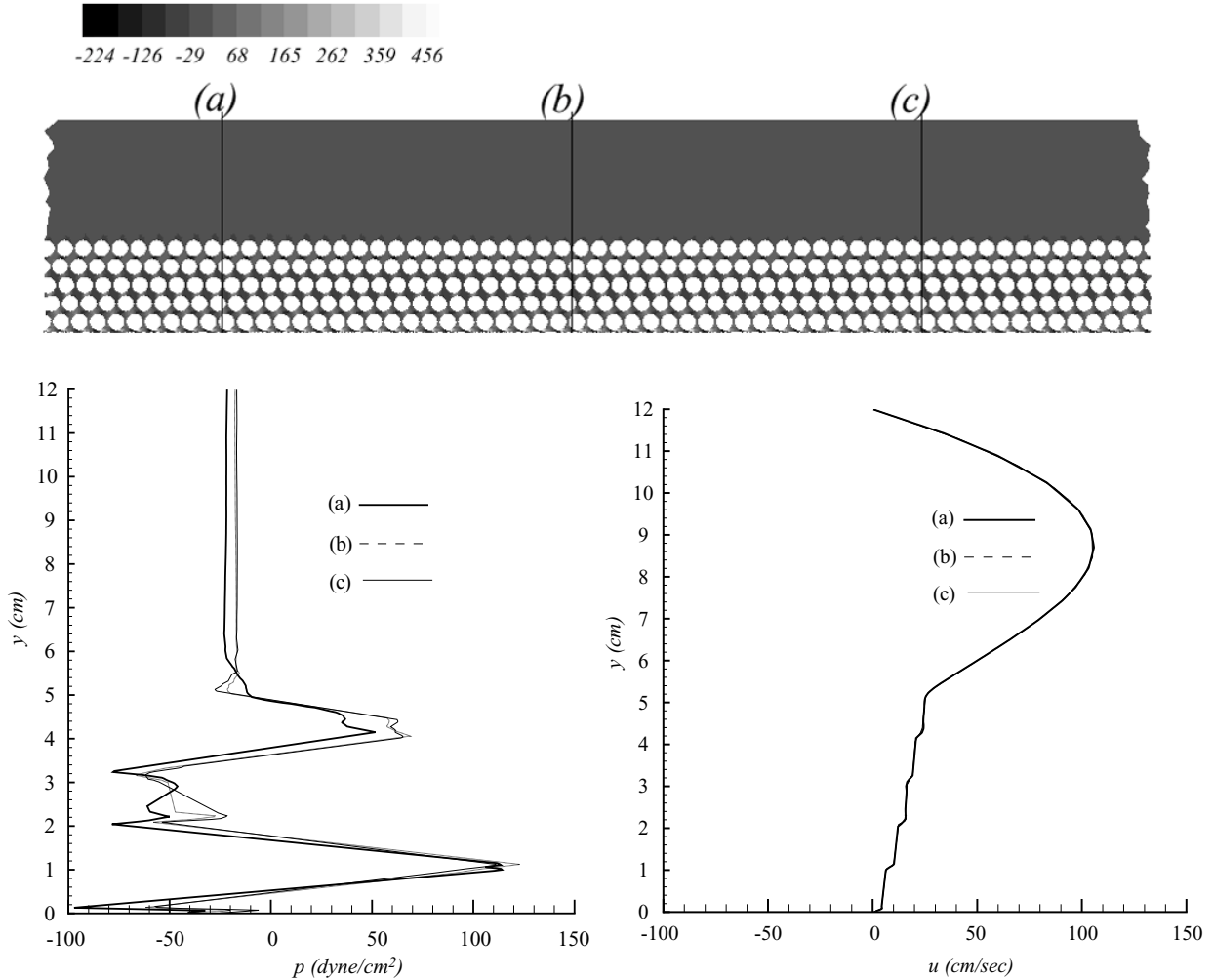


Figure XIV.6. Distribution of dynamic pressure  $p$  and streamwise velocity  $u$  at an early time ( $t = 0.9$  sec) when  $R = 120$ ,  $G = 0.08$  (cf. Figure XIV.5) at different cross sections of the channel. The dynamic pressure is stratified vertically but not horizontally. The rows of particles slide relative to one another moving like rigid bands separated by liquid.



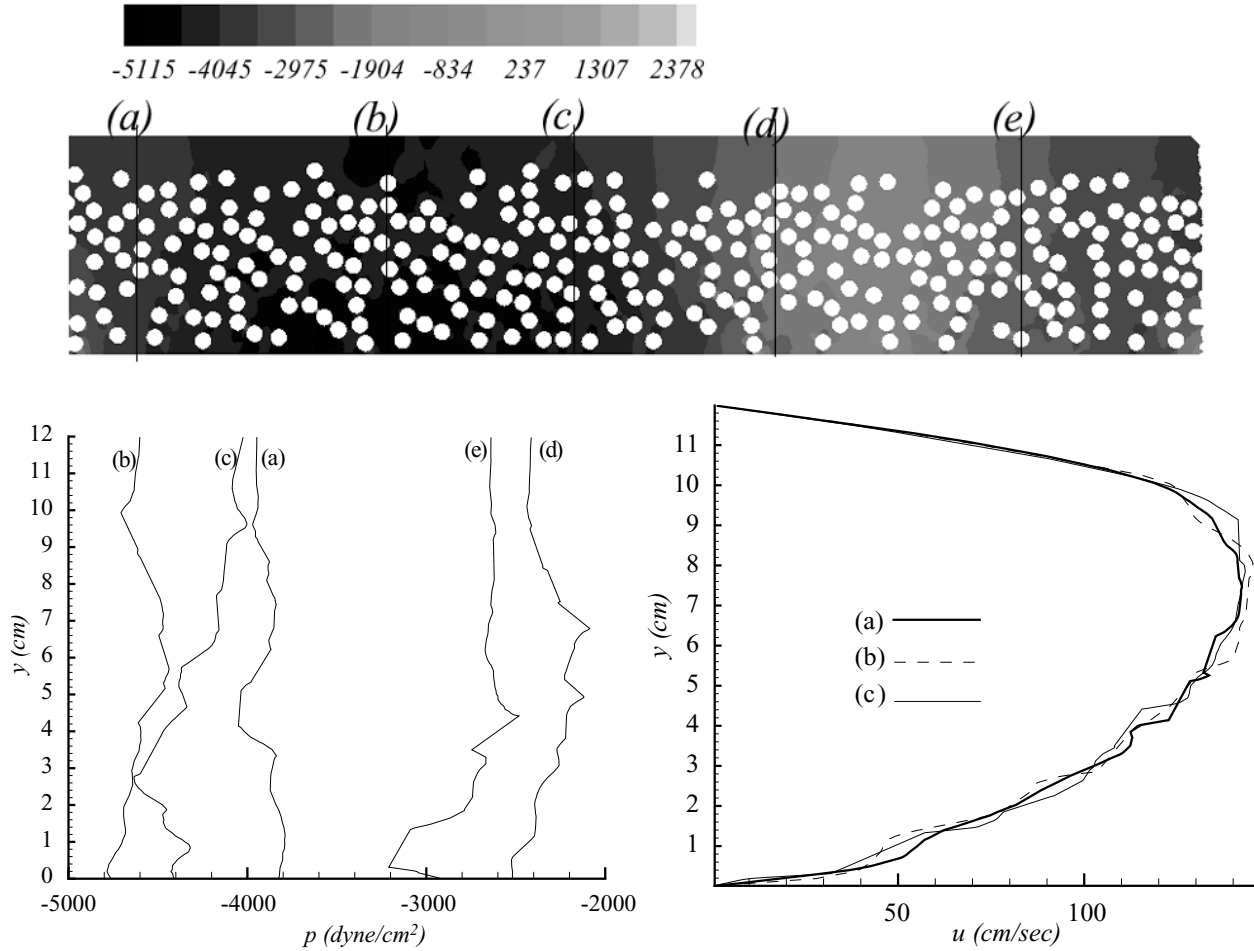


Figure XIV.7. Distribution of dynamic pressure  $p$  and streamwise velocity  $u$  after full fluidization ( $t = 27$  sec) when  $R = 120$ ,  $G = 0.08$ . The pressure  $p$  is stratified horizontally and not vertically; pressure pulses propagate horizontally.

