CAPILLARY COLLAPSE AND RUPTURE

T.S. Lundgren & D.D. Joseph

The breakup of a liquid capillary filament is analyzed as a viscous potential flow near a stagnation point on the centerline of the filament towards which the surface collapses under the action of surface tension forces. We find that the neck is of parabolic shape and its radius collapses to zero in a finite time. During the collapse the tensile stress due to viscosity increases in value until at a certain finite radius, which is about 1.5 microns for water in air, the stress in the throat passes into tension, presumably inducing cavitation there.

$$[u_z, u_r] = a(t)[z, -\frac{r}{2}] \quad \text{Stagnation flow}$$

$$\varphi = \frac{1}{2}az^2 - \frac{1}{4}ar^2 \qquad \text{Potential flow}$$

Bernoulli Equations:

$$\frac{\partial \varphi}{\partial t} + \frac{1}{2}(u_r^2 + u_z^2) + \frac{p}{\rho} = \frac{p_0}{\rho}$$

$$\frac{p - p_0}{\rho} = -\left(\frac{1}{2}\dot{a} + \frac{1}{2}a^2\right)z^2 + \left(\frac{1}{4}\dot{a} - \frac{1}{8}a^2\right)r^2$$

$$T_{zz} = -p + 2\mu\frac{\partial u_z}{\partial z} = -p + 2\mu a,$$

Stresses

$$T_{rr} = -p + 2\mu\frac{\partial u_r}{\partial r} = -p - \mu a$$

On $r = R(z,t) = R_0(t) + R_2(t)z^2 + O(z^4)$

$$-T_{nn} - p_a = \sigma\kappa$$

Normal

$$T_{nn} = n_r^2 T_{rr} + n_z^2 T_{zz}$$

Stress

$$\kappa = -\frac{\frac{\partial^2 R}{\partial z^2}}{\left(1+\left(\frac{\partial R}{\partial z}\right)^2\right)^{\frac{3}{2}}} + \frac{1}{R\left(1+\left(\frac{\partial R}{\partial z}\right)^2\right)^{\frac{1}{2}}}$$

balance

$$= \frac{1}{R_0} - 2R_2 + O(Z^2)$$

It's zero at $z = 0$

because $\partial u_z/\partial r = 0$

shear stress

$$u_r = \frac{\partial R}{\partial t} + u_z \frac{\partial R}{\partial z}$$

Kinematic Condition

$$-\tfrac{1}{2}aR = \frac{\partial R}{\partial t} + az\frac{\partial R}{\partial Z}$$

$$-\tfrac{1}{2}aR_0 = \overset{\circ}{R}_0 \qquad \text{Solve for } a$$

The parabola flattens

$$-\tfrac{5}{2}aR_2 = \overset{\circ}{R}_2 \qquad R_2 = CR_0^5$$

as it collapses

$$a = -\frac{2\overset{\circ}{R}_0}{R_0}$$

To lowest order in z^2

$$\frac{p_a - \sigma\kappa}{\rho} = \frac{T_{nn}}{\rho} = \frac{T_{rr}}{\rho} = -\frac{p}{a} - va$$

$$\frac{p_a}{\rho} + \frac{\sigma}{\rho}\left(\frac{1}{R_0} - 2R_2\right) = -\frac{p_0}{\rho} - \left(\frac{1}{4}\overset{\circ}{a} - \frac{1}{8}a^2\right)R_0^2 - va$$

$$R_2 = CR_0^5 \qquad\qquad a = 2\overset{\circ}{R}_0/R_0$$

"Rayleigh Plesset" equation:

$$\frac{p_0 - p_a}{\rho} - \frac{1}{2}R_0\ddot{R}_0 - \frac{2v\dot{R}_0}{R_0} = \frac{\sigma}{\rho}\left(\frac{1}{R_0} - 2CR_0^5\right).$$

$$\overset{\circ}{R}_0 = \frac{1}{2}\frac{\sigma}{\rho v}$$

Small R_0, balances

viscosity against inertia

to leading

$$R_0 = \frac{\sigma}{2\rho v}(t_* - t)$$

R_0 collapses to

order in

$$a = \frac{2}{t_* - t}$$

zero in finite time

$t_* - t$

$$\frac{p}{\rho} = \frac{p_0}{\rho} - \frac{3z^2}{(t_* - t)^2}$$

AXIAL STRESS (to leading order)

$$\frac{T_{zz}}{\rho} = -\frac{2p_0 - p_a}{\rho} + \frac{4v}{t_* - t} = -\frac{2p_0 - p_a}{\rho} + \frac{2\sigma}{\rho R_0(t)}$$

