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Friction factor improved correlations for laminar and turbulent gas–liquid flow in horizontal pipelines

F. García [a,*], J.M. García [b], R. García [c], D.D. Joseph [d]

[a] School of Mechanical Engineering, Central University of Venezuela, Caracas 1051, Venezuela
[b] Department of Thermodynamics and Transfer Phenomenons, Simón Bolívar University, Caracas 1080, Venezuela
[c] Applied Research Center, Florida International University, 10555 W. Flagler Street, EC 1278 Miami, FL 33174, USA
[d] Department of Aerospace Engineering and Mechanics, University of Minnesota, 107 Akerman Hall, 110 Union Street SE, Minneapolis, MN 55455, USA

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Abstract

We develop improved correlations for two-phase flow friction factor that consider the effect of the relative velocity of the phases, based on a database that includes 2560 gas–liquid flow experiments in horizontal pipes. The database includes a wide range of operational conditions and fluid properties for two-phase friction factor correlations. We classify the experiments by liquid holdup ranges to obtain composite analytical expressions for two-phase friction factor vs. the Reynolds number by fitting logistic dose curves to the experimental data with. We compute the liquid holdup values used to classify the experimental data using correlations proposed previously. The Reynolds number is based on the mixture velocity and the liquid kinematic viscosity. The Fanning friction factor for gas–liquid is defined in term of the mixture velocity and density. Additionally, we sort the experimental data by flow regime and obtain the two-phase friction factor improved correlations for dispersed bubble, slug, stratified and annular flow for different holdup ranges. We report error estimates for the predicted vs. measured friction factor together with standard deviation for each correlation. The accuracy of the correlations developed in this study is compared with that of other 21 correlations and models widely available in the specialized literature. Since different authors use different definitions for friction factors and Reynolds numbers, we present comparisons of the predicted pressure drop for each and every data point in the database. In most cases our correlations predict the pressure drop with much greater accuracy than those presented by previous authors.

Keywords: Friction factor; Gas–liquid; Power law; Pipe flow; Horizontal pipelines

1. Introduction

To predict the pressure drop of gas–liquid flow in pipeline is a problem of great interest in many industries, especially in the oil industry. Dimensionless pressure gradient is usually expressed as friction factor through

* Corresponding author.
E-mail address: garciaga@ucv.ve (F. García).

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the Euler number. The relation between pressure gradient and mass flux is expressed in dimensionless form as a relation between the friction factor and Reynolds number. This relation in one single fluid (one-phase) is depicted in the celebrated Moody diagram. The pipe roughness is an important factor in the Moody diagram; for turbulent flow in smooth pipes the data may be fit to the well-known power law of Blasius for which the friction factor increases with 0.25 power of the Reynolds number. The Moody diagram may be partitioned into the three regions: laminar, transition and turbulent.

Due to the complexity of multiphase flow systems, it is not possible to obtain the governing dimensionless groups uniquely, but various alternatives exist. For instance, Dukler et al. (1964) use one set, Beggs and Brill (1973) another set, Müller-Steinhagen and Heck (1986) another set and so on.

García et al. (2003) tested and evaluated various combinations of dimensionless parameters of friction factors vs. Reynolds numbers and they constructed an equivalent "Moody diagram" for gas–liquid flow in horizontal pipe in function of Fanning friction factor and mixture Reynolds number. Ranges were selected to reduce significantly the scatter in the data of more than 2400 gas–liquid flow experiments with a wide range of operational conditions and fluid properties. García et al. (2003) evaluated the predictions of their correlations together with the predictions of other four correlations, four homogeneous models and three mechanistic models reported in the literature comparing the scatter of the predictions against the real data through of six statistical parameters. Although the correlations developed by García et al. (2003) generated over all the best results, for annular and stratified flows obtained average absolute errors of 30% and 38%, respectively. These errors could decrease if the effect of the relative velocity of the phases, neglected by García et al. (2003), is considered.