▪ **Case 2:  $\eta = 0.2$  poise,  $R_G = 245$**

Figure XIV.8 shows the height of the center of the gravity of the 300 particles as a function of  $R = 6\bar{p}/\eta^2$ . The interpretation of figure XIV.8 is basically the same as figure XIV.1. Average values of the bed height  $\bar{H}$  cm, velocity  $\bar{U}$  cm/sec, and angular velocity  $\bar{\Omega}$  sec<sup>-1</sup> at full fluidization are given in table XIV.2. Snapshots of the evolution to full fluidization are shown in figures XIV.9 and XIV.10.

Table. XIV.2 Data for the forward motion of a fluidized suspension of 300 particles after the bed has fully inflated and the average height  $\bar{H}$  of all particles has stopped increasing ( $\eta = 0.2$ ).  $\bar{H} = \bar{H}_o = 2.65d$  at  $t = 0$ .  $\bar{U}$  and  $\bar{\Omega}$  are the average velocity and angular velocity of the particles.

$R$	$G$	$\bar{p}$	$\bar{H}$	$\bar{U}$	$\bar{\Omega}$
45	5.44	0.3	2.64	3.17	0.42
150	1.63	1.0	3.30	10.75	1.25
300	0.82	2.0	3.82	22.98	2.43
450	0.54	3.0	4.75	34.15	2.02

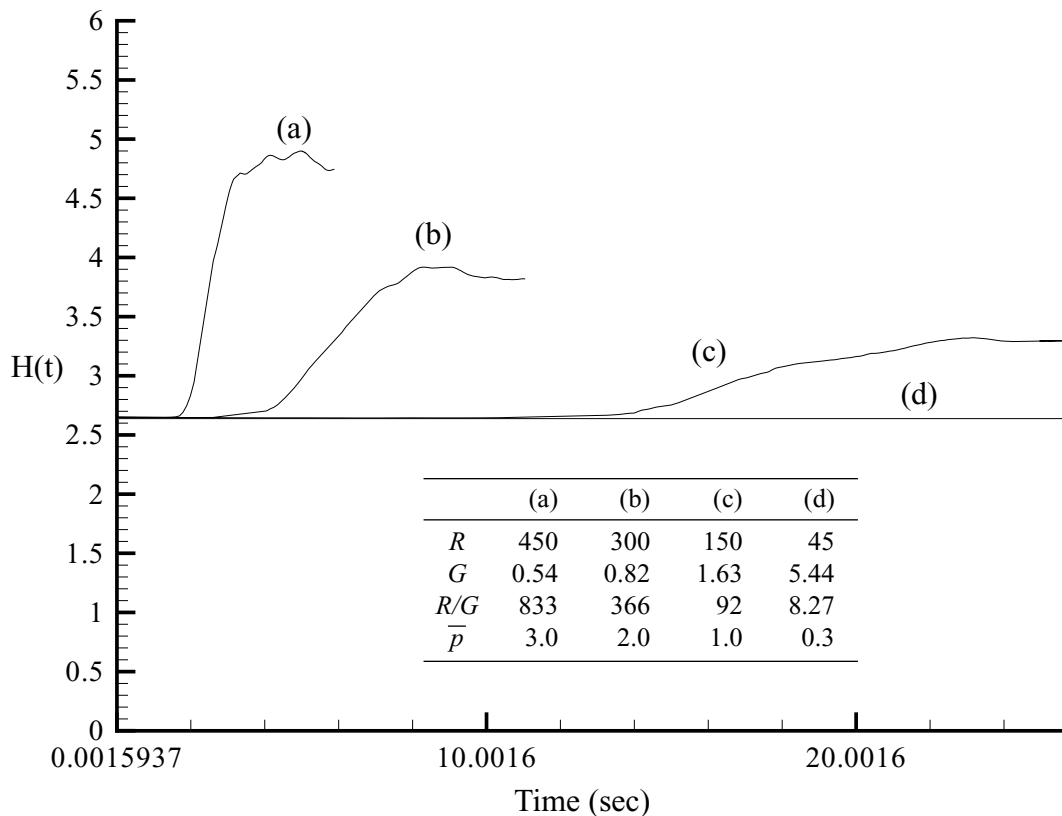


Figure XIV.8. Rise curves for the center of gravity of 300 circular particles fluidized by lift (fluid viscosity = 0.2,  $RG = 9.81/\eta^2 = 245$ .)  $\bar{p}$  is in  $\text{dyne/cm}^2$ . The bed is fully fluidized when the rise curves level off. The time to full fluidization is longer when the Reynolds number is smaller. The time to full fluidization is faster when  $\eta = 0.2$  than when  $\eta = 1$  (figure XIV.1). The time is scaled down by 5 and the center of gravity of the particles will eventually rise when  $(R, \bar{p}) = (45, 0.3)$ .

The description in the caption of figure XIV.2 applies also to figures XIV.9 and XIV.10 except that the particles are more mobile when the viscosity of the fluid is smaller. The particle laden region at  $t = 25$  sec in figure XIV.9, and  $t = 1.98$  in figure XIV.10 is separated from the particle free region by an "interface" which propagates like an interfacial wave. This interface disappears at a higher  $R = 450$ , figure XIV.10 for  $t > 2.7$  sec, because the stronger lift forces push

wave crests into the top of the channel; however, the pressure and associated void fraction wave persists.

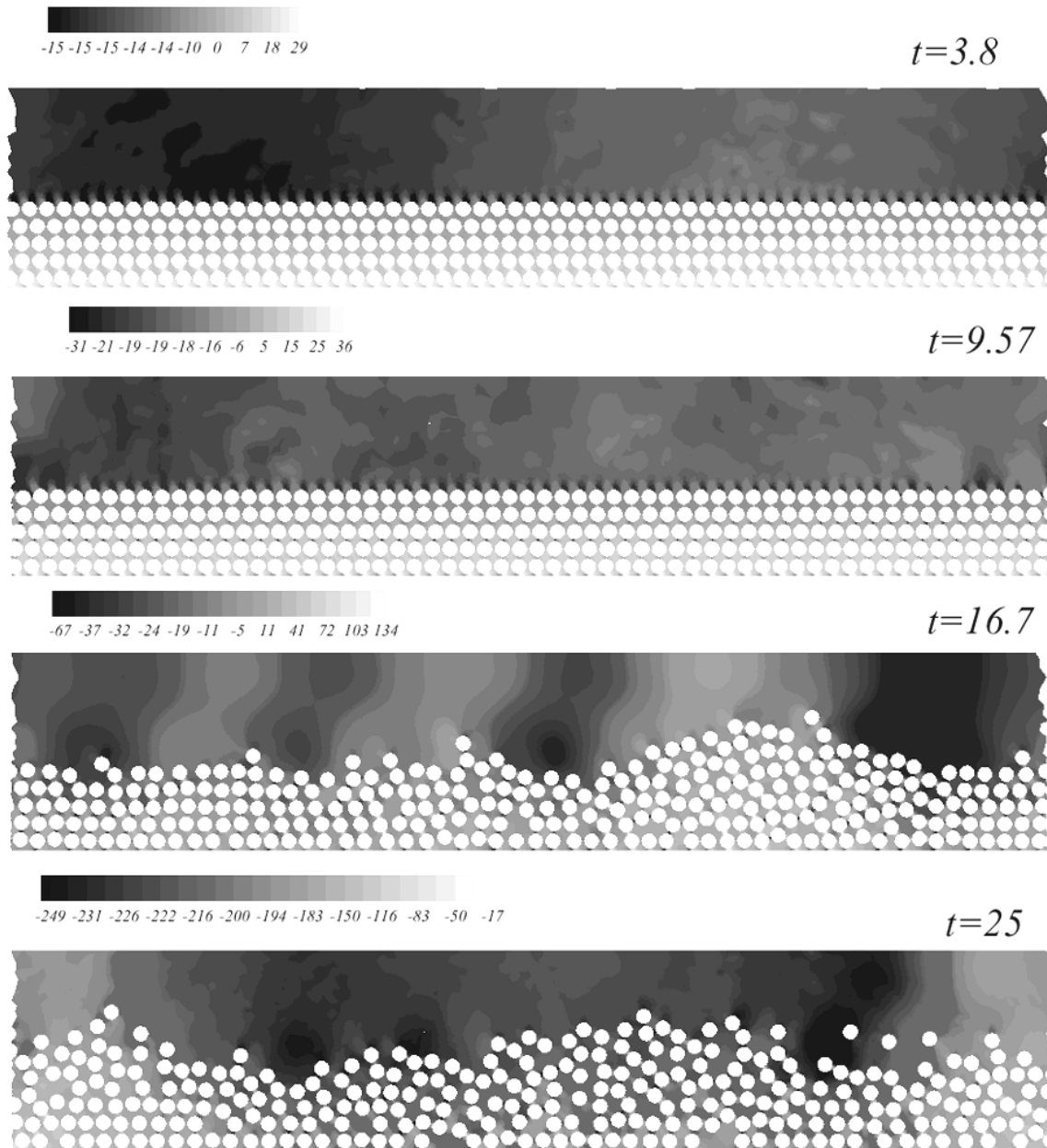


Figure XIV.9. Fluidization of 300 particles ( $\eta = 0.2$  poise,  $R = 150$ ,  $G = 1.63$ ). The final state of the fluidization at  $t = 25$  sec has not fully eroded. The particles that lift out of the bed can be described as saltating. A propagating "interfacial" wave is associated with the propagating pressure wave at  $t = 250$ .

