XIII Lift off of a single particle in plane-Poiseuille flow of an Oldroyd-B fluid

In this section we study the effect of fluid elasticity on the lift-off of circular particles in plane Poiseuille flow. There is limited literature on this problem.

Krishnan & Leighton 1995 calculated the lift force on a smooth sphere rotating and translating in a simple shear flow in contact with a rigid wall. Hu and Joseph 1999 extended their analysis to second-order fluids. Their results were valid at low Reynolds numbers. The non-dimensional lift R_G for an Oldroyd-B fluid is given by Hu and Joseph 1999

$$
\begin{aligned}
R_G &= \frac{3}{2\pi}\left(1.755R_U^2 + 0.1365R_\Omega^2 - 0.019R_U R_\Omega - 4.522R_U R + 0.303R_\Omega R + 2.314R^2\right) \\
&\quad + \frac{24}{5\varepsilon}E\left(1 - \frac{\lambda_2}{\lambda_1}\right)\left(R_U^2 + 0.25R_\Omega^2 - 0.25R_U R_\Omega\right), \qquad \text{(XIII.1)}
\end{aligned}
$$

$$
R_U = \frac{\rho_f U_p d}{\eta} \text{ and } R_\Omega = \frac{\rho_f \Omega_p d^2}{\eta}.
$$

The above expression is valid in the limit of slow and slowly varying flows so that the second-order fluid expansion is valid. For a freely translating and rotating sphere, R_U and R_Ω are functions of R and the gap size. The above calculations were performed for semi-infinite domains. Hence *W/d* is not a parameter of the problem.

The expression for the Newtonian case is obtained by substituting $E = 0$ in equation (XIII.1). The resulting expression is valid in the limit of zero gap size. A heavy particle freely translating and rotating in contact with a plane wall in simple shear flow of a Newtonian fluid is lifted from the wall and suspended in the fluid if the shear Reynolds number R is greater than a critical value. After the critical shear Reynolds number the particle rises from the wall to an equilibrium height at which the buoyant weight just balances the upward thrust from the hydrodynamic force. In a Newtonian fluid the case of zero separation distance corresponds to an infinite drag force due to the logarithmic singularities in the lubrication equations for drag and torque. This results in zero translational and rotational velocities of the particle Krishnan & Leighton 1995. For a particle in a viscoelastic fluid the elastic component of the lift force is also singular when the gap between the sphere and the wall approaches zero. This is an important qualitative feature that differentiates the lift force in a Newtonian and a viscoelastic fluid.

N. Patankar, Huang, Ko and Joseph 2001 studied the lift-off of a single particle in Newtonian and viscoelastic fluids by direct numerical simulation. They considered a particle heavier than the fluid driven forward on the bottom of a channel by a plane Poiseuille flow. After a certain critical Reynolds number the particle rises from the wall to an equilibrium height at which the buoyant weight just balances the upward thrust from the hydrodynamic force. A correlation for the critical shear Reynolds number for lift-off was obtained for the particle in Newtonian fluid.

Ko, Patankar and Joseph 2001 obtained lift-off correlations for a circular particle in a Poiseuille flow.

The set up is the same as that studied in Chapter IX and depicted by the cartoon in figure IX.2. The only difference is the dimensionless stress **T** is governed by Oldroyd-B rate equation

$$\mathbf{T} + \mathrm{De}\,\overset{\nabla}{\mathbf{T}} = \left[\mathbf{A} + \frac{\lambda_2}{\lambda_1}\mathrm{De}\,\overset{\nabla}{\mathbf{A}}\right],\ \mathbf{A} = 2\mathbf{D}[\mathbf{u}] \tag{XIII.2}$$

where λ_2 is the retardation time, λ_1 is the relaxation time and

$$\mathrm{De} = \lambda_1 \overset{\circ}{\gamma}_w = \lambda \frac{W}{12\eta}\overline{p} \tag{XIII.3}$$

is the Deborah number. The dimensionless scales are those introduced just below (XI.9) and the original work is by Patankar, Huang, Ko and Joseph 2001. The fluid and solid equations of motions are given by (IX.10) with $\nabla^2\mathbf{u}$ replaced by $\nabla\bullet\mathbf{T}$ and $\{-p\mathbf{1} + 2\eta\mathbf{D}[\mathbf{u}]\}$ replaced by $\{-p\mathbf{1} + \mathbf{T}\}$. Now there are six dimensionless groups

$$\frac{\rho_p}{\rho_f},\ \frac{2d}{W},\ R,\ G,\ \mathrm{De}, \frac{\lambda_1}{\lambda_2} \tag{XIII.4}$$

replacing the four given by (IX.13). For steady flow ρ_p/ρ_f is not an independent parameter. It is convenient to replace De and G with

$$R_G = RG,\ E = \mathrm{De}/\mathrm{R} = \lambda_1\eta/\rho_f d^2 \tag{XIII.5}$$

The gravity Reynolds number R_G and elasticity number E are both independent of $\dot{\gamma}_w$.

Shear thinning effects on lift-off are not reported here but they were considered in the cross-stream migration studies summarized in figure V.5. The effects of shear thinning may be addressed by replacing the constant viscosity in the dimensional form of the Oldroyd-B rate law with $\eta(\dot{\gamma})$ where

$$\frac{\eta\left(\overset{\circ}{\gamma}\right) - \eta_\infty}{\eta_0 - \eta_\infty} = \left[1 + \left(\lambda_3 \overset{\circ}{\gamma}\right)^2\right]^{\frac{n-1}{2}} \tag{XIII.6}$$

is expressed by the Bird-Carreau model, $\eta_0 = \eta\ (0)$ is the zero shear value, $\eta_\infty = \eta(\infty)$ is the plateau value and

$$\overset{\circ}{\gamma} = \sqrt{\frac{1}{2}\mathrm{tri}\,\mathbf{A}^2},\ \mathbf{A} = 2\mathbf{D}[\mathbf{u}]$$

is an invariant form which reduces to $\dot{\gamma} = du/dy$ pure shear. In forming dimensionless equations, η_0 is used as the scale for viscosity.

▪ Neutrally buoyant particle

Figure XIII.1 shows the equilibrium height vs. Deborah number for a neutrally buoyant particle in an Oldroyd-B fluid. The parameters are as specified in the figure. A freely rotating neutrally buoyant particle migrates to an equilibrium radius between the channel centerline and the wall as in the celebrated experiments of Segré & Silberberg 1961, 1962 with the caveat that the equilibrium radius here depends on the elasticity parameter. The particle rises more as the fluid elasticity is increased and non-rotating particles rise even more. In fact, at high enough Deborah numbers a non-rotating particle migrates all the way to the center of the channel; the centerline is then a stable position of equilibrium and the Segré-Silberberg effect does not occur.