In this work, we include the effect of the relative velocity of the phase in order to construct an "improved equivalent Moody diagram" for gas–liquid flow in horizontal pipes as function of the Fanning friction factor and mixture Reynolds number selected to reduce the scatter of 2560 experimental data for different liquid holdup ranges. The data are processed for power laws and composite expressions are found as a rational fraction of power laws which reduces to power law for low Reynolds numbers in the "laminar" range and a Blasius-like expression for large Reynolds numbers in the "turbulent" range. This leads to new improved Universal (for all flow patterns) composite (for all Reynolds numbers) correlations for gas–liquid friction factor for different liquid holdup ranges. Additionally, we sorted the experimental data by flow type to develop composite improved correlations for dispersed bubble, slug, stratified and annular flow for different holdup ranges. To sort the 2560 experiments, the liquid holdup was evaluated with the correlations developed by García et al. (2005).

The accuracy of the correlations developed in this paper is evaluated in two ways; first by comparing predictions with the data from which the predictions are derived and second by comparing the predictions of our correlations with predictions of other correlations and models available in the literature.

The comparison of our correlations with the other correlations and models is not conveniently carried out using friction factor vs. Reynolds number because different authors use different definitions of these quantities. An unambiguous comparison is constructed by comparing predicted pressure gradients against the experiments in our database. We compared our predicted pressure gradients with those obtained from the correlations of Lockhart and Martinelli (1949), Reid et al. (1957), Hoogendoorn (1959), Dukler et al. (1964), Beggs and Brill (1973), Kadambi (1981), Müller-Steinhagen and Heck (1986), Ortega et al. (2001) and Chen et al. (2002); the homogeneous models of McAdams et al. (1942), Cicchitti et al. (1960), Wallis (1969), Oliemans (1976), Beattie and Whalley (1982) and Ouyang (1998), and the mechanistic models of Xiao et al. (1990), Ouyang (1998), Gómez et al. (2000) and Padrino et al. (2002). Ouyang's models are for horizontal wells which reduce to pipelines when the inflow from reservoir is set to zero. The comparison of the accuracy of the pressure gradients prediction between different models and correlations against the 2560 experimental points is achieved by means of the Ripley range factor recommend by Govan (1988); however, other common statistical parameters are also included.

The correlations for separate flow patterns are more accurate, but possibly less useful than those for which knowledge of actual flow pattern is not required. It is important to bear in mind that in a typical field situation the flow pattern is unknown, and in such cases a correlation independent of the flow pattern is of topmost utility.

2. Dimensionless parameters

The mixture Fanning friction factor f_M and the mixture Reynolds number Re definitions are greatly important to develop an appropriate correlation of the experimental data. García et al. (2003) presented a mixture Fanning friction factor and mixture Reynolds number definition that allowed to reduce significantly the scatter of more than 2400 experiments when plotting f_M vs. Re. These definitions are adopted again in this work.

The Fanning friction factor for the gas–liquid mixture is defined as:

$$f_M = \frac{(\Delta p/L)D}{2\rho_M U_M^2} \tag{1}$$

where the pressure drop per unit length $(\Delta p/L)$ is related to the wall shear stress $\tau_w = (D\Delta p/4L)$, D is the pipe diameter, $U_M = U_{SG} + U_{SL}$ is the mixture velocity which is defined in terms of the superficial gas velocity ($U_{SG} = 4Q_G/\pi D^2$) and the superficial liquid velocity ($U_{SL} = 4Q_L/\pi D^2$). Q_G and Q_L are the gas and liquid flow rates, respectively. The mixture density $\rho_M = \rho_L\lambda_L + \rho_G(1 - \lambda_L)$ is a composite density weighted by the flow rate fraction, where $\lambda_L = Q_L/(Q_L + Q_G)$ is the liquid flow rate fraction.

The mixture Fanning friction factor f_M is correlated with a mixture Reynolds number defined by

$$Re = \frac{U_M D}{\nu_L} \tag{2}$$

where $\nu_L = \mu_L/\rho_L$ is the kinematic viscosity of the liquid; this definition acknowledges that the frictional resistance of the mixture is due mainly to the liquid.