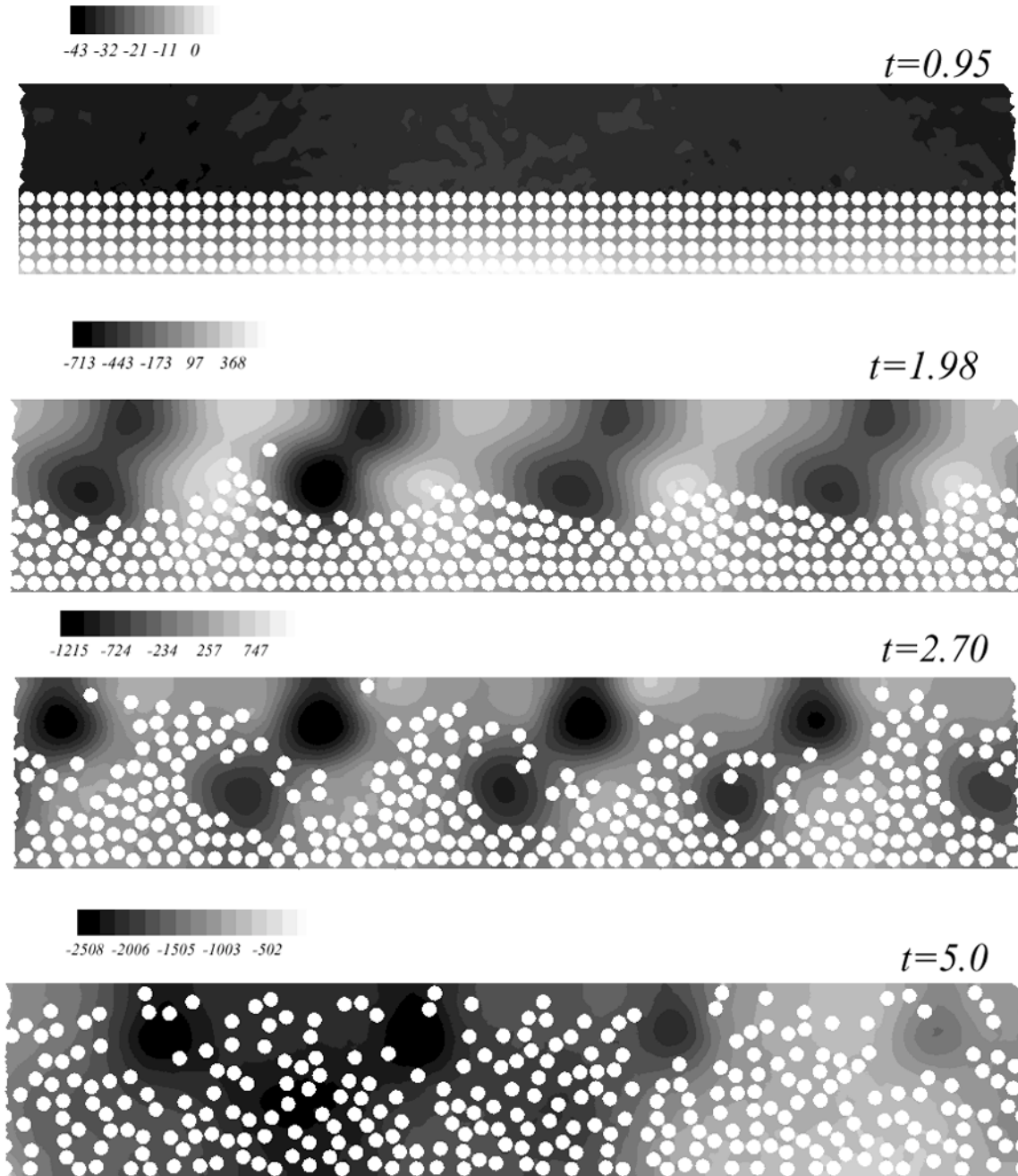


Figure XIV.10. Fluidization of 300 particles ( $\eta = 0.2$  poise,  $R = 450$ ,  $G = 0.54$ ). The flow is from left to right. The particles can be lifted to the top of the channel.

The pressure wave at  $t_0 = 4.204$  sec for the case ( $\eta = 0.2$ ,  $R = 450$ ) in figure XIV.10 is analyzed in figure XIV.11. The period of this wave is  $T = 0.56$  sec and its wavelength is 16 cm. The pressure wave is associated with a wave of solids fraction which could also be described as the passage of internal wave crests and troughs.

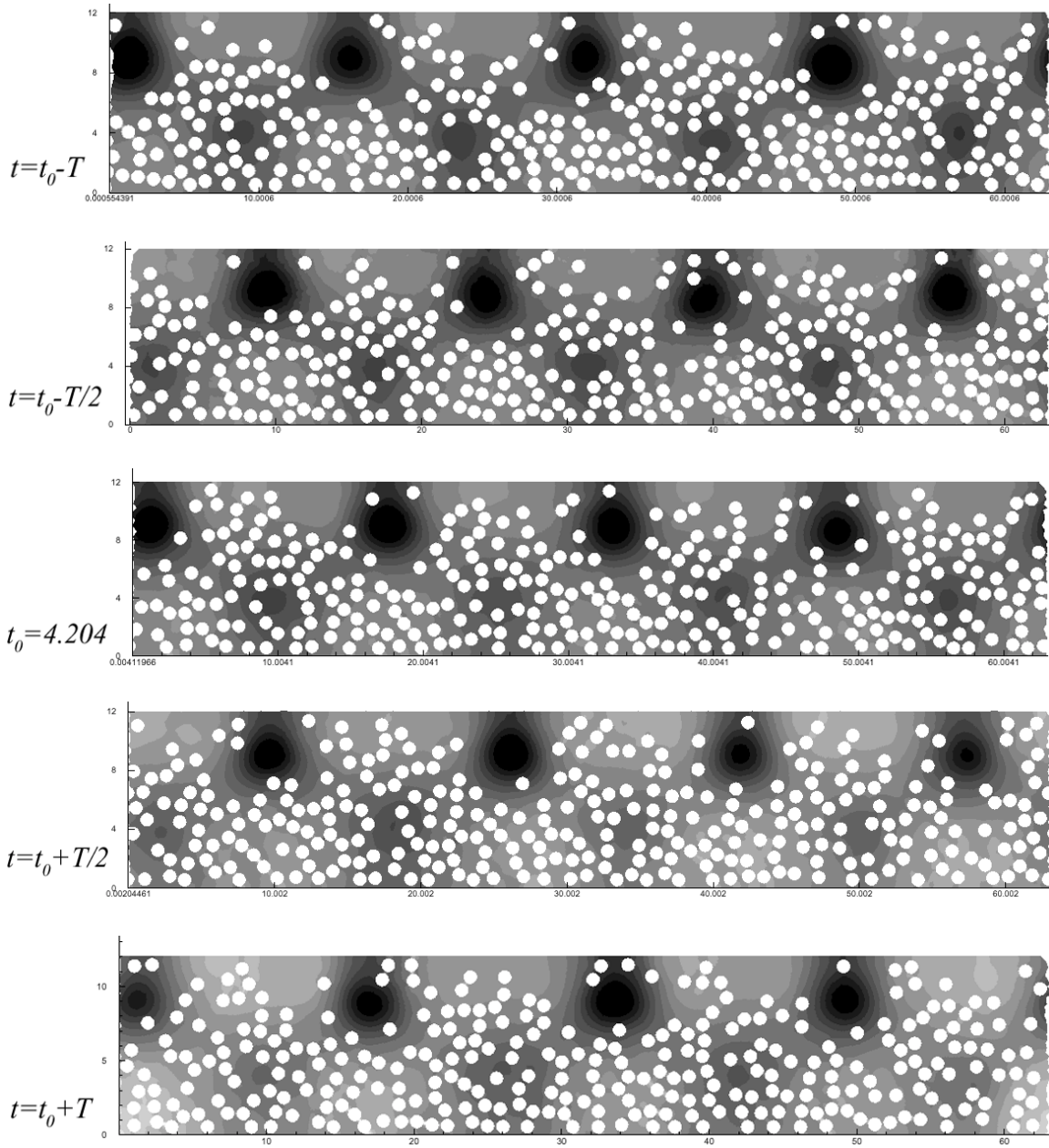


Figure XIV.11. Propagation of the pressure wave centered at  $t_0 = 4.204$  sec in case  $\eta = 0.2$  poise,  $R = 450$  shown in figure XIV.10. The period  $T$  of this wave is  $T = 0.56$  sec and its wavelength is 16 cm.