T_{zz} **turns positive for small** R_0

$$R_{ocr} = \frac{2\sigma}{2p_0 - p_a}$$

Estimating $p_0 = p_a = 10^6$ dynes/cm^2
$\sigma = 75$ dyne/cm

$R_0 = 1.5$ microns

At the collapse condition
the Reynolds number is
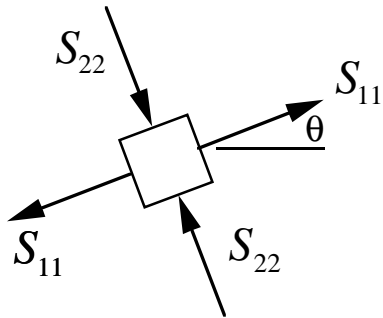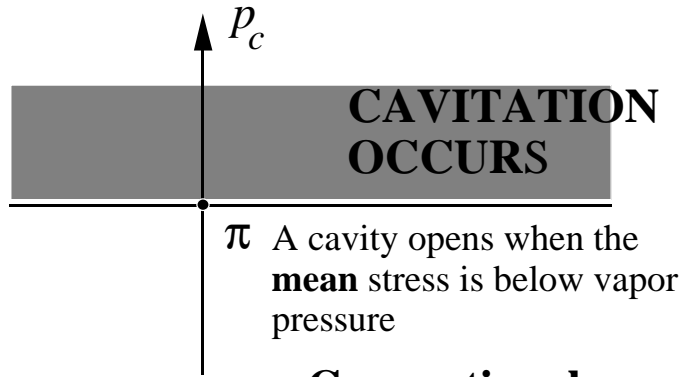about 55 based on $\overset{\circ}{R}_0$

The capillary thread cavitates before it collapses to zero.

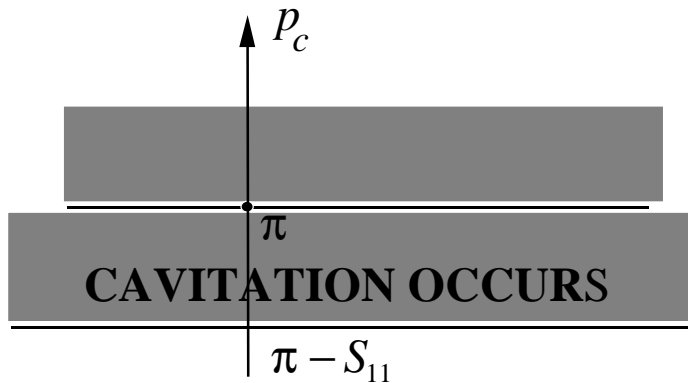# CAVITATION CRITERIA

2D: $S_{22} = -S_{11}$

principal axes

$$-\pi = \tfrac{1}{3}(T_{11} + T_{22} + T_{33})$$

π A cavity opens when the **mean** stress is below vapor pressure

Conventional

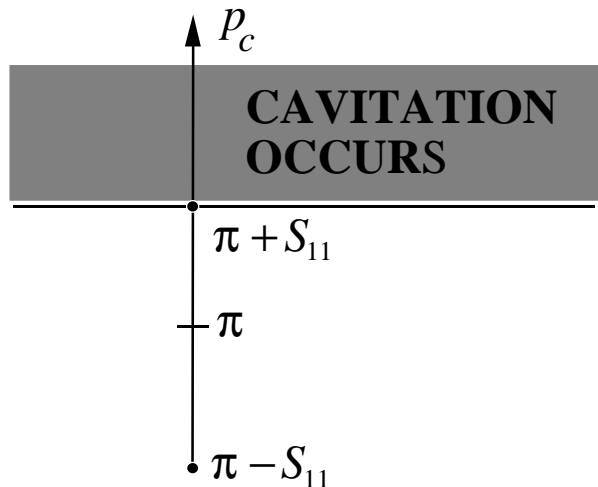p_c is the "vapor pressure", the nucleation threshold.