3. Augmented experimental database

In a previous paper, García et al. (2003) used a data base that included 2435 gas–liquid flow experiments in horizontal pipelines. Here we add 125 experimental results to reach an augmented data base containing 2560 experiments. The data were gathered from Intevep's databank, the Stanford multiphase flow database (SMFD), the database of the Tulsa University fluid flow projects (TUFFP) for gas–liquid flow in horizontal pipes and the specialized literature. These data are summarized in Tables 1–4. The columns in the tables are self-explanatory except that "No. Exp." means the number of experiments in each data set, ε is the average size of pipe wall roughness, FP means "flow pattern" and AN, DB, SL, SS and SW stand for annular, dispersed bubble, slug, stratified smooth and stratified wavy flow, respectively.

A summary of the augmented experimental database in order to evaluate the pressure gradients for gas–liquid flow in horizontal pipes is presented in Table 5.

To our knowledge, this database gathers the widest range of operational conditions and fluid properties compiled and used to develop friction factor correlations for gas–liquid flow in horizontal pipelines.

4. Universal (for all flow patterns) composite (for all Reynolds numbers) improved correlations for gas–liquid friction factor (FFIUC)

A fraction of the 2560 experimental data (2183 experimental points) was used to calculate the mixture friction factors f_M defined in Eq. (1). The points belonging to transition regions that come from Cabello's data (nine experimental points), Ortega's data (32 experimental points) and Johnson and Abou-Sabe's data (two experimental points) were excluded. The points that come from Rivero's data (74 experimental points), Eaton's data (51 experimental points) and SU199's data (209 experimental points) were excluded, because the pressure measurements had huge and unacceptable scatter for some points. The models and the correlations considered in Section 6, including ours, are tested against the entire data base (2560 experimental points).

Table 1
Intevep data

Source	No. Exp.	Fluid	μ_L (mPa s)	U_{SL} (m/s)	U_{SG} (m/s)	D (m)	ε (m)	FP
Rivero et al. (1995)	74	Air–water Air–oil	1–200	0.02–0.19	0.61–11.89	0.0508	0	SW
Ortega et al. (2000)	50 + 20[a]	Air–oil	500	0.10–2.77	0.02–38.24	0.0508	0	AN DB SL SS SW SL-AN SL-DB SS-SL SW-AN
Cabello et al. (2001)	26 + 9[a]	Air–kerosene	1	0.11–4.52	0.77–45.65	0.0508	0	AN DB SL SL-AN SL-DB
Ortega et al. (2001)	35 + 12[a]	Air–oil	1200	0.01–0.80	0.23–24.39	0.0508	0	AN SL SW SL-AN SW-AN SW-SL
Pereyra et al. (2001)[b]	94	Gas–HL[c]	8–400	2.69–0.58	0.26–12.91	0.0779	4.60×10^{-5}	SL
Mata et al. (2002)	31	Air–oil	100	0.11–1.49	0.06–3.43	0.0254	0	SL

[a] Transition points.
[b] The live oil viscosity is reported.
[c] HL: hydrocarbon liquid.

Initially we classify the 2183 experimental points of gas–liquid two-phase flow in horizontal pipes for different liquid holdup ranges calculated with the universal composite holdup correlations UCHC proposed by García et al. (2005). The liquid holdup ranges are selected in order to reduce the experimental data scatter of mixture friction factor f_M (Eq. (1)) vs. mixture Reynolds number Re (Eq. (2)). The different experimental data sets are fit with composite power law equations applying a technique described by Barree (Patankar et al., 2002). The equations are given by

$$f_M = F_2 + \frac{(F_1 - F_2)}{\left(1 + \left(\frac{Re}{t}\right)^c\right)^d} \tag{3}$$

where F_1 and F_2 are power laws defined as:

$$F_1 = a_1 Re^{b_1} \tag{4}$$

$$F_2 = a_2 Re^{b_2} \tag{5}$$

where a_1, b_1, a_2, b_2, c, d and t, are parameters that are computed simultaneously, fitting Eqs. (3)–(5) to the experimental data of each subsets minimizing the residual mean square using the nonlinear optimization method of Microsoft® Excel Solver. The parameters a_1, b_1, a_2, b_2, c, d and t for universal composite improved correlations are presented in Table 6.

Figs. 1 and 2 show the universal composite improved correlations for each liquid holdup range.