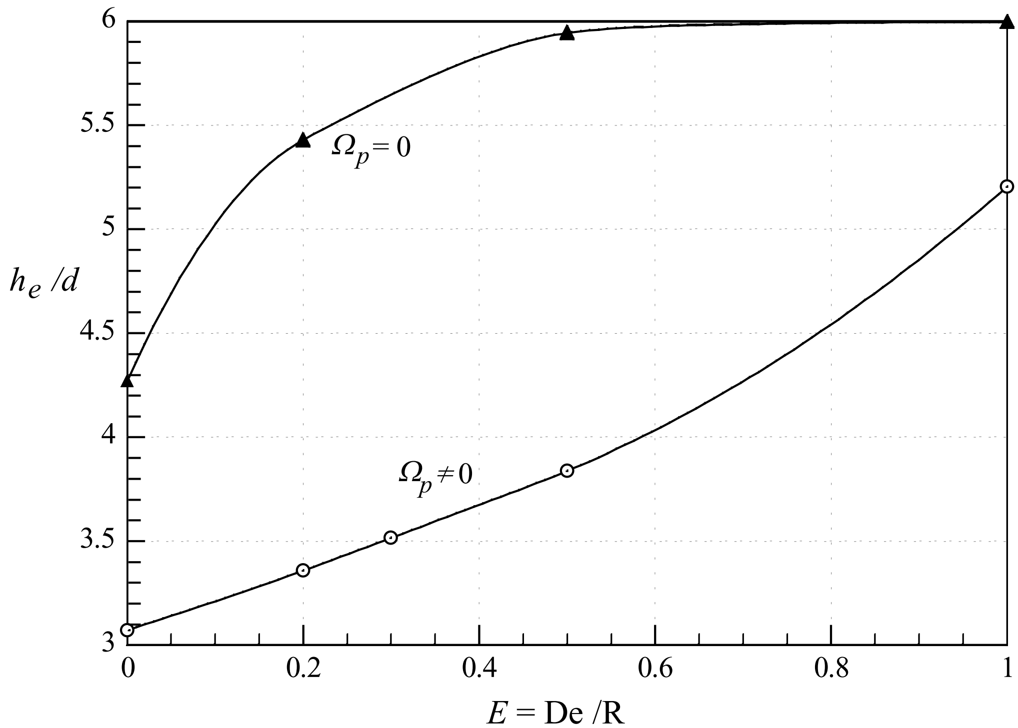

Figure XIII.1 Lift-off of a circular particle from a horizontal wall in a Poiseuille flow of an Oldroyd-B fluid ($W/d = 12$, $l/d = 22$, $\eta = 1.0$ poise, $d = 1.0$ cm, $R = 0.60$).

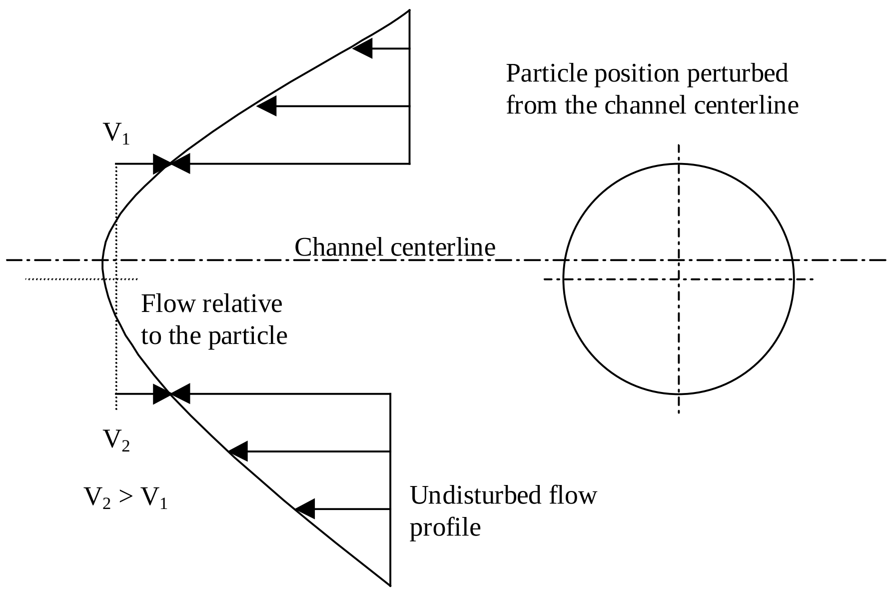

Figure XIII.2 Cartoon showing the fluid velocity relative to a non-rotating particle perturbed from the channel centerline in a plane Poiseuille flow.

Figure XIII.2 shows the flow relative to a non-rotating particle whose position is displaced from the channel centerline. Relative to the particle, the flow comes from the left. In a Newtonian fluid the effect of inertia is to reduce the average pressure on the bottom part of the channel since $V_2 > V_1$. As a result the particle is sucked further away from the channel centerline making it an unstable equilibrium position Feng *et al.* 1994. Joseph 1996 and Joseph & Feng 1996 argued that the elasticity of the fluid gives rise to a compressive stress that is proportional to the square of the shear-rate on the particle surface. This compressive stress acts through the pressure and would push the particle towards the channel centerline. At high enough Deborah numbers this effect would dominate making the channel centerline the stable equilibrium position. Our simulation results agree with this prediction.

Figure XIII.3 compares the effect of shear Reynolds number on the equilibrium height of a particle in a Newtonian and an Oldroyd-B fluid. The critical shear Reynolds number for lift-off in an Oldroyd-B fluid is smaller than that in a Newtonian fluid. The particle rises more at higher shear Reynolds numbers. At a given shear Reynolds number the particle rises more in an Oldroyd-B fluid. The fluid elasticity is seen to enhance the lift on a particle. Figure XIII.4 shows the equilibrium height vs. Deborah number at a fixed R for a heavy particle. A non-rotating particle rises more. In general the lift is seen to be greater at higher Deborah numbers.