Maximum Tension:

A cavity opens when **one** of the principal stresses is below vapor pressure
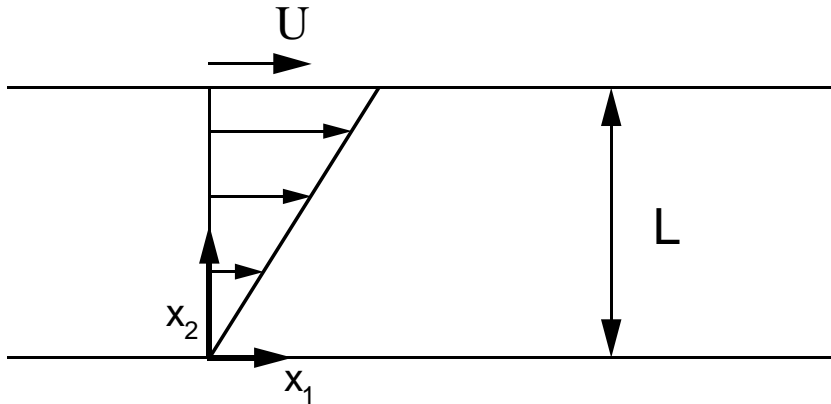

Minimum Tension:

A cavity opens when **all** of the principal stresses are below vapor pressure

CAVITATION IN SHEAR

The stress in this flow is given by

$$\begin{bmatrix} T_{11} & T_{12} & 0 \\ T_{12} & T_{22} & 0 \\ 0 & 0 & T_{33} \end{bmatrix} = -\pi \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \eta \begin{bmatrix} 0 & \frac{U}{L} & 0 \\ \frac{U}{L} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $\pi = \frac{1}{3}(T_{11} + T_{22} + T_{33})$ is determined by the "pressurization" of the apparatus. The angle which diagnonalizes $\mathbf{T}$ is given by

$$\theta = 45°$$

(In the break-up of viscous drop experiments in plane shear flow done by G.T. Taylor [1934], the drops first extend at 45° from the direction of shearing.)

In principal coordinates, we have

$$\begin{bmatrix} T_{11} + \pi & 0 & 0 \\ & T_{22} + \pi & 0 \\ 0 & 0 & T_{33} + \pi \end{bmatrix} = \eta \frac{U}{L} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where

$$T_{11} + \pi = S_{11} = \eta \frac{U}{L}$$

is a tension.

This tension is of the order of one atmosphere of pressure if

$$\eta \frac{U}{L} = 10^6 \frac{\text{dynes}}{\text{cm}^2} = 10^5 \text{Pa}$$

If $\eta = 1000$ poise, $U = 10$ cm/sec and $L = 10^{-1}$cm, we may achieve such a stress. *A shear stress of this magnitude is enough to put the liquid into tension.*

COMPETITION BETWEEN INERTIA & VISCOELASTICITY

- The shear rate is large where the velocity is large.

- The pressure of inertia is largest near stagnation points and smallest where the velocity is large.