Table 2
Stanford data

Source	No. Exp.	Fluid	μ_L (mPa s)	U_{SL} (m/s)	U_{SG} (m/s)	D (m)	ε (m)	FP
Alves (1954)	28	Air–oil	80	0.02–1.78	0.12–13.16	0.0266	4.57×10^{-5}	AN SL SW
Govier and Omer (1962)	57	Air–water	1	0003–1.53	0.05–16.57	0.0261	0	AN SL SS SW
Eaton (1966)	51	Gas–water	1	0.04–2.24	0.28–22.42	0.0508	4.06×10^{-5}	SL SS SW
Agrawal (1971)	19	Air–oil	5	0.01–0.06	0.11–6.16	0.0258	0	SS
Yu (1972)	15	Air–oil	5	0.10–0.32	0.07–0.62	0.0258	0	SL
Mattar (1973)	8	Air–oil	5	0.31–1.55	0.30–7.83	0.0258	0	SL
Aziz et al. (1974)	128	Air–oil	5	0.03–1.68	0.02–3.75	0.0258	0	DB SL
Companies[a]	141		3–19	0.07–6.26	0.32–63.44	0.0232	1.51×10^{-6}	
	146		3–19	0.07–5.96	0.28–57.09	0.0237	1.52×10^{-6}	
	61	Air–HL[b]	1–25	0.02–3.40	0.10–24.05	0.0381	4.57×10^{-5}	AN
	209	Air–water	1	0.001–1.04	0.09–61.30	0.0455	0	SL
	470	Air–oil	3–15	0.03–7.25	0.04–69.56	0.0502	1.53×10^{-6}	SS
	131		3–22	0.03–7.10	0.16–59.52	0.0909	1.53×10^{-6}	SW
	156		3–20	0.07–6.07	0.11–24.47	0.1402	1.53×10^{-6}	

[a] Data sets are identified as: SU28, SU29, SU184–187, SU199, SU24, SU25 and SU26.
[b] HL: hydrocarbon liquid.

Table 3
Tulsa data

Source	No. Exp.	μ_L (mPa s)	U_{SL} (m/s)	U_{SG} (m/s)	D (m)	ε (m)	FP
Beggs (1972)		1				0	AN
	21		0.03–2.62	0.31–24.97	0.0254		DB
	22		0.02–1.60	0.37–15.12	0.0381		SL SS SW
Cheremisinoff (1977)	151	1	0.02–0.07	2.58–24.01	0.0635	0	SS SW
Andritsos (1986)						0	AN
	92	1–70	0.001–0.06	4.49–30.09	0.0252		SL
	111	1–80	0.001–0.19	4.29–29.51	0.0953		SS SW
Mukherjee (1979)	44	1	0.03–3.40	0.23–24.06	0.0381	1.16×10^{-6}	AN SL SS SW
Kokal (1987)	10	8	0.03–0.06	1.18–11.51	0.0512	0	SS
	13		0.05–0.15	1.01–9.01	0.0763		SW

In order to compare predicted friction factor ($f_{M,pred}$) with experimental data ($f_{M,exp}$), we use the following eight commonly used statistical parameters (Gregory and Fogarasi, 1985; Xiao et al., 1990; Ouyang, 1998; García et al., 2003; García et al., 2005). The statistical parameters are defined as:

Table 4
Experimental data obtained of the specialized literature

Source	No. Exp.	Fluid	μ_L (mPa s)	U_{SL} (m/s)	U_{SG} (m/s)	D (m)	ε (m)	FP
Johnson and Abou-Sabe (1952)	33 + 2[a]	Air–water	1	0.32–4.95	0.63–30.79	0.0221	0	AN DB SL SW SL-AN
Johnson (1955)	17	Air–oil	25	0.35–2.60	1.02–43.03	0.0187	0	AN SL
Reid et al. (1957)	15 28	Air–water	1	0.75–1.52 0.65–1.70	2.86–16.16 1.32–6.74	0.1023 0.1541	4.57×10^{-5}	SL
Dos Santos (2002)	30	Air–oil	100	0.17–3.19	0.02–1.94	0.0254	0	DB SL

[a] Transition points.