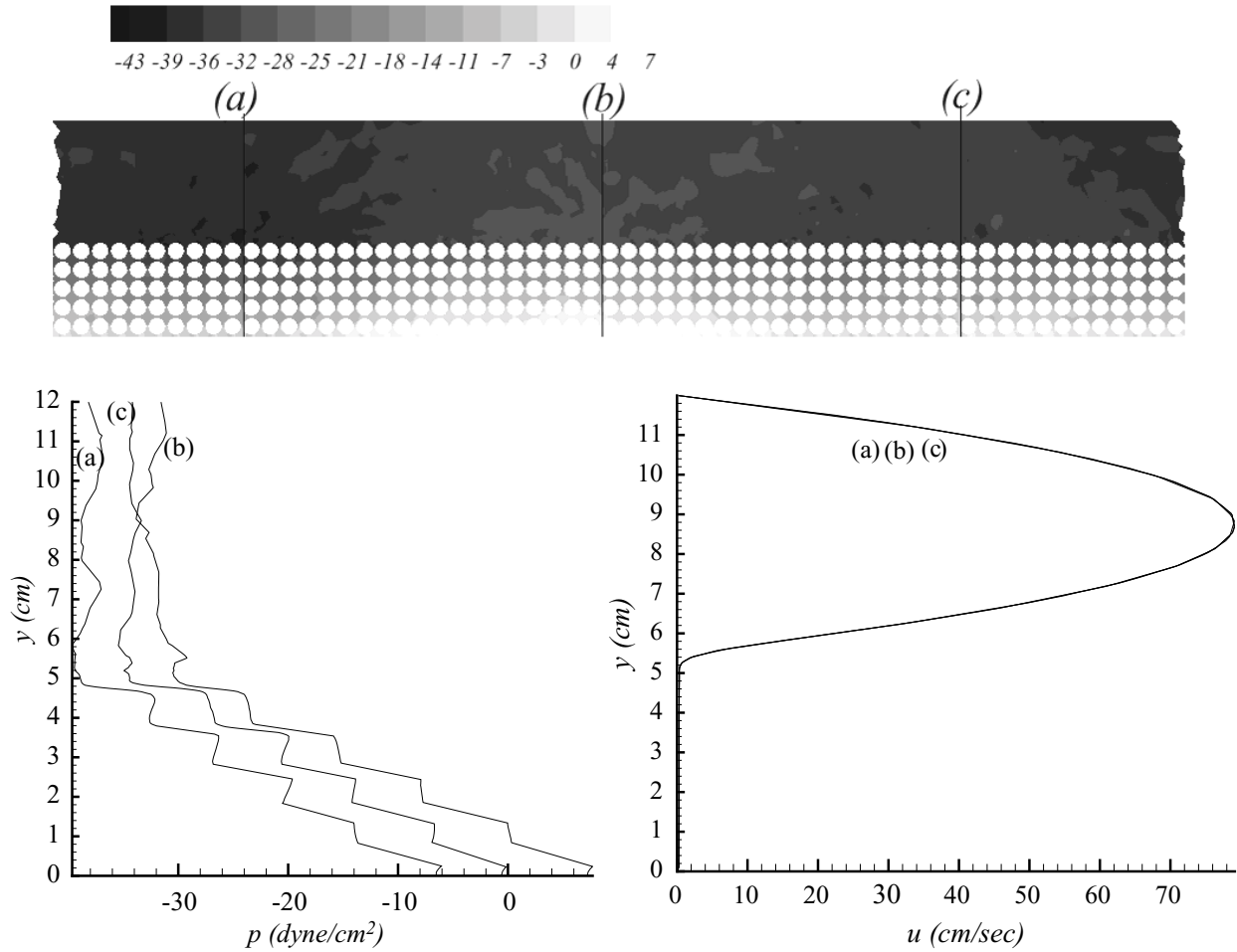


Figure XIV.12. Distribution of dynamic pressure  $p$  and streamwise velocity  $u$  at  $t = 0.1$  sec when  $R = 350$ ,  $\eta = 0.2$  poise. The distribution is basically vertical. This kind of distribution is typical of the earliest times. The vertical steps show how the fluid supports rows of particles. The particles move forward together, with only a small velocity.

Figures XIV.12 and XIV.13, like figures XIV.6 and XIV.7, show the evolution of dynamic pressure from an essentially vertical stratification at early time ( $t = 0.1$  sec) to propagating horizontal waves in the fully developed suspension at  $t = 4.95$  sec.



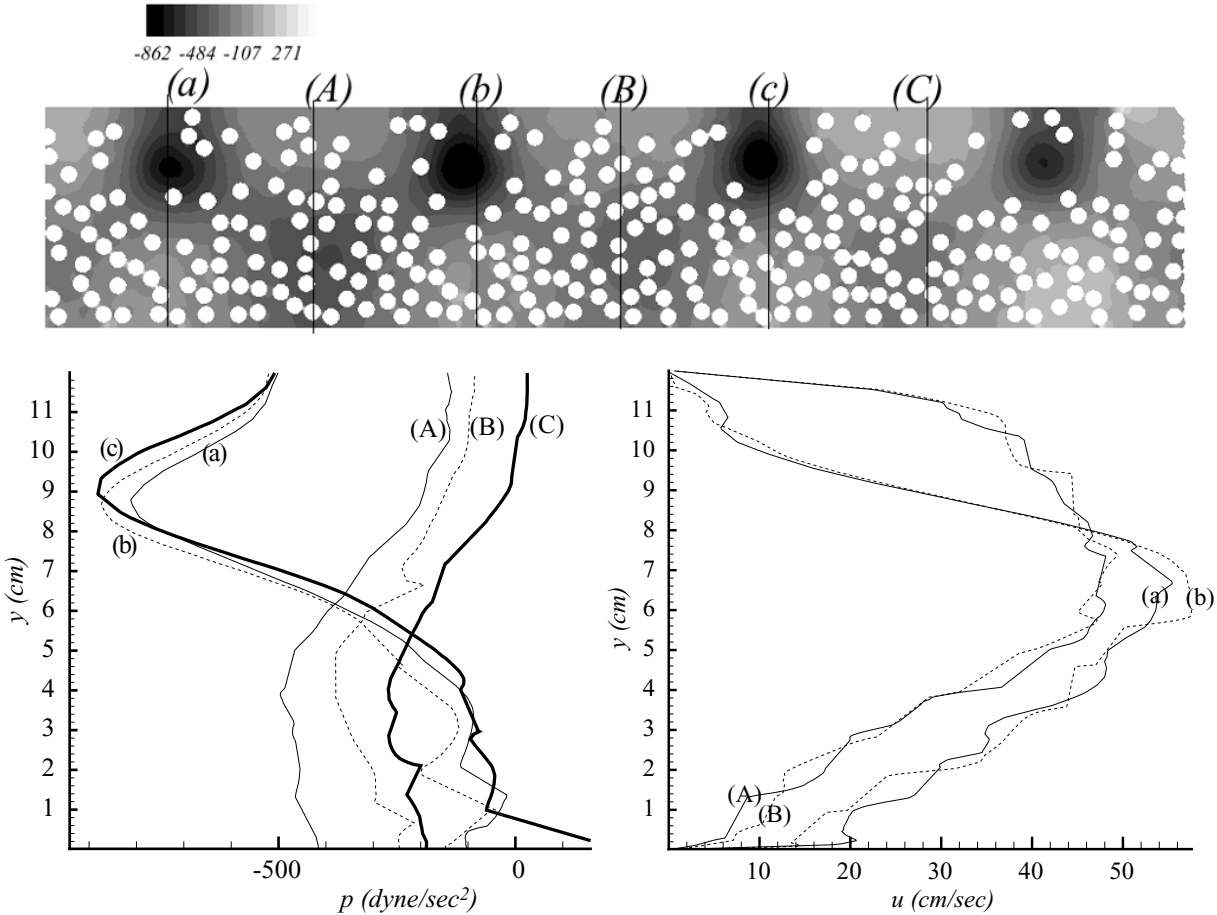


Figure XIV.13. Distribution of dynamic pressure  $p$  and streamwise velocity  $u$  at  $t = 4.95$  when  $R = 350$ ,  $\eta = 0.2$  poise (see figure XIV.10). The pressure and velocity distribution can be associated with "crest" and "trough" propagating void fractions.