- The viscoelastic pressure is large where the inertial pressure is small. The viscoelastic pressure is small where the inertial pressure is large. This is the reason why the microstructure in Newtonian and viscoelastic fluids are maximally different.
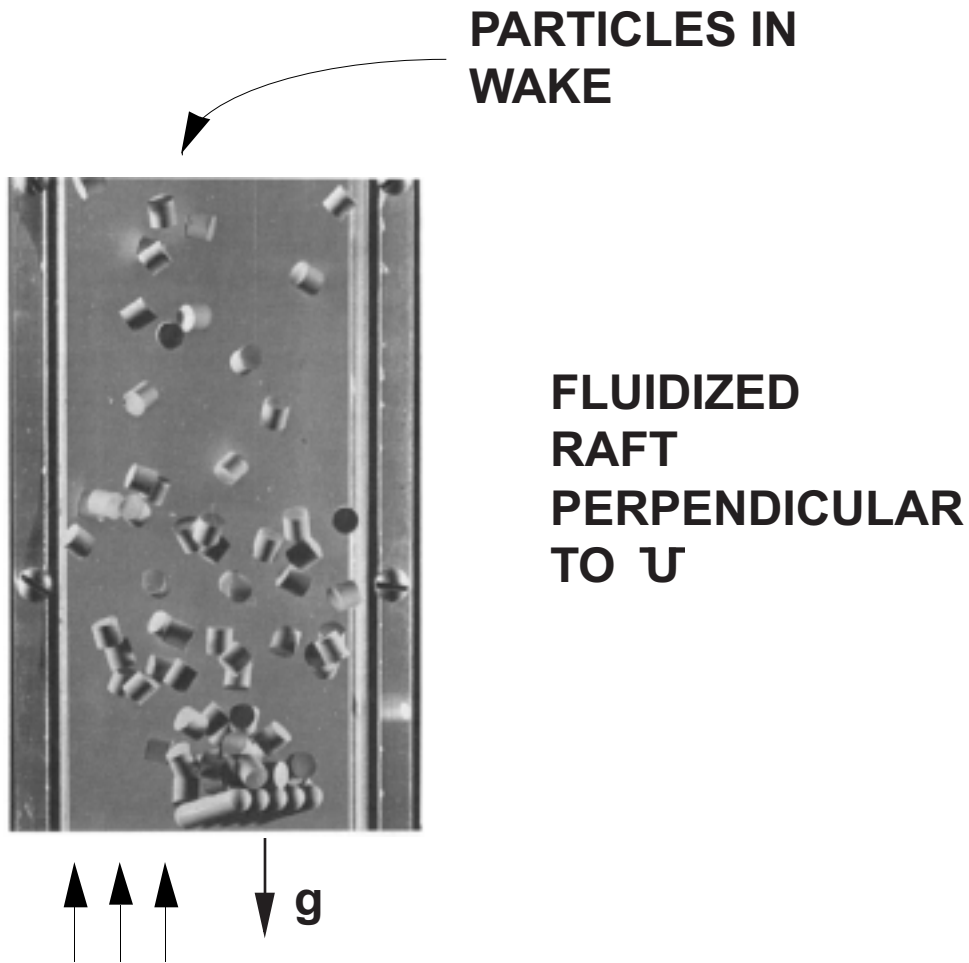
PARTICLES IN WAKE

FLUIDIZED RAFT PERPENDICULAR TO U

g

Two Phase Flow Models Do Not Predict

- Long bodies stable broad side on
- Drafting, Kissing, Tumbling
- Across the stream arrays of spheres
- Nested wake structure, "Flying Birds"
- Doublets, Triplets, Quadraplets
- Fluidized Raft, Wake Aggregates

VIDEO 3 - TURBULENT PING PONG BALLS

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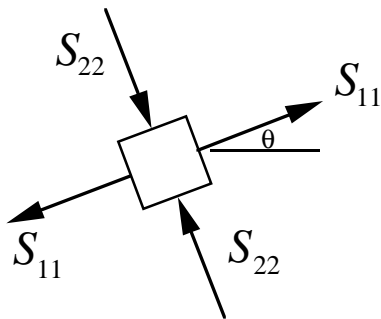- Fluidized Raft, Wake Aggregates