Table 5
Summary of the 2560 experimental data processed for friction factor

Variable	Mean	Standard deviation	Minimum	Median	Maximum
U_{SL} (m/s)	1.158	1.481	0.001	0.544	7.254
U_{SG} (m/s)	7.682	10.648	0.015	3.637	69.558
U_M (m/s)	8.839	10.290	0.098	5.590	69.602
μ_L (mPa s)	47.5	144.1	0.7	4.8	1118.2
D (m)	0.0545	0.0321	0.0187	0.0502	0.1541
ε (m)	5.51E−06	1.38E−05	0.0	1.51E−06	4.60E−05
dp/dz (Pa/m)	2588.8	4291.1	1.2	816.7	37421.0

Table 6
Parameters of the universal composite improved correlations for mixture friction factor

Range	a_1	b_1	a_2	b_2	c	d	t
$1 > H_L \geqslant 0.5$	16.0019	−0.9904	0.4548	−0.3448	2.4848	0.0597	295
$0.5 > H_L \geqslant 0.4$	16.0077	−0.9815	0.3082	−0.2790	2.4364	0.0445	295
$0.4 > H_L \geqslant 0.3$	16.0136	−0.9198	0.0399	−0.1611	5.2260	0.1142	295
$0.3 > H_L \geqslant 0.2$	15.9999	−0.8619	0.1786	−0.2649	3.2130	0.1364	10,000
$0.2 > H_L \geqslant 0.1$	16.0110	−0.8385	0.0158	−0.0949	4.8460	0.1079	10,000
$0.1 > H_L \geqslant 0.05$	16.2165	−0.7890	0.1421	−0.2986	2.7274	0.2588	10,000
$0.05 > H_L > 0$	15.8771	−0.7408	0.5794	−0.0647	0.5051	0.0232	1,000,000

$$E_1 = \frac{1}{n}\sum_{i=1}^{n} r_i \tag{6}$$

$$E_2 = \frac{1}{n}\sum_{i=1}^{n} |r_i| \tag{7}$$

$$E_3 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(r_i - E_1)^2} \tag{8}$$