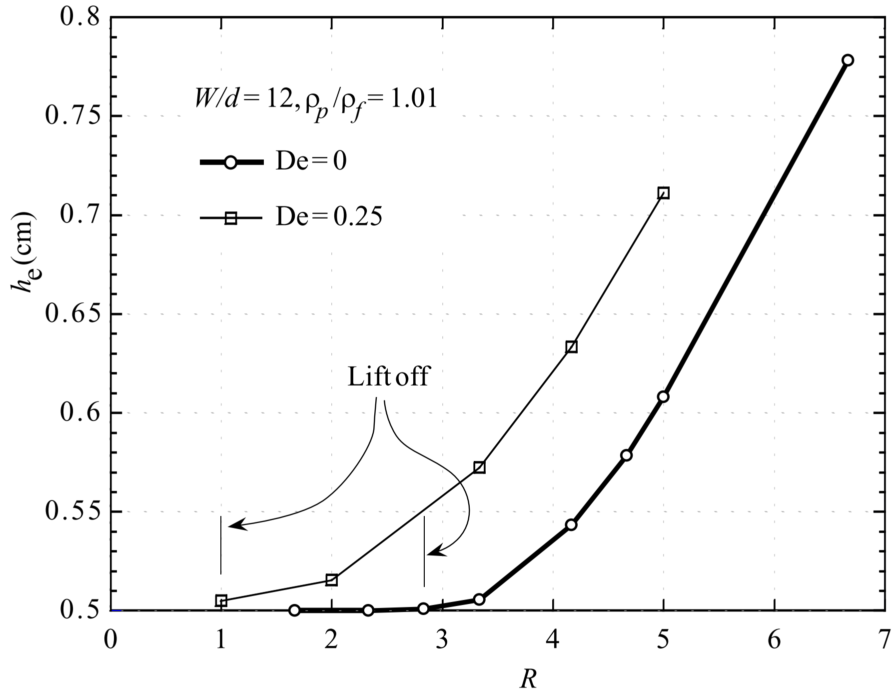

Figure XIII.3 Equilibrium height vs. shear Reynolds number for a particle in a Poiseuille flow of an Oldroyd-B fluid.

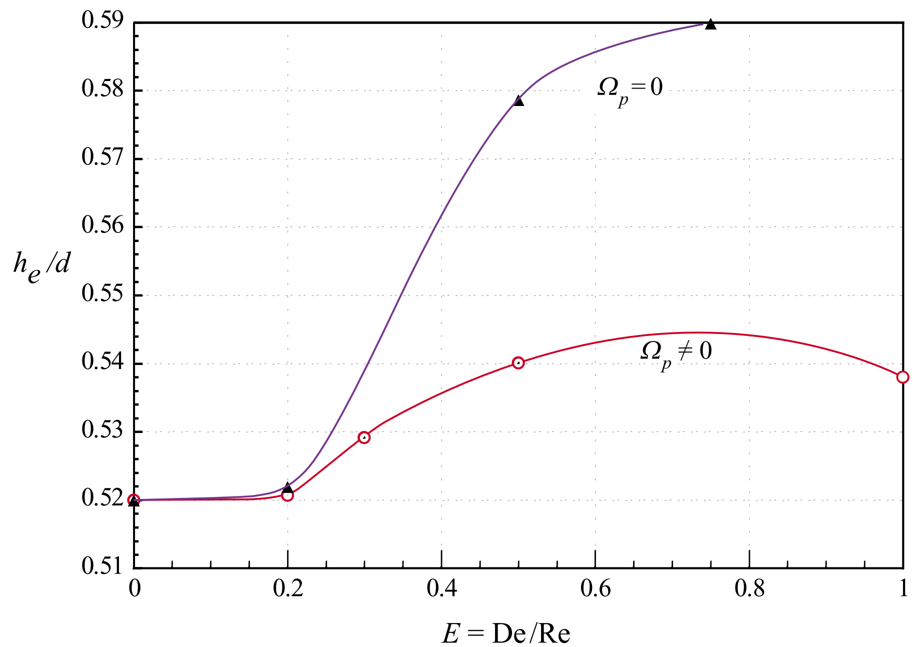

Figure XIII.4 Lift-ff of a circular particle from a horizontal wall in a Poiseuille flow of an Oldroyd-B fluid (W/d = 12, l/d = 22, η = 1.0 poise, d = 1.0 cm, ρ_p = 1.001 g/cc).

Pressure shear and viscoelastic lift forces

The stress at any point in an Oldroyd-B fluid can be decomposed as $\mathbf{T} = -p\mathbf{I}+\eta\mathbf{A}+\boldsymbol{\tau}_e$, where $\boldsymbol{\tau}_e$ is the elastic stress. The elastic component of lift L_e on a particle is given by

$$L_e = \oint_{\partial P} \boldsymbol{\tau}_e \cdot \mathbf{n} \, d\Gamma . \qquad \text{(XIII.7)}$$

Only the tangential (or shear) component of $\boldsymbol{\tau}_e \cdot \mathbf{n}$ on a rigid surface is non-zero for an Oldroyd-B fluid Huang, Hu & Joseph 1998 and Patankar 1997. The lift fractions are defined as

$$\begin{aligned}
\Phi_p &= \frac{L_p}{L_p + L_s + L_e}, \\
\Phi_s &= \frac{L_s}{L_p + L_s + L_e}, \\
\Phi_e &= \frac{L_e}{L_p + L_s + L_e}, \\
\Phi_p &+ \Phi_s + \Phi_e = 1.
\end{aligned} \qquad \text{(XIII.8)}$$

Figure XIII.5 shows the lift fractions vs. shear Reynolds number for the cases shown in figure XIII.3. Maximum contribution to the lift force on a particle in an Oldroyd-B fluid comes from the pressure whereas the elastic stress makes the least contribution. The pressure lift fraction in an Oldroyd-B fluid is typically larger than that in a Newtonian fluid.

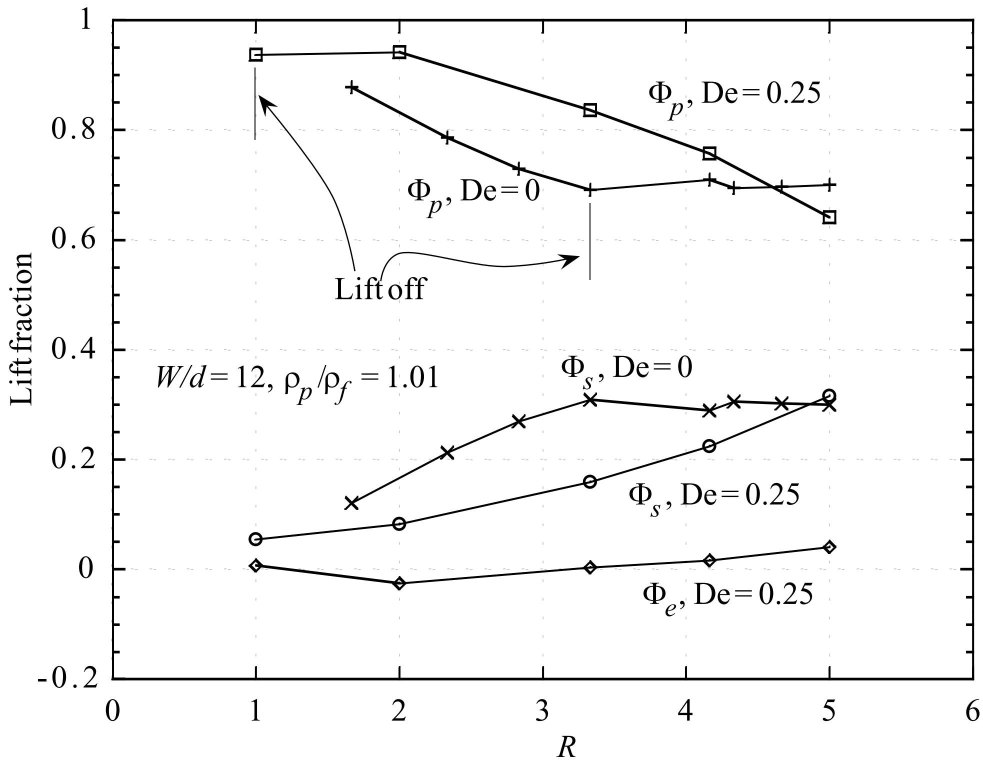

Figure XIII.5. Lift fractions vs. shear Reynolds number for a particle in a Poiseuille flow of an Oldroyd-B fluid.