▪ **Case 3:  $\eta = 0.1$  poise,  $R_G = 9.81/\eta^2 = 9.8 \times 10^3$**

Figure XIV.4 gives the height of the center of gravity of 300 particles in a liquid with  $\eta = 0.1$  poise as a function of  $R = 6\bar{p}/\eta^2 = 600\bar{p}$ . The interpretation of figure XIV.4 is basically the same as figure XIV.1. Snapshots of the evolution to full fluidization are shown in figures XIV.15 to XIV.21. Notice the overshoot in the bed height at early times. The final bed height is an increasing function of  $\bar{p}$  (or  $R$ ). Data for the simulation after the bed settles down to its final height is shown in table XIV.3.

Table XIV.3. Data for the forward motion of a fluidized suspension of 300 particles after the bed has fully inflated and the average height  $\bar{H}$  of all particles has stopped increasing ( $\eta = 0.1$ ).  $\bar{H} = \bar{H}_0 = 2.65d$  at  $t = 0$ .  $\bar{U}$  and  $\bar{\Omega}$  are the average velocity and angular velocity of the particles.

$R$	$G$	$\bar{p}$	$\bar{H}$	$\bar{U}$	$\bar{\Omega}$
120	8.18	0.2	2.93	2.82	0.32
180	5.45	0.3	3.07	5.21	0.58
300	3.27	0.5	3.2	8.01	0.84
420	2.34	0.7	3.28	12.43	1.31
600	1.64	1.0	3.53	18.92	1.94
1200	0.82	2.0	3.96	43.94	3.67
1800	0.55	3.0	4.34	72.94	4.17

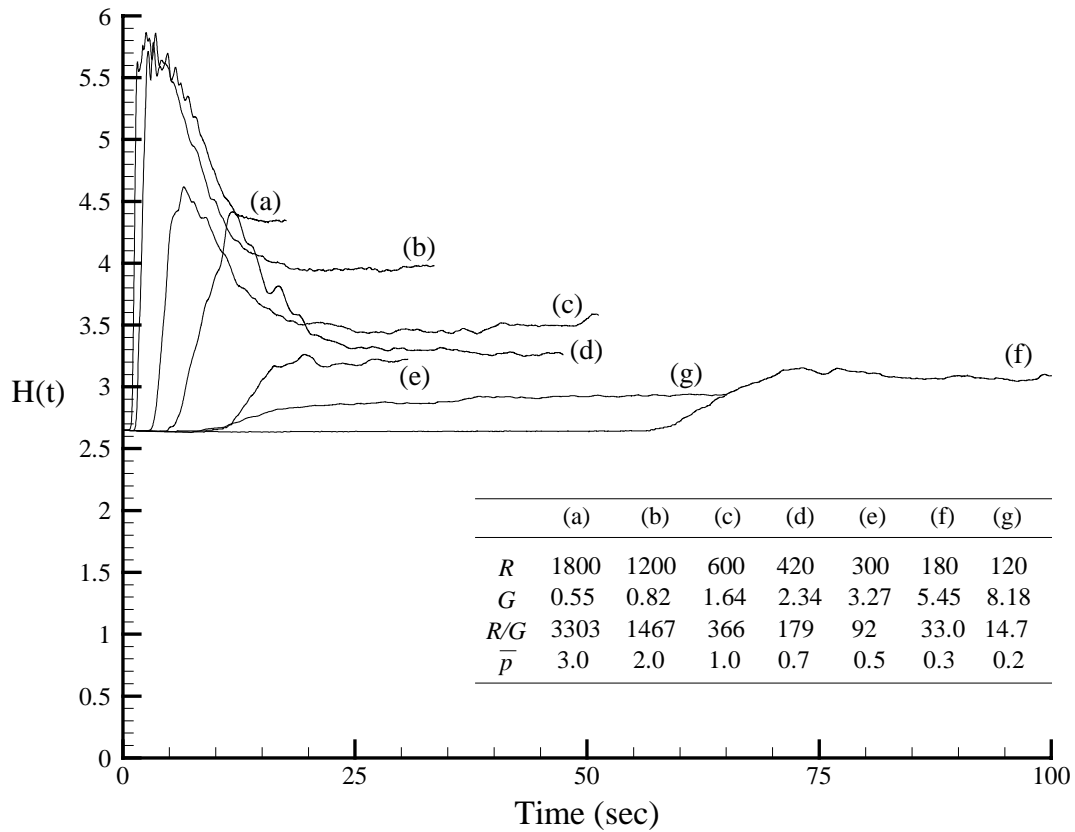


Figure XIV.14. Rise curves for the center of gravity of 300 circular particles fluidized by lift (fluid viscosity = 0.1 poise,  $R = 600 \bar{p}$   $R_G = 9.81/\eta^2 = 9.8 \times 10^3$ ,  $\bar{p}$  is in dynes/cm<sup>2</sup>). Notice the overshoot at early times. The final bed height is an increasing function of  $\bar{p}$ .

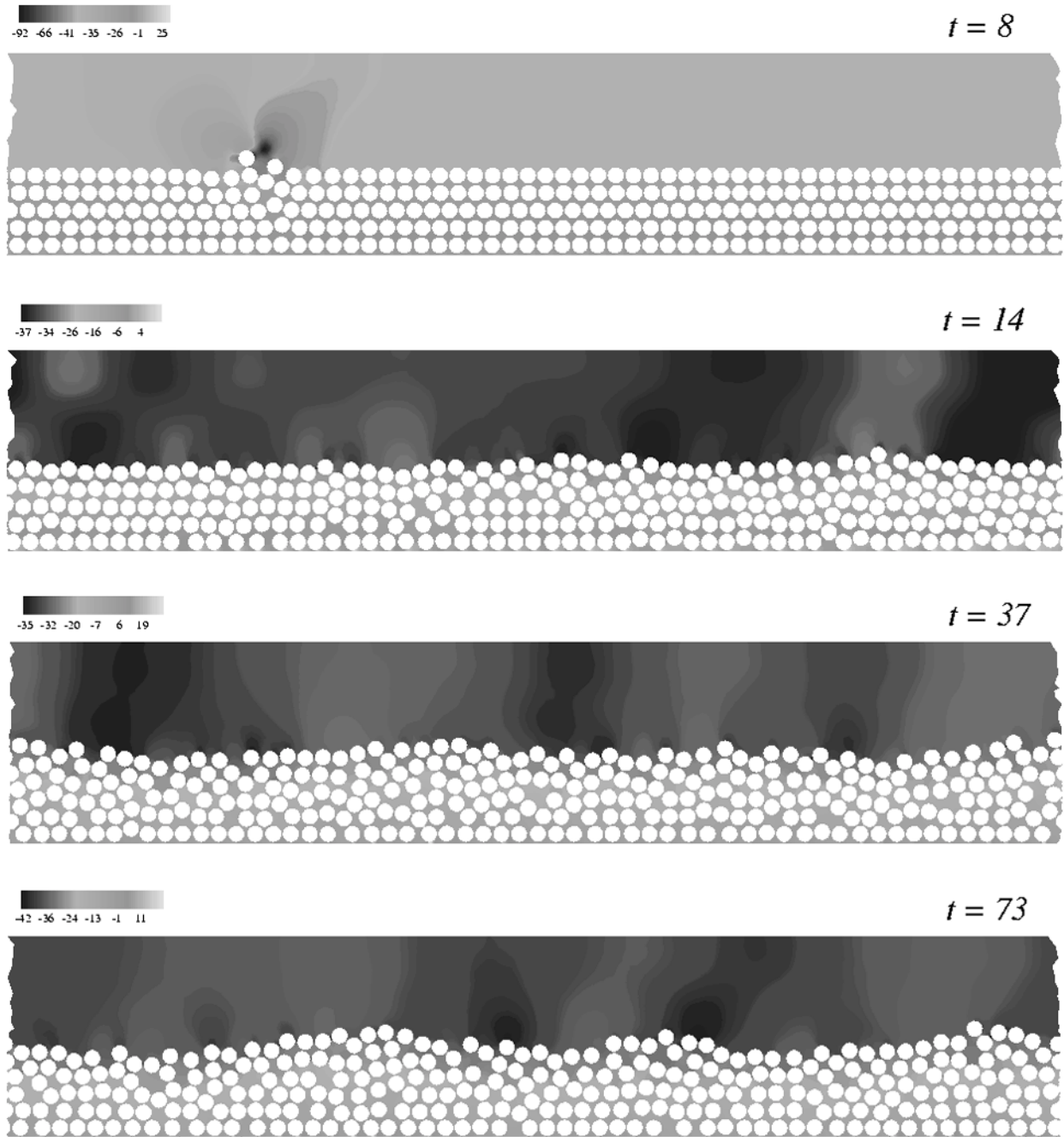


Figure XIV.15. Fluidization of 300 particles ( $\eta = 0.1$  poise,  $R = 120$ ,  $G = 8.18$ ). The flow is from left to right. The bed height rises slightly. The final condition at  $t = 73$  can be described as a partially eroded bed.

