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# **Two-Phase Flows of Rheologically Complex Fluids**

Final Report

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## **Memorandum of Transmittal**

# Final Progress Report

## Foreword

This is the final report for the grant to Daniel D. Joseph, University of Minnesota on “Two-phase Flows of Rheologically-Complex Fluids,” ARO contract DA/DAAH04-95-1-0106. This final report gives a summary of all the works which were produced under this grant.

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## **Research Accomplishments**

This project was funded in March 1, 1995 for work on three problems in two-phase flow.

### **Problem One**

The first problem was to simulate the motions of particles in a viscoelastic fluid in a direct, two-dimensional simulation. The main results are described in the following three abstracts.

#### **Dynamic Simulation of sedimentation of solid particles in an Oldroyd-B fluid**

*J. Feng, P.Y. Huang, D.D. Joseph*

##### **Abstract**

In this paper we present a two-dimensional numerical study of the viscoelastic effects on the sedimentation of particles in the presence of solid walls or another particle. The Navier-Stokes equations coupled with an Oldroyd-B model are solved using a finite-element method with the EVSS formalism, and the particles are moved according to their equations of motion. In a vertical channel filled with a viscoelastic fluid, a particle settling very close to one side wall experiences a repulsion from the wall; a particle farther away from the wall is attracted toward it. Thus a settling particle will approach an eccentric equilibrium position, which depends on the Reynolds and Deborah numbers. Two particles settling one on top of the other attract and form a doublet if their initial separation is not too large. Two particles settling side by side approach each other and the doublet also rotates till the line of centers is aligned with the direction of sedimentation. The particle-particle interactions are in qualitative agreement with experimental observations, while the wall repulsion has not been documented in experiments. The driving force for lateral migrations is shown to correlate with the pressure distribution on the particle's surface. As a rule, viscoelasticity affects the motion of particles by modifying the pressure distribution on their surface. The direct contribution of viscoelastic normal stresses to the force and torque is not important.

## **Direct simulation of the motion of solid particles in Couette and Poiseuille flows of viscoelastic fluids**

*P.Y. Huang, J. Feng, H.H. Hu and D.D. Joseph*

### **Abstract**

This paper reports the results of direct numerical simulation of the motion of a two-dimensional circular cylinder in Couette flow and Poiseuille flow of an Oldroyd-B fluid. Both neutrally buoyant and non-neutrally buoyant cylinders are considered. The cylinder's motion and the mechanisms which cause the cylinders to migrate are studied. The stable equilibrium position of neutrally buoyant particles varies with inertia, elasticity, shear-thinning and the blockage ratio of the channel in both shear flows. Shear-thinning promotes the migration of the cylinder to the wall while inertia causes the cylinder to migrate away from the wall. The cylinder moves closer to the wall in a narrower channel. In a Poiseuille flow, the effect of elastic normal stresses is manifested by an attraction toward the nearby wall if the blockage is strong. If the blockage is weak, the normal stresses act through the curvature of the inflow velocity profile and generate a lateral force that points to the centerline. In both cases, the migration of particles is controlled by elastic normal stresses which in the limit of slow flow in two dimensions are compressive and proportional to the square of the shear rate on the body. A slightly buoyant cylinder in Couette flow migrates to an equilibrium position nearer the centerline of the channel in a viscoelastic fluid than in a Newtonian fluid; On the other hand, the same slightly buoyant cylinder in Poiseuille flow moves to a stable position farther away from the centerline of the channel in a viscoelastic fluid than in a Newtonian fluid. Marked effects of shear thinning are documented and discussed.