Figures XIII.6a-6d show that a freely moving particle in a Newtonian fluid does not lift-off at the given parameters whereas it lifts off in an Oldroyd-B fluid under similar conditions. The dominant contribution to the lift force comes from the high pressure in the third quadrant (figure XIII.6d). The additional upward thrust on the particle in the Oldroyd-B fluid comes from the pressure in the bottom half of the particle, where the shear-rate is larger (figure XIII.6d) – in agreement with the argument of Joseph 1996 and Joseph & Feng 1996. Figures XIII.6e and 6f show that the contribution to lift from the viscous shear stress is more for a non-rotating particle than for a freely rotating one. The contribution from the pressure is still dominant.

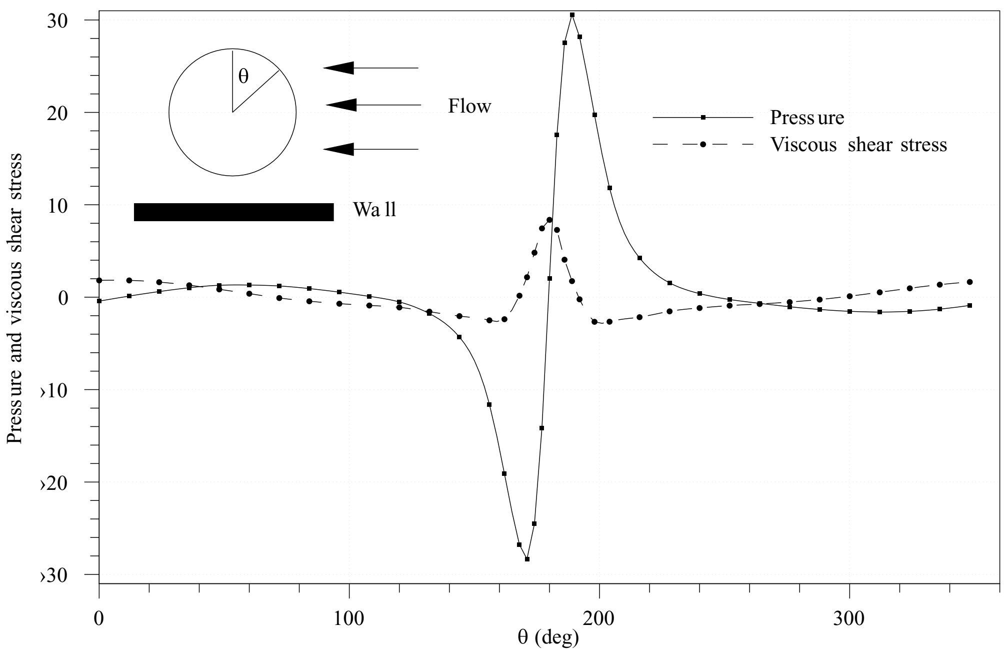

Figure XIII.6(a). Distributions of pressure and viscous shear stress on the surface of a freely rotating circular particle in a Poiseuille flow of a Newtonian fluid. W/d = *12,* L/d = *22,* d = *1.0 cm,* ρ_p/ρ_f *= 1.001,* R = *0.6. The particle does not lift off.*

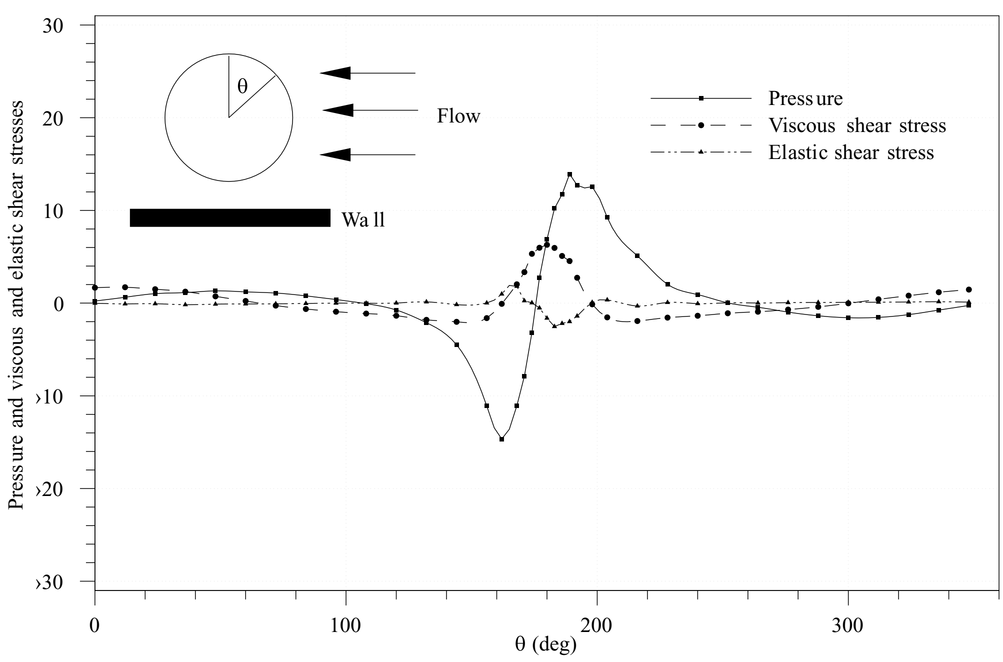

Figure XIII.6(b). Distributions of pressure and viscous and elastic shear stresses on the surface of a freely rotating circular particle in a Poiseuille flow of a Newtonian fluid. W/d = *12,* L/d = *22,* d = *1.0 cm,* ρ_p/ρ_f *=1.001,* R = *0.6,* E = *0.5. The particle lifts off.*

