Cavitation in a flowing liquid

by

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In this letter, I am going to propose that the cavitation threshold in a flowing liquid could be associated with the maximum tension that the fluid can sustain before undergoing cohesive fracture at a point. My criterion is not isotropic; I am thinking that a liquid will break if the tension in one direction exceeds a threshold, independent of the value of the other principal stresses. The other thought is that if a liquid breaks, it is a cohesive fracture in which the liquid molecules disassociate into vapor and recondense as mist.

Suppose that the vapor pressure of the liquid at a certain temperature θ is $p_s(\theta)$. In general, $p_s(\theta)$ is an increasing function so that the cavitation threshold

$$p < p_s(\theta), \tag{1}$$

where p is the pressure in the liquid is raised with the temperature [1–2]. The pressure in a liquid at rest is the mean normal stress. For a liquid in motion, the stress is given by

$$\mathbf{T} = -p\mathbf{1} + \mathbf{S}, \tag{2}$$

where **S** is the extra stress due to motion. Here p is an extra variable needed to satisfy the constraint of incompressibility, and it is not determined by a constitutive equation or equation of state.

The considerations just raised pose the problem of how to pose the cavitation threshold condition in a moving liquid. In fact, the liquid does not understand the decomposition into $p\mathbf{1}$ and **S**. If we cut the liquid, the traction vector on the cut is given by $\mathbf{n} \cdot \mathbf{T}$, where $\mathbf{n}$ is normal to the cut. The fluid feels tractions $\mathbf{n} \cdot \mathbf{T}$ on cuts and the stress $\mathbf{T}$ in the bulk.

There is a very substantial literature aimed at determining the maximum tension that a liquid may withstand, and it is found that if nucleation sites are eliminated, large tensions can be maintained [3–5]. In practical applications, $p_s(\theta)$ in (1) can be replaced with an empirical criterion, say, $\tilde{p}_s(\theta)$, which could be a limit associated with degassing or impurities [2–6].

The interpretation of the maximum tension in a liquid at point requires that we compare the components on the diagonal of $\mathbf{T}$ in a coordinate system in which it is diagonal. Supposing then that

$$\mathbf{T}_{11} \geq \mathbf{T}_{22} \geq \mathbf{T}_{33}, \tag{3}$$

we may state that the liquid at point will break if

$$\mathbf{T}_{11} \geq \mathbf{T}_m, \tag{4}$$

where T_m is the breaking threshold. If we think that the breaking stress is determined by the cavitation threshold, then $\mathbf{T}_m \geq -\tilde{p}_s(\theta)$ and we should see vapor whenever and wherever

$$\mathbf{T}_{11} \geq -\tilde{p}_s(\theta). \tag{5}$$

This criterion implies (3.1) if the fluid is at rest, but in a Newtonian fluid for which

$$\mathbf{T}_{11} = -p + 2\mu \frac{\partial u}{\partial x}, \tag{6}$$

we can expect vapor and mist when and where the rate of stretching

$$\dot{s} = \frac{\partial u}{\partial x} \tag{7}$$

is large enough

$$\dot{s} > \frac{p - \tilde{p}_s(\theta)}{2\mu}. \tag{8}$$

Equation (8) is our proposal for a cavitation threshold and it can lead cavity formation under conditions greatly different than (1).

Professor Barenblatt has brought to my attention that the concept of a breaking threshold in cavitation proposed here is to a certain extent analogous to the theoretical strength of solids -- the maximum tensile strength between two planes. Apparently, as in the fracture of solids, this idealized criterion could be modified if the fluid contains defects: impurities, bubbles, etc. In this case, a criterion like

$$\mathbf{T}_{11}\sqrt{a} < K_{cr}(\theta) \tag{9}$$

can be proposed, analogous to similar criteria in the theory of fracture mechanics. Here, a is the size of the defect, $K_{cr}(\theta)$ is a temperature dependent material property.

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