## **Direct simulation of the sedimentation of elliptic particles in Oldroyd-B fluids**

*P.Y. Huang, H.H. Hu and D.D. Joseph*

### **Abstract**

Cross stream migration and stable orientations of elliptic particles falling in an Oldroyd-B fluid in a channel are studied. We show that the normal component of the extra stress on a rigid body vanishes; lateral forces and torques are determined by the pressure. Inertia turns the longside of the ellipse across the stream and elasticity turns it along the stream; tilted off-center falling is unstable. There are two critical numbers; elasticity and Mach numbers. When the elasticity number is smaller than critical the fluid is essentially Newtonian with broadside-on falling at the centerline of the channel. For larger elasticity numbers the settling turns the longside of the particle along the stream in the channel center for all velocities below a critical one, identified with a critical Mach number of order one. For larger Mach numbers the ellipse flips into broadside-on falling again. The critical numbers are functions of the channel blockage ratio, the particle aspect ratio and the retardation/relaxation time ratio of the fluid. Two ellipses falling nearby, attract, line-up and straighten-out in a long chain of ellipses with longside vertical, all in a row. Stable, off-center tilting is found for ellipses falling in shear thinning fluids and for cylinders with flat ends in which particles tend align their longest diameter with gravity.

### **Problem Two**

The second problem was to identify new physics that are revealed about the motion and mixing when account is taken of the fact that the density of mixtures of incompressible liquids must change when they are diffusing. These ideas are thoroughly explored in the abstract of the following paper.

## **Non-solenoidal velocity effects and Korteweg stresses in simple mixtures of incompressible liquids**

*Daniel D. Joseph, Adam Huang, Howard Hu*

### **Abstract**

We study some basic problems of fluid dynamics of two incompressible miscible liquids modeled as a simple mixture in which the volume of the mixture does not change on mixing. In general, the expansion  $\Delta = \text{div} \mathbf{u}$  in these problems does not vanish. The velocity in such a mixture can be

decomposed into a solenoidal and an expansion part. The expansion velocity is induced by diffusion which is proportional to the gradient of the volume fraction in a simple mixture. The expansion can be large at certain times and places. We have carried out an analysis of transient or dynamic interfacial tension for the problem of smoothing of an initial discontinuity of composition across a plane and spherical surface. The dynamic tension at the spherical interface decays as  $t^{-1/2}$ ; it has two terms, one term arises from the Korteweg stress and it gives rise to a stress opposing the internal pressure as in the case of equilibrium tension if the Korteweg coefficient has the appropriate sign. The other term arises from the expansion velocity and is proportional to the rate of change of viscosity with volume fraction. This term has the wrong sign for interfacial tension in the case of glycerin and water solutions but has the right sign when the light fluid is more viscous. In the context of the new theory, we derive a new and elementary solution which describes diffusion of binary species along a pipe. An analysis of this solution to small disturbances leads to a nonseparable problem which is solved by a finite element method. The numerical study indicates stability under all circumstances.

We found that diffusion layers develop at side walls when the densities of incompressible constituents of miscible liquids are different. This is explained in the abstract of the following paper.

### **Sidewall effects in the smoothing of an initial discontinuity of concentration**

*By T.Y. Liao and D.D. Joseph*

#### **Abstract**

The velocity field of a binary mixture of incompressible miscible liquids is non-solenoidal when the densities of the two liquids are different. If the mixture density is linear in the volume fraction, as in the case of simple (ideal) mixtures or very nearly for glycerin and water, then the velocity can be decomposed into a solenoidal and an expansion part. In the context of this theory, we derive a new solution which describes the smoothing of an initial plane discontinuity in concentration across a channel bounded by side walls. The requirement that the velocity vanishes on the side wall introduces a different initial discontinuity not present in the solenoidal theory. The problem may be reduced to a partial differential equation in two similarity variables one for the smoothing of a concentration

discontinuity without sidewalls and the other for the smoothing the velocity discontinuity at the sidewall. The similarity equations are solved explicitly in a special case.

### **Problem Three**

The third problem was to study breakup of thickened liquids at high Mach numbers. We built a shock tube to study this problem and found funds from an NSF equipment grant to buy a drum camera. We made the first movies of drop breakup in these extreme conditions. It appears that the shock tube is the only way presently known to obtain drop breakup data at low pressures. The results are reported in the following abstract.