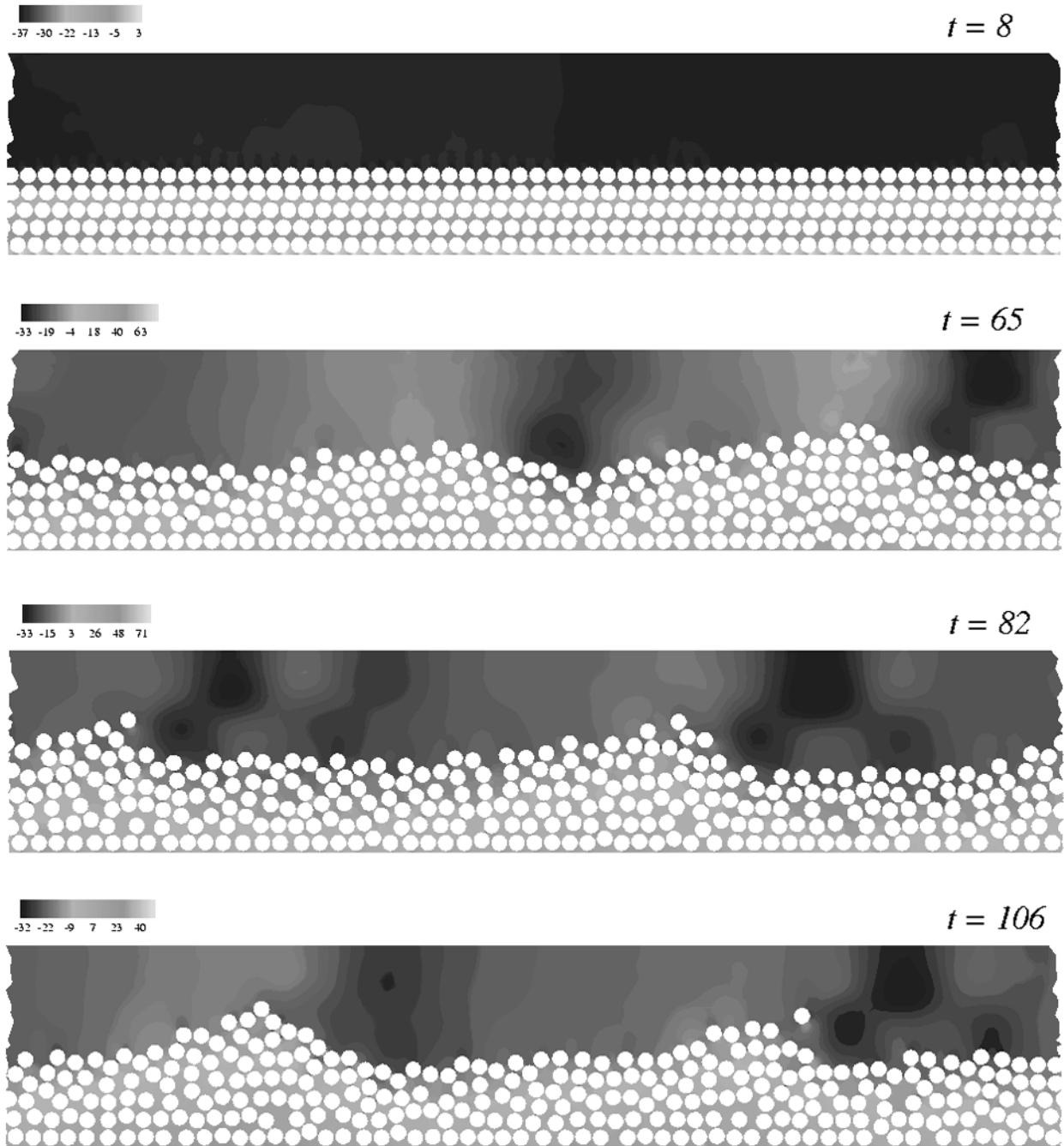


Figure XIV.16. Fluidization of 300 particles ( $\eta = 0.1$  poise,  $R = 180$ ,  $G = 5.45$ ). The bed has eroded more. At  $t = 106$  sec. The particles at the top are moved by saltation waves.

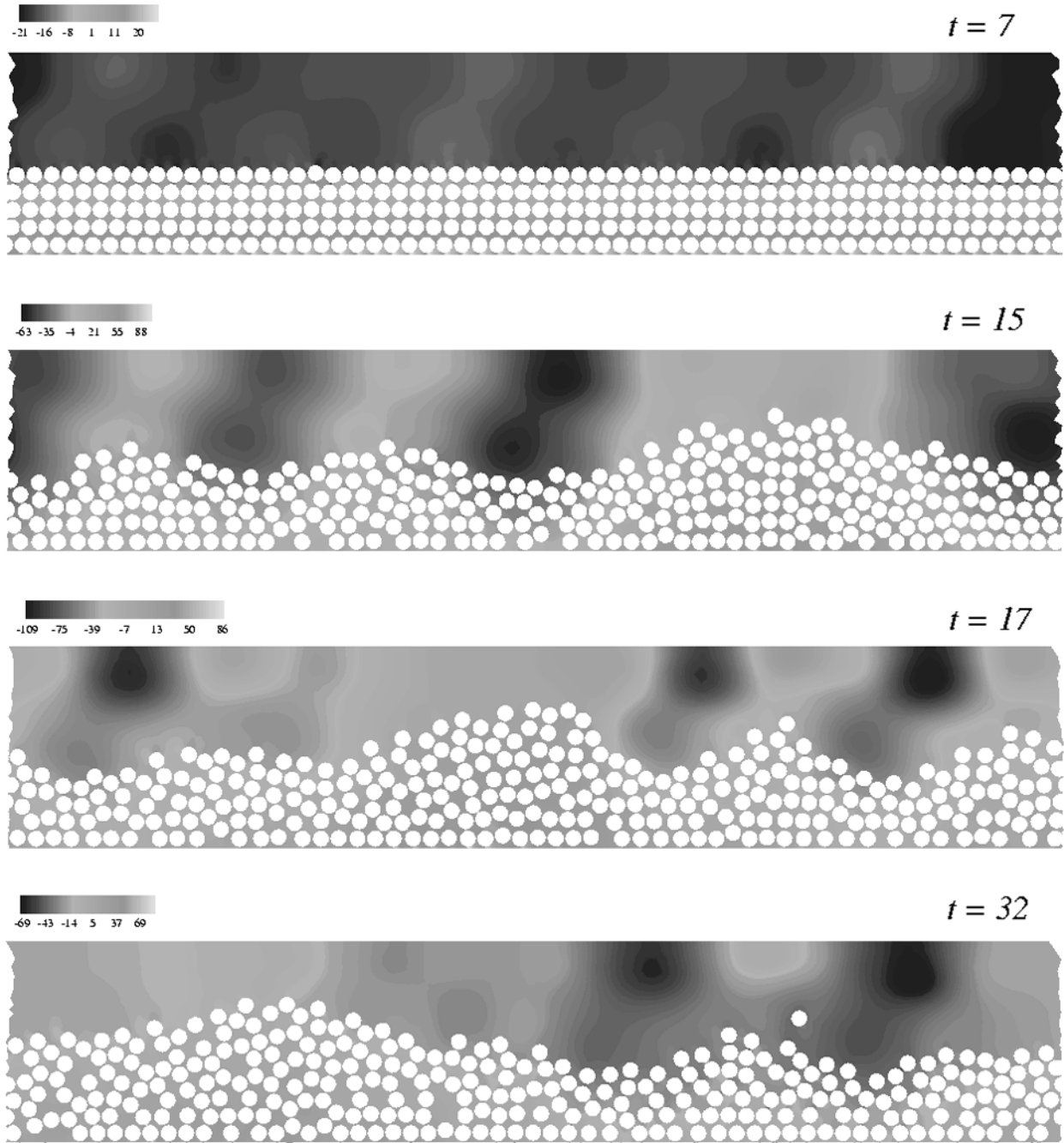


Figure XIV.17. Fluidization of 300 particles ( $\eta = 0.1$  poise,  $R = 300$ ,  $G = 3.27$ ).