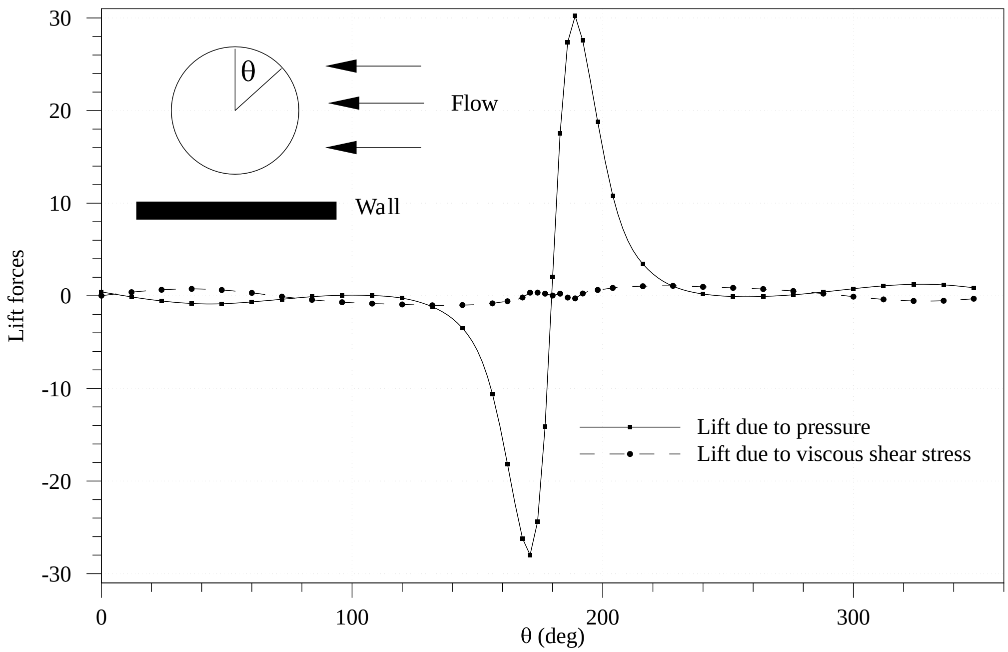

Figure XIII.6(c). The distribution of lift forces on the surface of a freely rotating particle. W/d = *12,* L/d = *22,* d = *1.0 cm,* ρ_p/ρ_f *=1.001,* R = *0.6, Newtonian fluid. The particle does not lift off.*

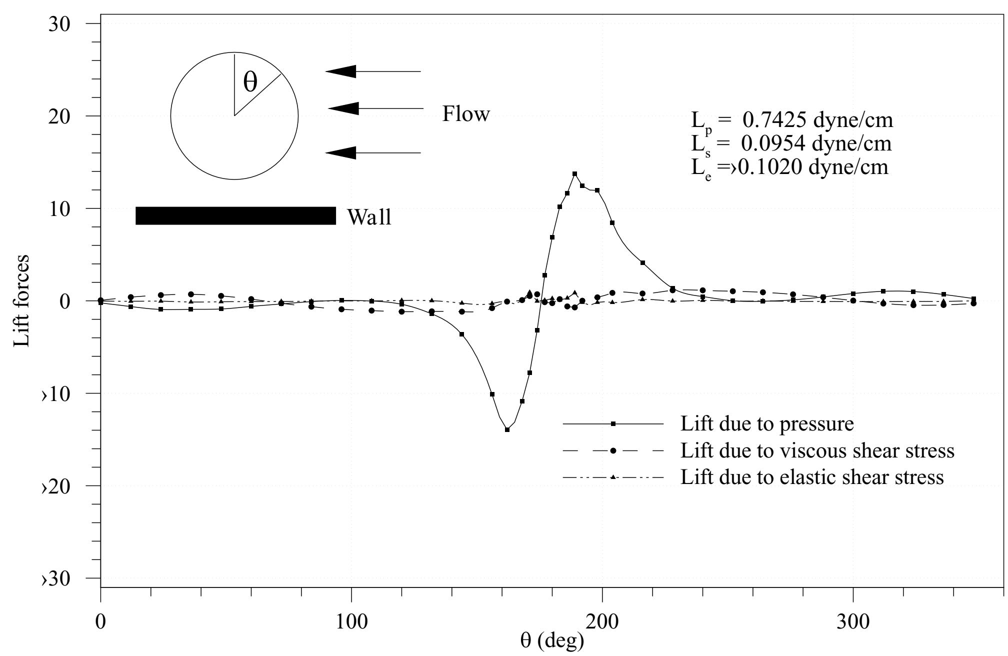

Figure XIII.6(d). The distribution of lift forces on the surface of a freely rotating particle. W/d = *12,* L/d = *22,* d = *1.0 cm,* ρ_p/ρ_f *=1.001,* R = *0.6,* E = *0.5. The particle lifts off.*

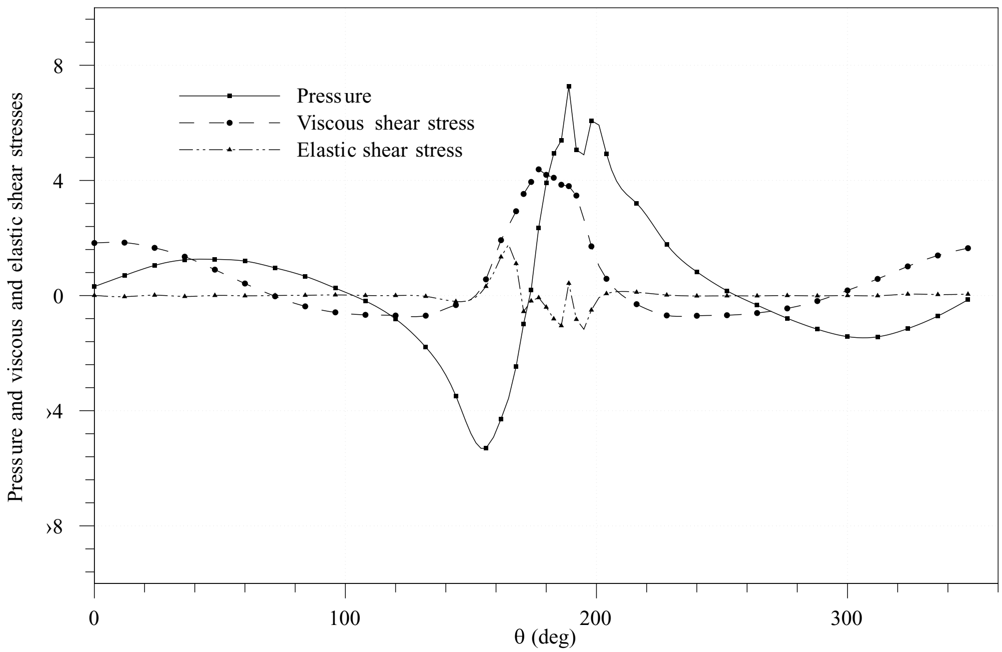

Figure XIII.6(e). Distributions of pressure and viscous and elastic shear stresses on the surface of a lifted circular particle in a Poiseuille flow of an Oldroyd-B fluid. W/d = *12,* L/d = *22,* d = *1.0 cm* ρ_p/ρ_f *= 1.001,* R = *0.6,* E = *0.5,* Ω_p *= 0.*