### **Breakup of a liquid drop suddenly exposed to a high-speed airstream**

*By Daniel D. Joseph, J. Belanger & G.S. Beavers*

Submitted to *J. Fluid Mech*

#### **Abstract**

The problem is to study how drops of various liquids, Newtonian and non-Newtonian, break in the high speed airstream created by a shock wave in a shock tube. We constructed high speed movies of breakup with a rotating drum camera giving one photograph every  $5\mu\text{s}$ . Drops of the order of one millimeter are reduced to droplet clouds, and possibly to vapor, in a time less than  $500\mu\text{s}$ . The entire fragmentation history is now available in movie form ([http://www.aem.umn.edu/research/Aerodynamic\\_Breakup](http://www.aem.umn.edu/research/Aerodynamic_Breakup)). The movies reveal sequences of breakup events which have universal features which occur in all the liquids and specific features which differ from one type of liquid to another. In particular, the viscoelastic fluids give rise to stringlike and clumped drop fragments not seen in Newtonian fluids. Rayleigh-Taylor instabilities are endemic to the breakup; for example, the acceleration from gas to liquid required for this instability is of the order  $10^5$  times the acceleration of gravity in the flow behind Mach 3 shock waves. The process of drop breakup can be characterized in part as the inflation of the drop by pumping in fingers of hot air under the action of Rayleigh-Taylor instability.

### **Awards and honors**

Joseph was given the Timoshenko Medal of the ASME in May 1995 and he got the Thomas Baron Fluid-Particle Systems Award of the AIChE and Shell in Nov. 1996. He got the Illinois



Institute of Technology Professional Achievement Award in 1997 and he was the Croco Lecturer in Mechanical Engineering at Princeton University in Oct 1995.

### **List of Recent Publications (Since 1995)**

220. "A three-dimensional computation of the force and moment on an ellipsoid settling slowly through a viscoelastic fluid" (with J. Feng, R. Glowsinski and T.W. Pan). *J. Fluid Mech.* **283**, 1-16 (1995).
221. "Stability of eccentric core-annular flow" (with A. Huang). *J. Fluid Mech.* **282**, 233-245 (1995).
222. "Dynamic simulation of the motion of capsules in pipelines" (with J. Feng). *J. Fluid Mech.* **286**, 201-207 (1995).
223. "Dynamics of fluidized suspensions of spheres of finite size" (with P. Singh). *International Journal of Multiphase Flow* **21**, 1-26 (1995).
224. "Propagation of voidage wave in a two-dimensional liquid-fluidized bed" (with M. Poletto and R. Bai). To appear in *Int. J. Multiphase Flow* **39**, 323-344 (1995).
225. "The negative wake in a second order fluid" (with J. Feng). *J. Non-Newtonian Fluids* **57**, 313-320 (1995).
226. "A Maxwell memory model for the delayed weather reponse to solar heating" (with K.R. Sreenivasan). Submitted to *Proc. Roy. Soc. A* (1995).
227. "Boundary layer flow of air over water on flat plate" (with J. Nelson and A. Alving). *J. Fluid Mech.* **284**, 159-169 (1995).
228. "Cavitation in a flowing liquid." *Physical Review E* **51**(3), R1649 (1995).
229. "Motions of particles settling in a viscoelastic fluid." To appear in *Proceedings of the Second International Conference on Multiphase Flow*, Kyoto, Japan, April 3-7, 1995.
230. "Two-dimensional cusp at the trailing edge of an air bubble rising in a viscoelastic liquid" (with Y.J. Liu and T.Y. Liao). *J. Fluid Mech.* **304**, 321-342 (1995).
231. "Cement-lined pipe for water lubricated transport of hydrocarbons" (with M. Arney, G. Ribeiro, E. Guevarra and R. Bai). To appear in *Int. J. Multiphase Flow* (1995).
232. "The effective density and viscosity of a suspension" (with M. Poletto). *J. Rheology* **39**(2), 323-343 (1995).
233. "The unsteady motion of solid particles in creeping flows" (with J. Feng). *J. Fluid Mech.* **303**, 83-102 (1995).
234. "Non-solenoidal velocity effects and Korteweg stresses in simple mixtures of incompressible liquids", (with A. Huang & H. Hu), *Physica D*, **97**, 104-125 (1996).
235. "Vaporization of a liquid drop suddenly exposed to a high-speed air stream" (with A. Huang and G.V. Candler), *J. Fluid Mech.*, **318**, 223-236 (1996).
236. "Direct simulation of interfacial waves in a high viscosity ratio of axisymmetric core annular flow" (with R. Bai and K. Kelkar). *J. Fluid Mech.*, **327**, 1-34 (1996).
237. "Flow characteristics of concentrated emulsions of very viscous oil in water" (with G. Núñez, H. Rivás, Maria Briceño and Clara Mata), *J. Rheology*, **40** (3), 405-423 (1996).
238. "A note on the forces that move particles in a second-order fluid" (with J. Feng). *J. Non-Newtonian Fluid Mech.*, **64** (2-3), 299-302 (1996).
239. "Core-annular flows" (with R. Bai, K.P. Chen, and Y.Y. Renardy), *Annual Reviews of Fluid Mech*, **29**, Jan. (1997).