$$E_4 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(r_i)^2} \tag{9}$$

$$E_5 = \frac{1}{n}\sum_{i=1}^{n} e_i \tag{10}$$

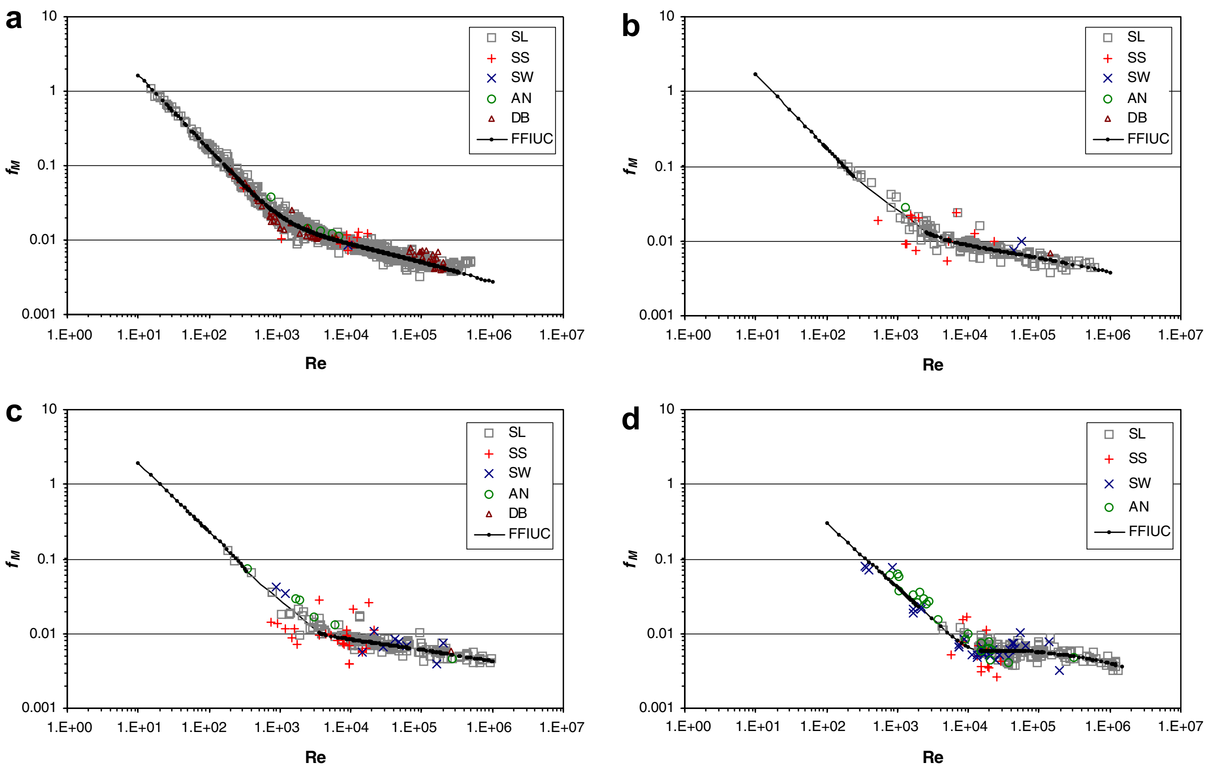

Fig. 1. Mixture friction factor improved correlations for: (a) $1 > H_L \geqslant 0.5$, (b) $0.5 > H_L \geqslant 0.4$, (c) $0.4 > H_L \geqslant 0.3$, and (d) $0.3 > H_L \geqslant 0.2$.

$$E_6 = \frac{1}{n}\sum_{i=1}^{n}|e_i| \tag{11}$$

$$E_7 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(e_i - E_5)^2} \tag{12}$$

$$E_8 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(e_i)^2} \tag{13}$$

where $r_i = \left[\frac{f_{M,pred} - f_{M,exp}}{f_{M,exp}}\right] \times 100$, $e_i = f_{M,pred} - f_{M,exp}$ and n is the number of the experimental data. E_1 is the average percent error, E_2 is the average absolute percent error, E_3 is the standard deviation of the average absolute percent error, E_4 is the root mean square percent error, E_5 is the average error, E_6 is the average absolute error, E_7 is the standard deviation of the average absolute error and E_8 is the root mean square error.

The average percent error E_1 is a measure of the agreement between predicted and measured data. It indicates the degree of overprediction (positive values) or underprediction (negative values). Similarly, the average absolute percent error E_2 is a measure of the agreement between predicted and measured data. However, in this parameter the positive errors and the negative errors do not cancel each other. For this reason, the average absolute percent error is a key parameter to evaluate the prediction capability of models and correlations. The standard deviation percent error E_3 indicates how large the errors are on the average. The root mean square percent error E_4 indicates how close the predictions are to the experimental data. The statistical parameters E_5, E_6, E_7 and E_8 are similar to E_1, E_2, E_3 and E_4 but the difference is that they are not based on the errors relative to the experimental mixture friction factor.

The statistical parameters E_1–E_8 for each correlation are presented in Table 7.