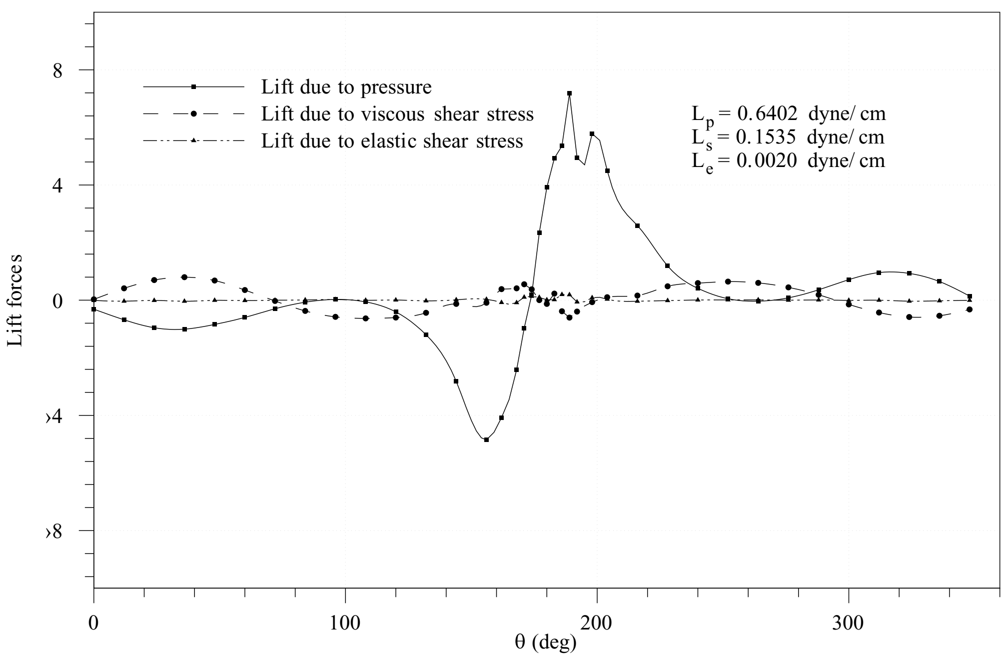

Figure XIII.6(f). The distribution of lift forces on the surface of a lifted circular particle in an Oldroyd-B fluid. W/d = *12,* L/d = *22,* d = *1.0 cm,* ρ_p/ρ_f *=1.001,* R = *0.6,* E = *0.5,* Ω_p *= 0.*

- ## Power law correlations for lift-off in an Oldroyd B fluid.

Correlations like those shown in figure IX.5 were obtained for Oldroyd B fluids by Ko, Patankar and Joseph 2001. The critical lift-off Reynolds number is the minimum shear Reynolds number required to lift a particle to an equilibrium height greater than 0.501*d*; 0.001*d* can be viewed as surface roughness with a dimensionless gap size $\varepsilon = \delta/d$. Figure IX.5 shows that channel width-particle diameter ratio *W/d* is close to asymptotic large *W/d* limit when *W/d* > 12. In the simulations reported below, *W/d* = 12.

Figures XIII.7a and XIII.7b show the plot of R_G vs. the critical shear Reynolds number for lift-off at different values of the elasticity number and λ_2/λ_1, respectively. It is seen that larger *R* is required to lift a heavier particle. The fluid elasticity enhances the lift on the circular particle. The data from the simulations can be represented by a power law equation given by $R_G = aR^n$, where the values of *a* and *n* are given in the figures. We observe that the slopes, *n*, for a Newtonian and an Oldroyd-B fluid are different. The slope does not vary significantly for Oldroyd-B fluids with non-zero elasticity numbers and when $\lambda_2/\lambda_1 \neq 1$. The value of *a* changes as *E* and λ_2/λ_1 changes. This is in agreement with equation (XIII.1).

At low Reynolds numbers (XIII.1) predicts that *a* depends on $E(1-\lambda_2/\lambda_1)$ in a three-dimensional case. Figure XIII.8 shows the plot of *a* vs. $E(1-\lambda_2/\lambda_1)$. We observe an almost linear variation of *a* with respect to $E(1-\lambda_2/\lambda_1)$; in qualitative agreement with the prediction for a three-dimensional case in equation (XIII.1).

In the above simulations the critical shear Reynolds number for lift-off is defined for a given equilibrium height corresponding to $\varepsilon = 0.001$. Equation (XIII.1) predicts that in general R_G is also a function of the gap size. Figures XIII.9a and XIII.9b show the plot of R_G vs. the shear Reynolds number for a Newtonian and an Oldroyd-B fluid, respectively. The parameters are as defined in the figure. The slope *n* does not significantly change with the gap size whereas the value of *a* does depend on ε. Figure XIII.10 compares the variation of *a* vs. ε for the given parameters for a Newtonian and an Oldroyd-B fluid. It is seen that the value of *a* increases rapidly as the gap size tends to zero in an Oldroyd-B fluid suggesting that the lift force is singular in this limit, in qualitative agreement with the theoretical predictions of equation (XIII.1). Such behavior is not observed for the Newtonian case.