240. "Dynamic simulation of sedimentation of solid particles in an Oldroyd-B fluid" (with J. Feng and P.Y. Huang). *J. Non-Newtonian Fluid Mech*, **63** (1), 63-89 (1996).
241. "The motion of solid particles suspended in viscoelastic liquids under torsional shear" (with J. Feng), *J. Fluid Mech.*, **324**, 199-222 (1996).
242. "The motion of a solid sphere suspended by a Newtonian or viscoelastic jet" (with J. Feng), *J. Fluid Mech.*, **315**, 367-387 (1996).
243. "Flow induced microstructure in Newtonian and Viscoelastic fluids", Keynote presentation (paper no. 95a), Proceedings of the 5th World Congress of Chemical Engineering, Particle Technology Track. Second Particle Technology Forum, San Diego, July 14-18, 1996, AIChE New York, **6**, 3-16 (1996).
244. "Stability of annular flow and slugging", (with Antonio Bannward and J. Lui), *Int. J. of Multiphase Flow*, **22**(6), 1247-1254, (1996).
245. "Infiltration of an initially dry, deformable porous media", *Int. J. of Multiphase Flow*, **22**(6), 1205-1222, (1996).
246. "The motion and interaction of solid particles in viscoelastic liquids" (with J. Feng and P.Y. Huang), *Rheology and Fluid Mechanics of Nonlinear Materials*, **217**, 123-133 (1996).
247. "Letter to the Editor: Steep wave fronts on extrudates of polymer melts and solutions" (with Y.J. Lui), *J. Rheol.* **40**(2), 317-320 (1996).
248. "Direct simulation of the motion of solid particles in Couette and Poiseuille flows of viscoelastic fluids", (with P.Y. Huang, J. Feng and H. Hu), *J. Fluid Mech.*, **343**, 73-94 (1997).
249. "Sidewall effects in the smoothing of an initial discontinuity of concentration" (with T.Y. Liao), *J. Fluid Mech.*, **342**, 37-51 (1997).
250. "Steep wave fronts on extrudates of polymer melts and solutions: Lubrication layers and boundary lubrication", *J. Non-Newtonian Fluid Mech.* **70**, 187-203 (1997).
251. "Direct simulation of the sedimentation of elliptic particles in Oldroyd-B Fluids" (with P. Huang & H. Hu), *J. Fluid Mech*, **362**, 297-326 (1998).
252. "Cavitation and the state of stress in a flowing liquid", accepted in *J. Fluid Mech* (1997).
253. "How bubbly mixtures foam and foam control using a fluidized bed" (with Jose Guittian), *Int. J. Multiphase Flow*, **24** (1), 1-16 (1998).
254. "Foam control using a fluidized bed of hydrophobic particles" (with Clara Mata), submitted to *Int. J. Multiphase Flow*, (1998).
255. "A Distributed Lagrange Multiplier fictitious-domain method for flow" (with Glowinski, Pan and Hesla), accepted in *Int. J. Multiphase Flow* (1998).
256. "Questions in fluid mechanics: Understanding foams & foaming," Technical Forum, *J. Fluid Eng.*, 119, 497 (1997).

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