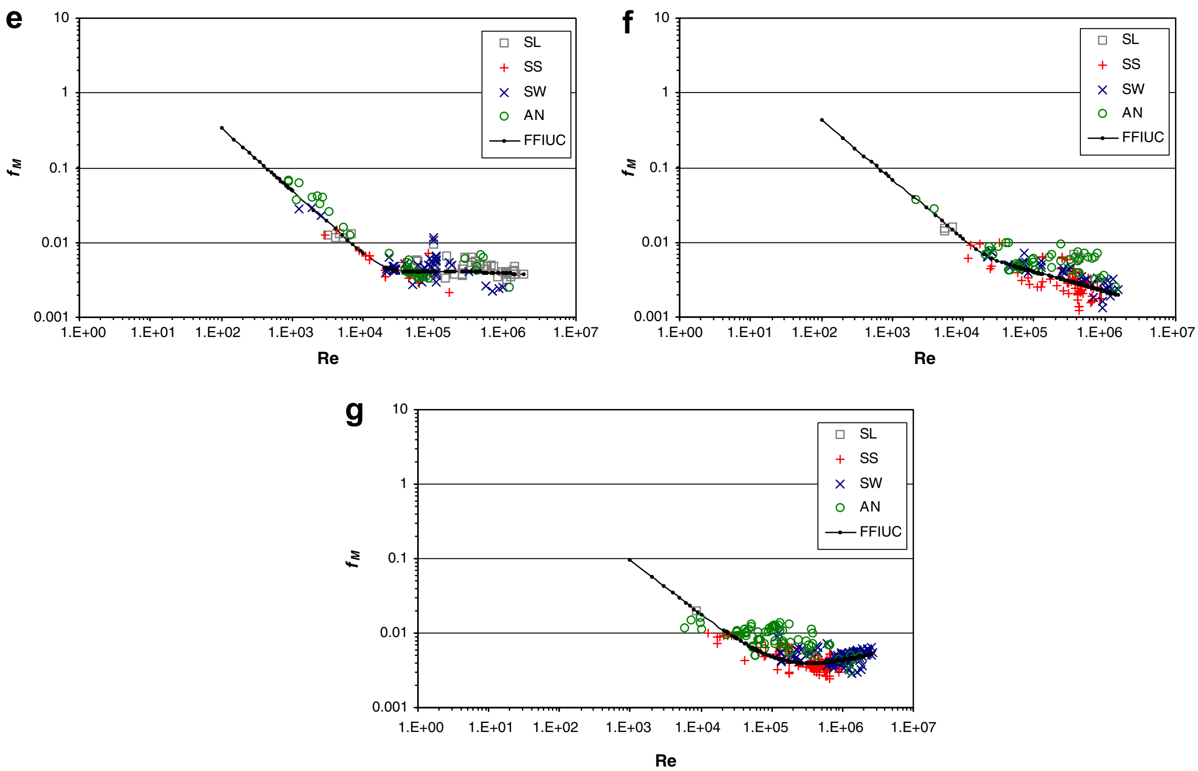

Fig. 2. Mixture friction factor improved correlations for: (e) $0.2 > H_L \geqslant 0.1$, (f) $0.1 > H_L \geqslant 0.05$, and (g) $0.05 > H_L > 0$.

Table 7
Statistical parameters of the universal improved correlations for gas–liquid friction factor

Range	E_1 (%)	E_2 (%)	E_3 (%)	E_4 (%)	E_5	E_6	E_7	E_8
$1 > H_L \geqslant 0.5$	−0.51	10.05	14.11	14.12	-8.35×10^{-4}	2.46×10^{-3}	6.80×10^{-3}	6.85×10^{-3}
$0.5 > H_L \geqslant 0.4$	−0.03	14.59	23.64	23.64	-9.52×10^{-4}	2.13×10^{-3}	4.22×10^{-3}	4.33×10^{-3}
$0.4 > H_L \geqslant 0.3$	−1.51	15.94	24.97	25.02	-9.37×10^{-4}	2.04×10^{-3}	4.03×10^{-3}	4.14×10^{-3}
$0.3 > H_L \geqslant 0.2$	0.80	18.28	25.29	25.30	-4.58×10^{-4}	2.02×10^{-3}	4.60×10^{-3}	4.62×10^{-3}
$0.2 > H_L \geqslant 0.1$	−4.20	19.55	25.68	26.03	-9.70×10^{-4}	1.81×10^{-3}	3.81×10^{-3}	3.93×10^{-3}
$0.1 > H_L \geqslant 0.05$	−5.40	25.94	33.68	34.12	-6.59×10^{-4}	1.18×10^{-3}	1.61×10^{-3}	1.74×10^{-3}
$0.05 > H_L \geqslant 0$	−6.28	21.85	28.26	28.95	-7.59×10^{-4}	1.54×10^{-3}	2.43×10^{-3}	2.55×10^{-3}

Universal improved correlations for gas–liquid friction factor in horizontal pipelines have an average error of −1.9% and an average absolute error of 15.4%. 85.2% of the points (1859 experimental points) are in the band between ±30% and 74.5% of the points (1626 experimental points) are in the band between ±20%. The best agreements are obtained for slug and dispersed bubble flow data, with an average absolute error of 10.6% and 14.5%, respectively. The worst agreements are obtained for annular and stratified flow data, with an average absolute error of 29.8% and 22.8%, respectively.