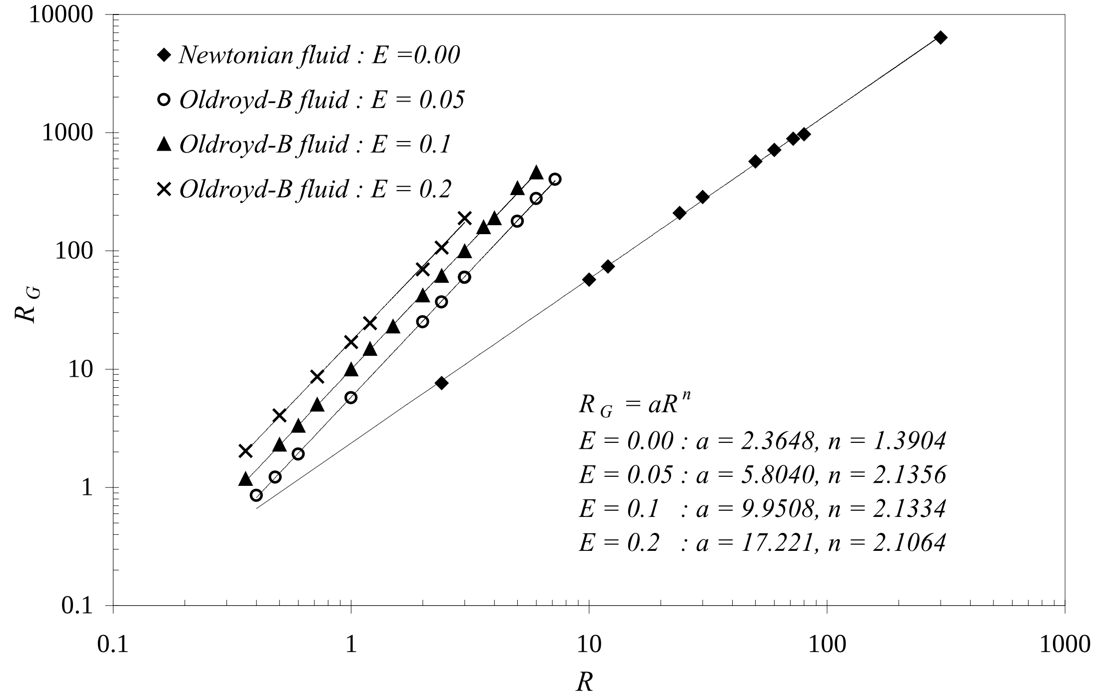

Figure XIII.7(a). The plot of R_G vs. the shear Reynolds number R for lift-off on a logarithmic scale for a Newtonian and an Oldroyd-B fluid at different elasticity numbers. (W/d = 12, ε = 0.001, λ_2/λ_1 = 0.125)

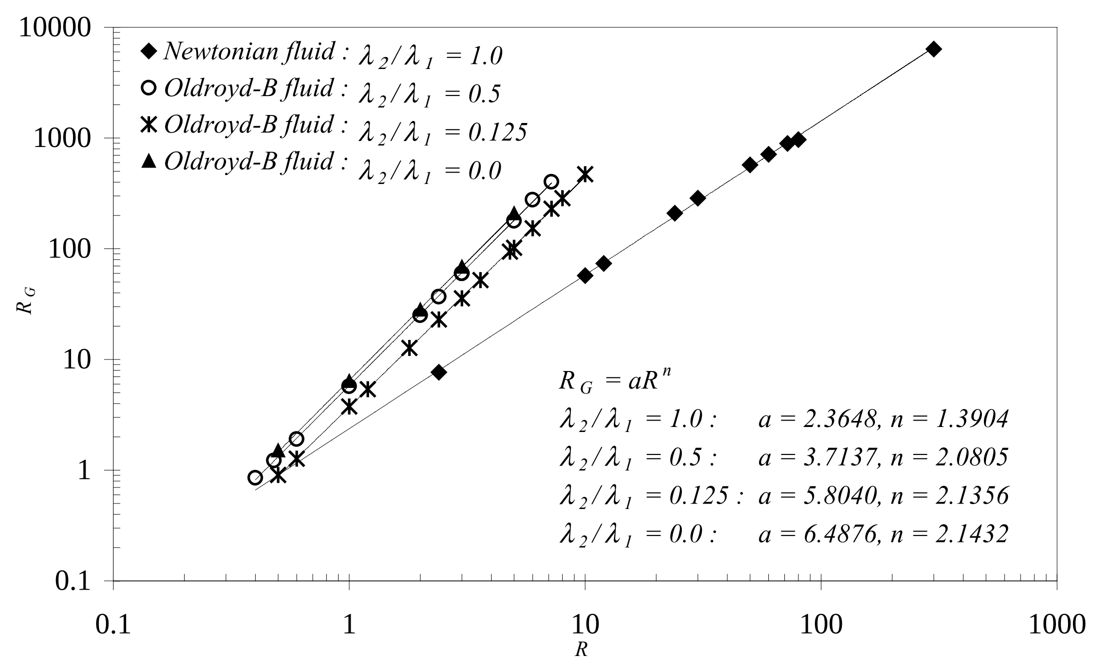

Figure XIII.7(b). The plot of R_G vs. the shear Reynolds number R for lift-off on a logarithmic scale for a Newtonian and an Oldroyd-B fluid at different values of l2/l1. (W/d = 12, ε = 0.001, E = 0.05)