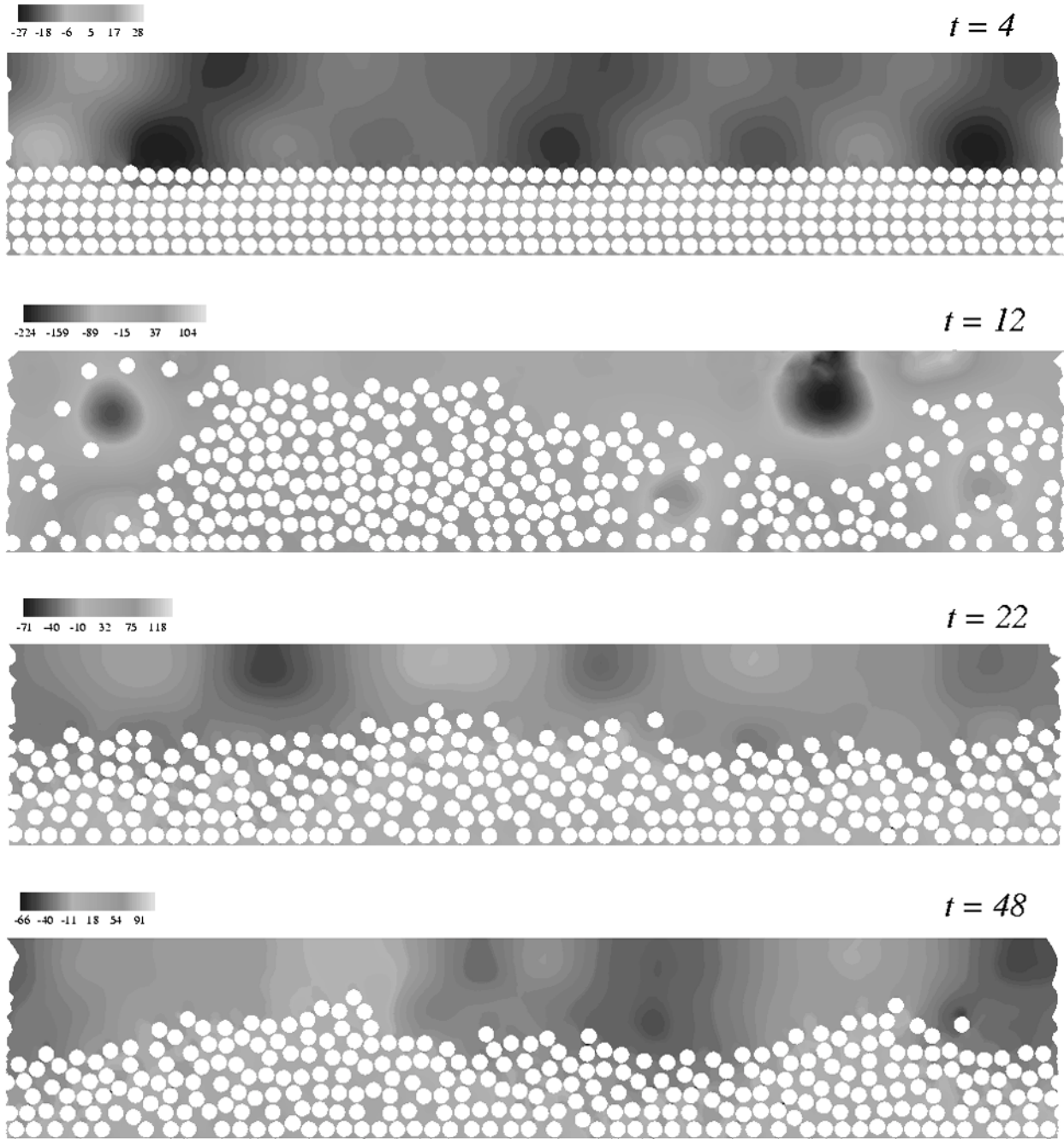


Figure XIV.18. Fluidization of 300 particles ( $\eta = 0.1$  poise,  $R = 420$ ,  $G = 2.34$ ). The flow is from left to right. The bed height increases with  $R$ . At  $R = 420$  the bed has nearly eroded.



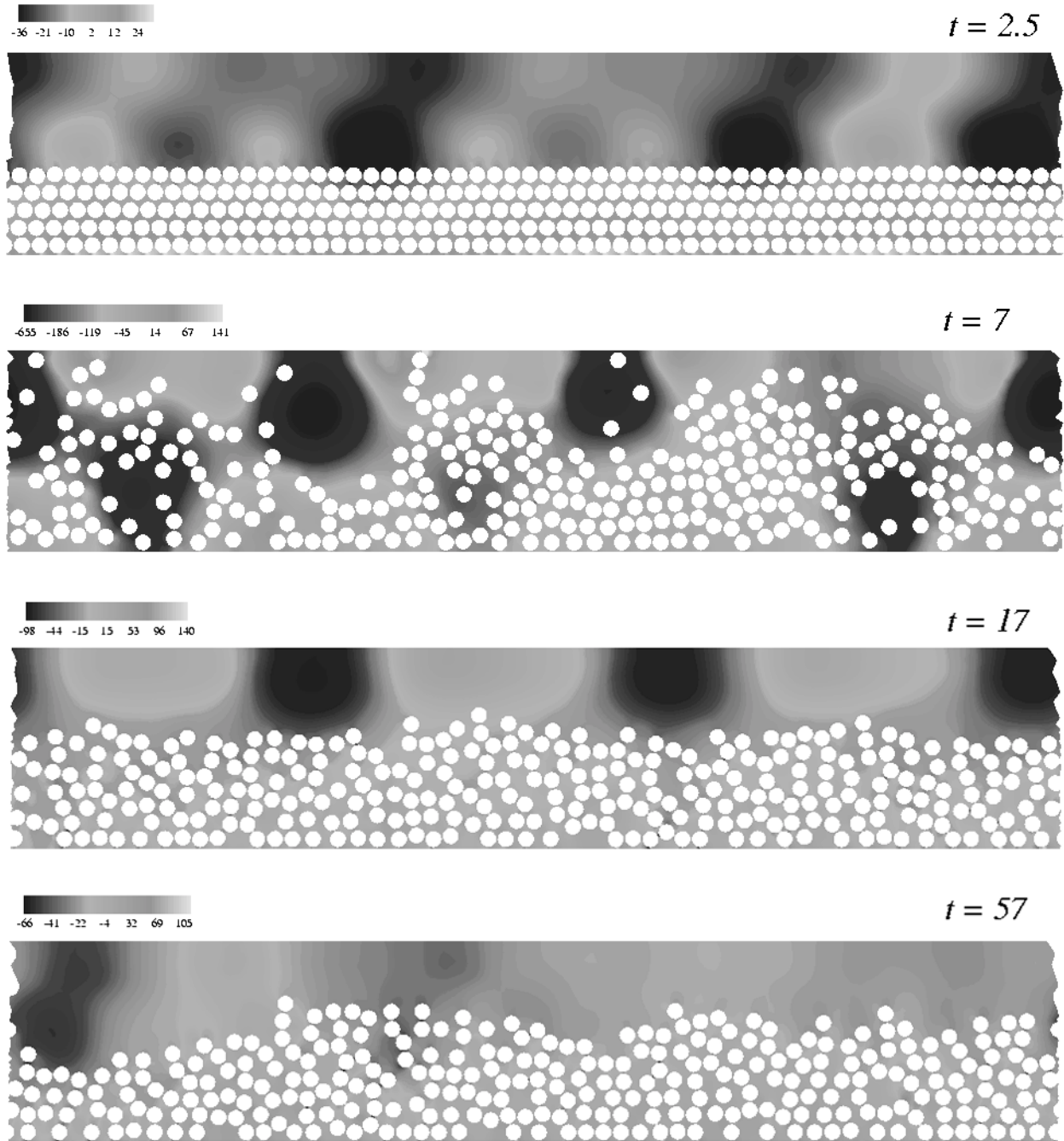


Figure XIV.19. Fluidization of 300 particles ( $\eta = 0.1$  poise,  $R = 600$ ,  $G = 1.64$ ). The fluidization of the particle is accomplished by pressure waves. At  $R = 600$  even the bottom row of particles has eroded.