VIDEO 3 - TURBULENT PING PONG BALLS

8
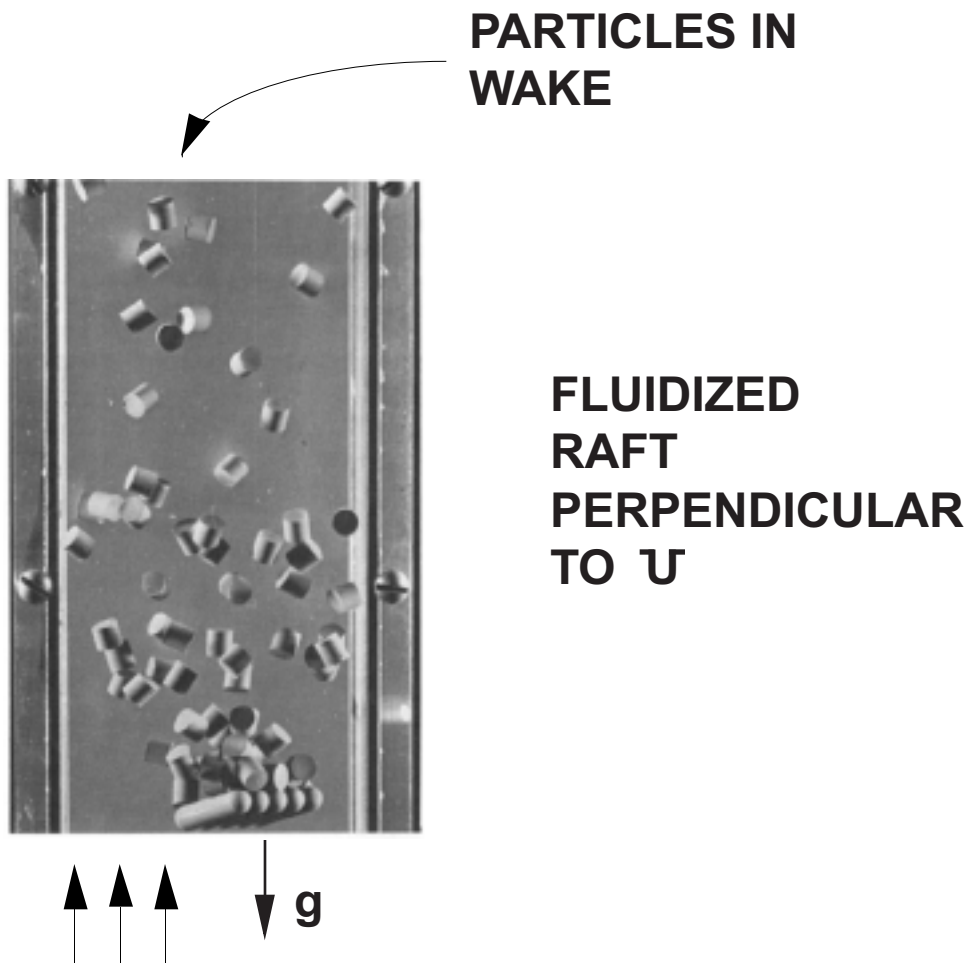
- **Mean normal stress and deviator**

$$\mathbf{T} = -p\mathbf{1} + \mathbf{S}$$

$$p = -\frac{T_{11} + T_{22}}{2}$$ **the fluid cannot average stresses**

$$\mathbf{S} = \begin{bmatrix} S_{11} & 0 \\ 0 & S_{22} \end{bmatrix}, \quad S_{11} + S_{22} = 0$$

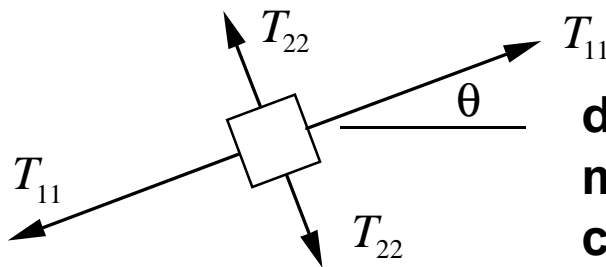**the extra stress is good because it has positive & negative components**

STRESS, PRINCIPAL AXES, DEVIATOR

- ## Stress in two dimensions

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{12} & T_{22} \end{bmatrix}$$

"pressure" cannot be recognized in a liquid; it sees a state of stress

- ## Principal coordinates

direction of maximum tension. A cavitation must open in the direction θ; then it can rotate away

BREAKING STRENGTH OF POLYMER STRANDS

(Wagner, Schulze and Gottfert [1996])

The strand breaks at the thinnest cross section of the strand when the tensile stress

$$\sigma = \frac{FV}{A_0} \approx 10^6 \, \text{Pa} = 10 \text{ atmospheres}$$

for many kinds of polymeric liquids. They say that the breaking stress is a material constant.