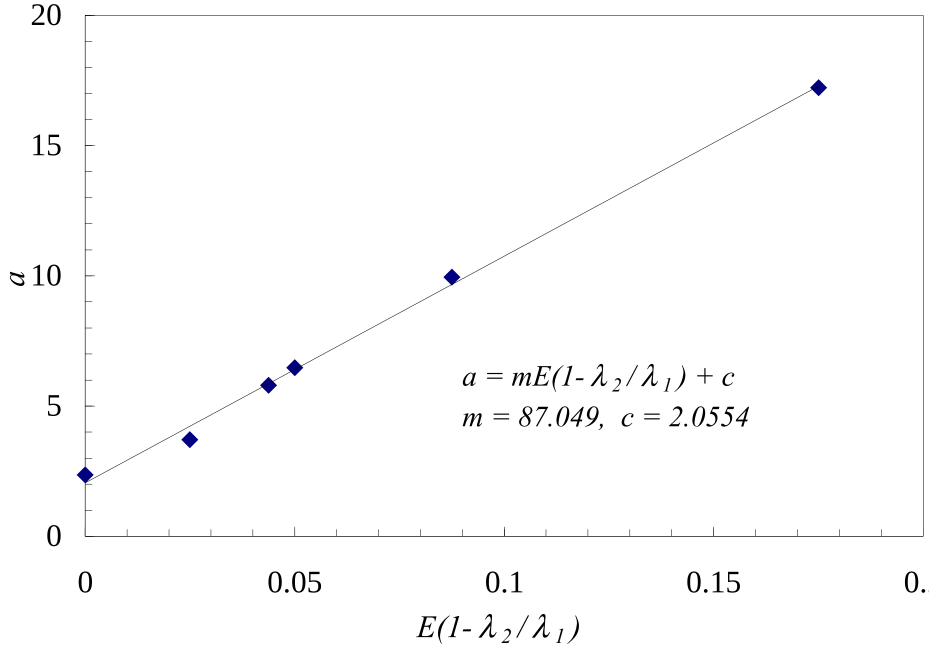

Figure XIII.8. The plot of a *vs.* E*(1-* λ_2/λ_1*). (*W/d *= 12,* ε *= 0.001)*

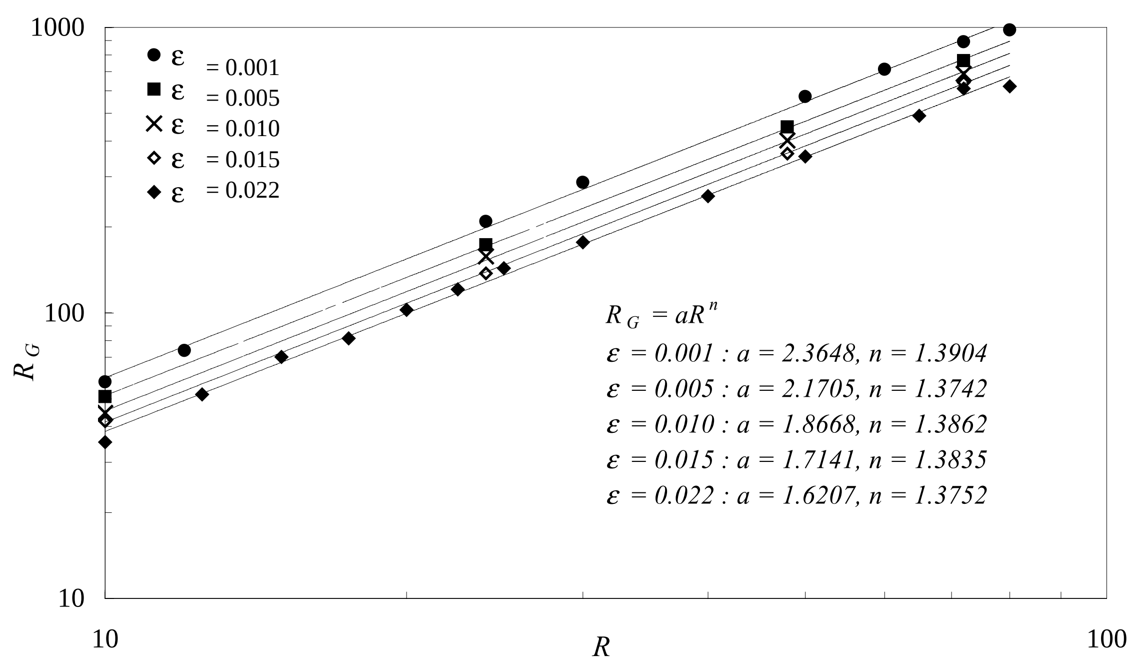

Figure XIII.9(a). The plot of R_G *vs. the critical shear Reynolds number R for lift-off on a logarithmic scale for a Newtonian fluid for various gap sizes (*W/d *= 12).*

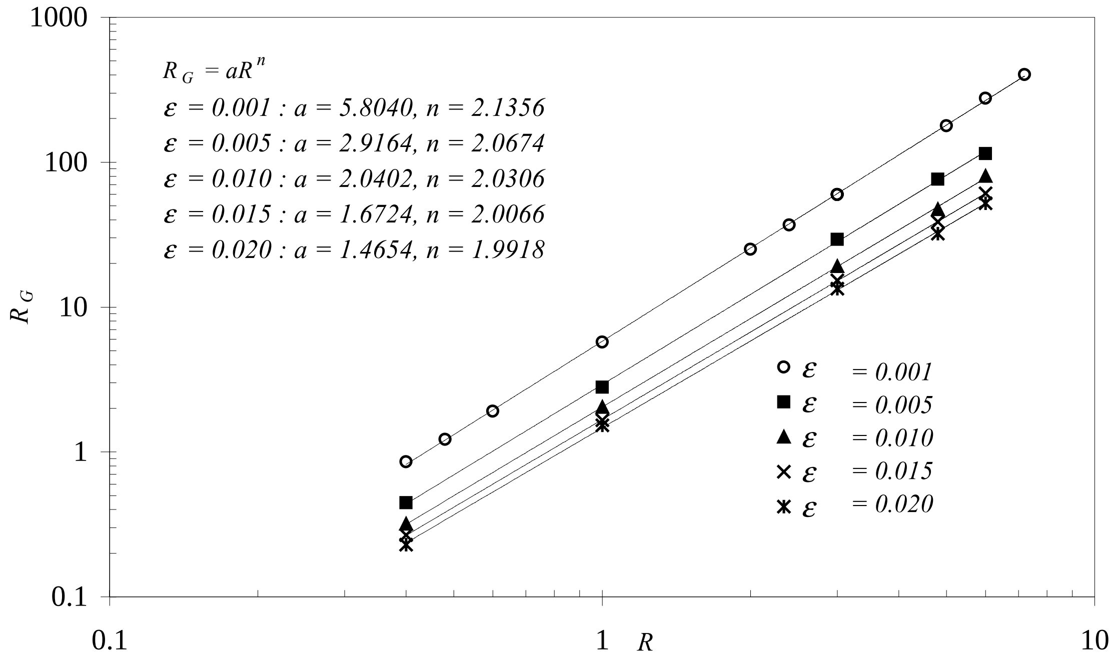

*Figure XIII.9(b). The plot of R_G vs. the critical shear Reynolds number R for lift-off on a logarithmic scale at Oldroyd-B fluid for various gap sizes. (*W/d *= 12, E = 0.05, λ_2/λ_1 = 0.125)*

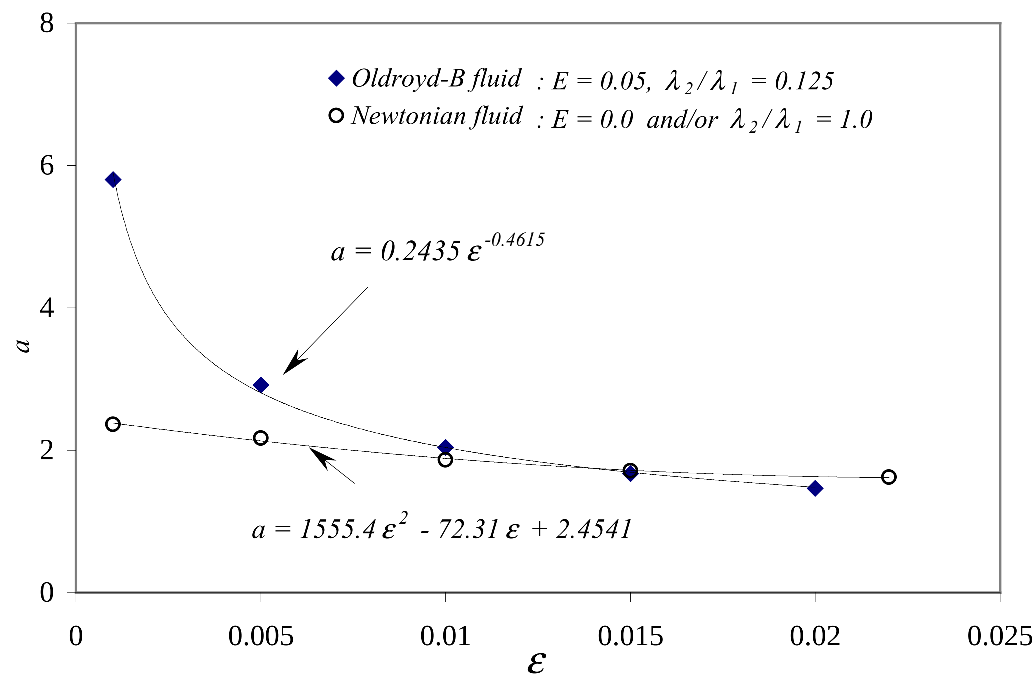

Figure XIII.10. The comparison of the variation of a *vs. ε for a Newtonian and an Oldroyd-B fluid with λ_2/λ_1 = 0.125 and E = 0.05 (*W/d *= 12).*

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