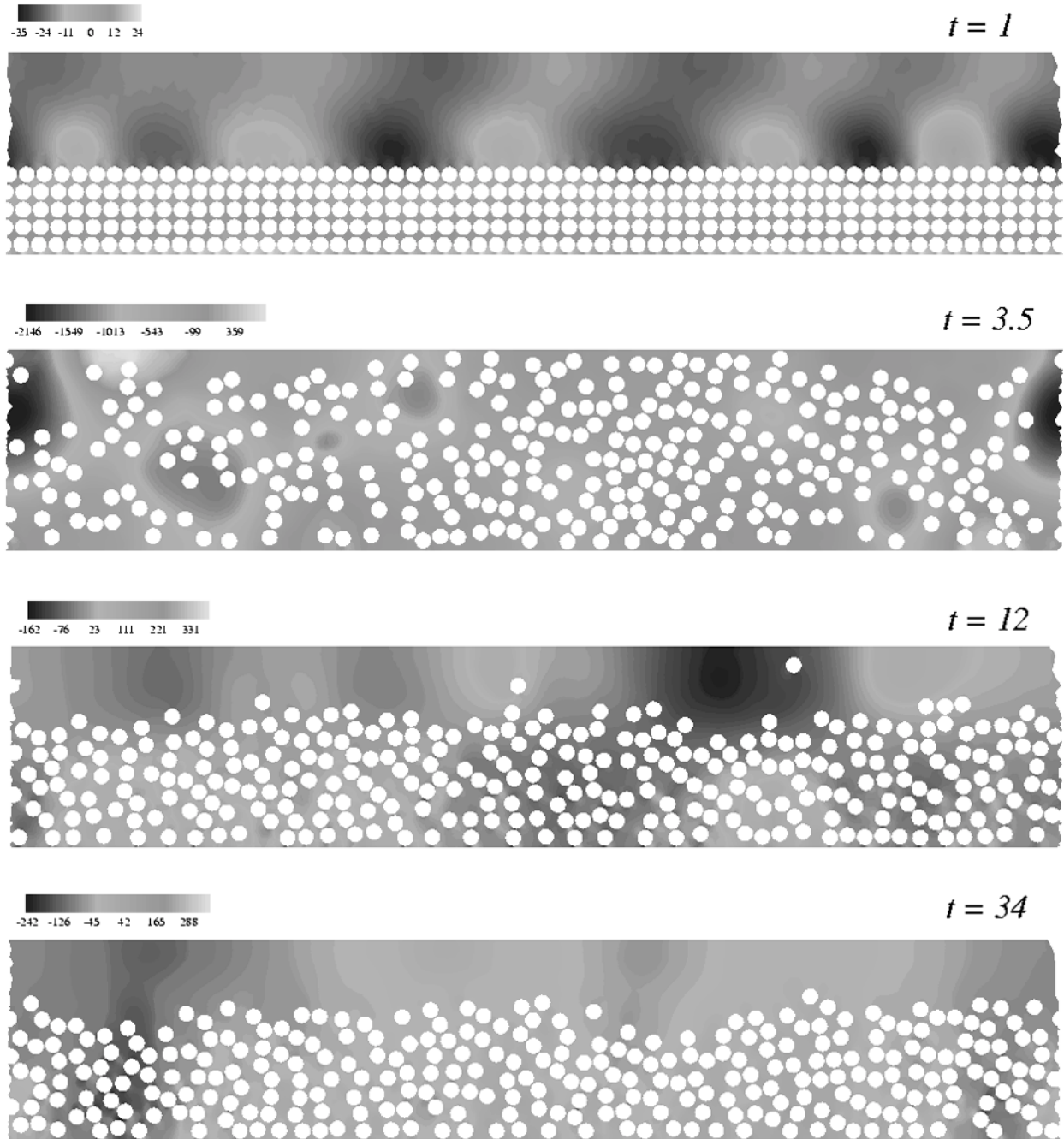


Figure XIV.20. Fluidization of 300 particles ( $\eta = 0.1$  poise,  $R = 1200$ ,  $G = 0.82$ ). The flow is from left to right. The bed rises at  $t = 3.5$  to the top wall where  $\dot{\gamma}$  for the flow without particles is negative. The environment at the top wall is unfavorable to the maintenance of positive fluid or particle rotations required for lift. The lift then at the top wall must decrease and the bed settle into its final fully eroded condition.

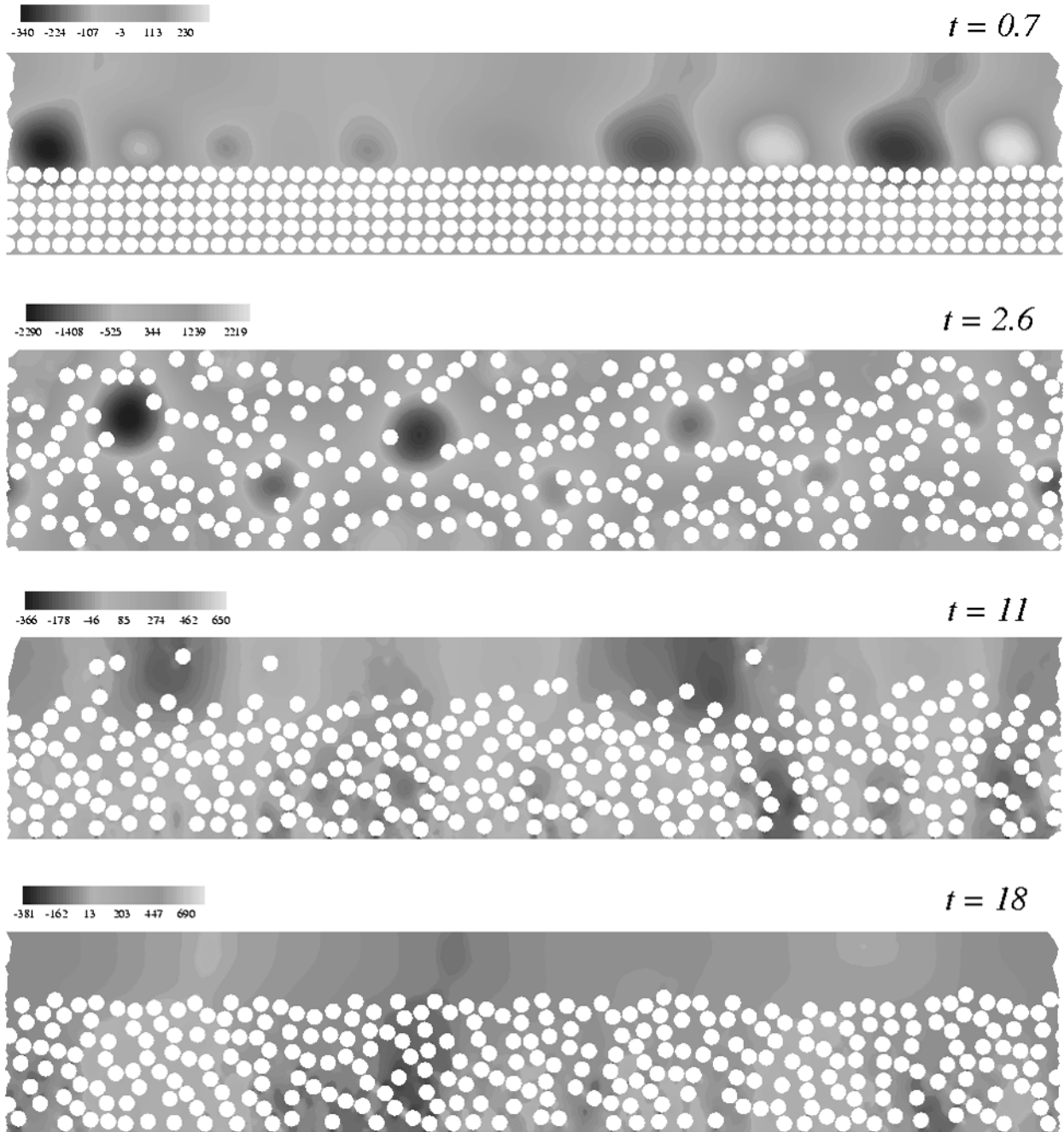


Figure XIV.21. Fluidization of 300 particles ( $\eta = 0.1$  poise,  $R = 1800$ ,  $G = 0.55$ ).

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