BREAKING TIME & FLOW TIME

Cavitation of ultrathin (nanometer) films of

Israelachvili, Chen, Kuhl & Ruths

Their experiments show that the formation of cavities open in tension at a threshold value of the extensional stress $2\eta \overset{\circ}{S}$

$$\left(\begin{array}{c} \text{which I estimate as} \\ 3.6 \times 10^5 \, \text{Pa} < 2\eta \overset{\circ}{S} < 3.6 \times 10^6 \, \text{Pa} \end{array} \right)$$

and that the formation of cavities is analogous to the fracture of solids except *after* fracture, vapor flows into the cavity "...When a cavity initially forms and grows explosively, it is essentially a VACUUM CAVITY since dissolved solute molecules or gases have not had time to enter the rapidly growing cavity."

COMPUTATIONAL DOMAINS AND PROBLEMS

Finite representation of an infinite domain

This is appropriate when tracking the motion of a cluster of particles, say in a pipe or channel.
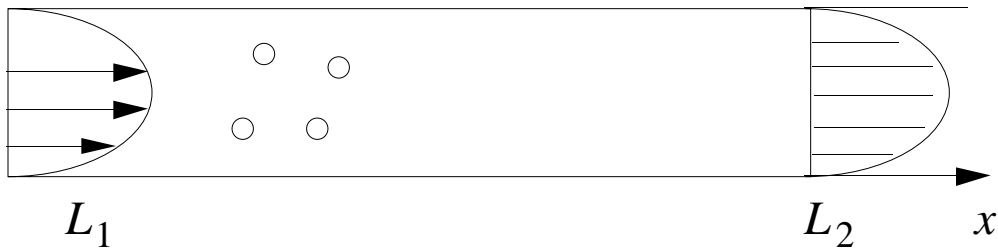
If we have Poiseuille flow at $x = \pm \infty$, then we choose a finite domain, far away from the particles say at L_1 and L_2. You must prescribe conditions at L_1 and L_2; this is subtle. For example, you could say that you must have Poiseuille flow at L_1 and L_2, which is not strictly correct.

To do the computation, you must move L_1 and L_2 with the particles so that no particle will leave the computational domain.

Periodic flow

Every cell is the same as another. You compute everything by computing in one cell.
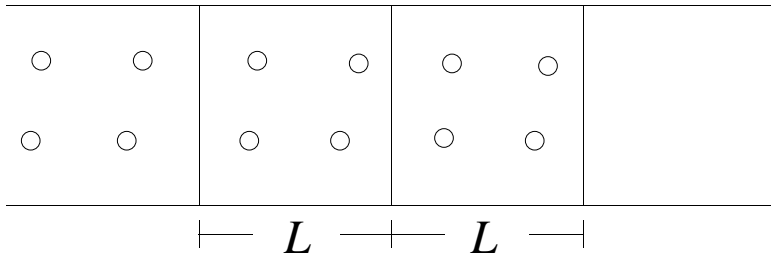
Of course, the dynamics must give rise to periodic solutions under the conditions assumed in the calculations, otherwise you could have garbage. The problem of existence of periodic solutions of particulate flows is deep, unsolved and virgin.

Finite Domains

We prescribe conditions at the inlet and outlet and along the sides of the computational domain. This setup is straightforward. For example, we have considered sedimentation of heavier than water particles against the solid bottom of a container of water.

DIRECT NUMERICAL SIMULATION (DNS)

Solves the dynamic problem of the motion of particles in a fluid exactly (in principle). The particles are moved by Newton's laws under the action of hydrodynamic forces computed from the numerical solution of the fluid equations (Navier-Stokes equations for water, etc.)

I. Since solutions are exact, they reveal the properties that control transport:

   A. Lubrication layers
   B. Aggregation or dispersion
   C. Lift off
   D. Slip velocity
   E. Concentration gradients
   F. Microstructural features

- These simulations complement and compete with experiments. Experiments tell the truth in all detail. The simulations, though potentially exact, have limitations of various kinds

    G. Number of particles

    H. Modeling of collisions

    I. Turbulence

    J. High Deborah numbers

    K. 3D effects for large numbers of particles

Advantages of simulations:

  I. They are much cheaper

  II. You can isolate effects

    A. Turn shearing on or off

    B. Turn on normal stresses with or without shear thinning

    C. Test scale up

Aims of simulation:

I. Produce understanding of mechanisms underlying propant transport

II. Provide data for correlation which formerly were obtained by experiments. For example, we can make a program to generate Richardson-Zaki correlations for transport models from DNS

III. Provide a standard for validating continuum models of propant transport

The production of cavitation in pure shear appears to have been realized recently (1997)

"FRACTURE" PHENOMENA IN SHEARING FLOW OF VISCOUS LIQUIDS

L.A. Archer, D. Ternet and R. Larson

ABSTRACT

In startup of steady shearing flow of two viscous unentangled liquids, namely low-molecular-weight polystyrene and α-D-glucose, the shear stress catastrophically collapses if the shear rate is raised above a value corresponding to a critical initial shear stress of around 0.1 --- 0.3 Mpa. The time-dependence of the shear stress during this process is similar for the two liquids, but visualization of samples in situ and after quenching reveals significant differences. **For α-D-glucose, the stress collapse evidently results from debonding of the sample from the rheometer tool, while in polystyrene, bubbles open up within the sample; as occurs in cavitation.** Some similarities are pointed out between these phenomena and that of "lubrication failure" reported in the tribology literature.

We have adhesive and cohesive fracture, 0.1--0.3 Mpa = 1--3 atm. This is enough to put the sample into tension 45° from the direction of shearing.

PRINCIPLES OF CAVITATION IN A FLOWING LIQUID

- Conventional cavitation based on pressure
I.   What is pressure?
    A. Newtonian fluids
    B. Non-Newtonian fluids
- Nonconventional cavitation based on principal stresses
II. Maximum tension
- Mean normal stress and deviator
- An angle is always involved
- Cavitation in shear
- Tensile strength of liquids.
    A. Polymer strands
    B. Breaking time and flow time, vacuum cavities
    C. Capillary collapse and rupture
III. Outgassing is cavitation of liquid gas in solution

All *books on cavitation have sections on the "tensile strength of liquids." A stress tensor is **never** introduced.*

Knapp et al. [1970] note that:

"If a cavity is to be created in a homogeneous liquid, the liquid must be ruptured, and the stress required to do this is not measured by the vapor pressure but is the *tensile strength* of the liquid at that temperature. The question naturally arises then as to the magnitudes of tensile strengths and the relation these have to experimental findings about inception.

CONVENTIONAL CAVITATION

A fluid will cavitate when the local pressure falls below the cavitation pressure

- The cavitation pressure is the vapor pressure in a pure liquid

- Natural liquids have nucleation sites defined by impurities and may cavitate at higher pressures

WHAT IS PRESSURE?

- In an incompressible Newtonian fluid "pressure" is the mean normal stress. A fluid cannot average its stresses, even though you can. The fluid knows its state of stress at a point.

- In non-Newtonian fluids the pressure is an unknown flow variable, usually not even the mean normal stress, and the definition of it is determined by the constitutive equation. This "pressure" has no intrinsic significance. The fluid doesn't recognize such a "pressure" and knows its state of stress.

NONCONVENTIONAL CAVITATION BASED ON PRINCIPAL STRESSES

Look at the state of stress at each point in the fluid in principal axis coordinates. Identify the largest of the stresses. Suppose a static fluid cavitates at zero pressure. It will cavitate in flow wherever the maximum tensile stress is positive.

If it cavitates statically when the pressure falls below the vapor pressure, it will cavitate in flow even when the maximum tensile stress is only slightly negative.

Suppose you do an experiment in your lab where the ambient pressure is

one atmosphere = 10^5 Pa

Then if you get tensile stresses due to flow larger than this, the fluid will cavitate.

MAXIMUM TENSION

*All books on cavitation have sections on "tensile strength of liquids." A stress tensor is **never** introduced.*

"If a cavity is to be created in a homogeneous liquid, the liquid must be ruptured, and the stress required to do this is not measured by the vapor pressure but is the *tensile strength* of the liquid at that temperature." (Knapp et al. 1970)

FIRST THE FLUID RUPTURES
THEN VAPOR FILLS THE CAVITY