5. Improved correlations for gas–liquid friction factor sorted by flow pattern (FFIPC)

The 2183 experimental points were classified by flow type: 1393 slug flow SL, 67 dispersed bubble flow DB, 532 stratified flow and 191 annular flow. The experimental data for slug flow, stratified flow and annular flow were classified by liquid holdup ranges and mixture friction factor improved correlations were created for each flow type. The liquid holdup is calculated to each one of the experimental points, according to the pattern of

Table 8
Parameters of the mixture friction factor improved correlations for each flow pattern

FP	Range	a_1	b_1	a_2	b_2	c	d	t
SL	$1 > H_L \geqslant 0.5$	16.9998	−1.0035	1.5254	−0.3688	2.8377	0.0153	295
	$0.5 > H_L \geqslant 0.3$	15.9841	−0.9561	0.5170	−0.2897	2.3547	0.0315	599
	$0.3 > H_L > 0$	15.9997	−0.8634	0.2259	−0.2692	1.1037	0.2618	10,000
DB	$1 > H_L > 0$	16.3996	−0.9915	0.0666	−0.2171	9.0732	0.1527	500
ST	$1 > H_L \geqslant 0.3$	16.0007	−1.2324	0.1568	−0.1523	−0.0084	0.2875	100,000
	$0.3 > H_L \geqslant 0.1$	15.7516	−0.9694	0.1370	−0.2088	−0.0551	0.3733	100,000
	$0.1 > H_L \geqslant 0.05$	15.9780	−0.9231	0.2090	−0.1239	−0.0619	0.1093	100,000
	$0.05 > H_L > 0$	15.9744	−0.7569	0.1448	−0.0647	0.5017	0.0502	100,000
AN	$1 > H_L \geqslant 0.1$	15.1288	−0.8583	0.7169	−0.151	0.1652	0.0304	100,000
	$0.1 > H_L \geqslant 0.05$	15.1898	−0.7883	0.7024	−0.3373	1.3049	0.3505	100,000
	$0.05 > H_L > 0$	15.1896	−0.7825	1.2561	−0.4108	1.0571	1.5895	100,000

Table 9
Statistical parameters of the mixture friction factor improved correlations for each flow pattern

FP	Range	E_1 (%)	E_2 (%)	E_3 (%)	E_4 (%)	E_5	E_6	E_7	E_8
SL	$1 > H_L \geqslant 0.5$	−0.94	9.56	13.18	13.21	-6.07×10^{-4}	2.38×10^{-3}	7.08×10^{-3}	7.11×10^{-3}
	$0.5 > H_L \geqslant 0.3$	−0.67	10.66	15.28	15.30	-3.91×10^{-4}	1.46×10^{-3}	3.15×10^{-3}	3.17×10^{-3}
	$0.3 > H_L > 0$	0.64	15.73	20.77	20.78	-2.45×10^{-4}	9.77×10^{-4}	1.49×10^{-3}	1.51×10^{-3}
DB	$1 > H_L > 0$	−4.08	10.25	13.29	13.91	-6.49×10^{-4}	1.30×10^{-3}	2.08×10^{-3}	2.18×10^{-3}
ST	$1 > H_L \geqslant 0.3$	−15.44	33.67	37.12	40.26	-4.16×10^{-3}	5.18×10^{-3}	8.29×10^{-3}	9.30×10^{-3}
	$0.3 > H_L \geqslant 0.1$	−8.24	26.47	32.92	33.94	-2.00×10^{-3}	2.81×10^{-3}	6.44×10^{-3}	6.75×10^{-3}
	$0.1 > H_L \geqslant 0.05$	−2.56	19.01	23.51	23.65	-3.20×10^{-4}	7.64×10^{-4}	1.12×10^{-3}	1.16×10^{-3}
	$0.05 > H_L > 0$	−2.03	15.32	19.89	19.99	-2.65×10^{-4}	8.21×10^{-4}	1.14×10^{-3}	1.17×10^{-3}
AN	$1 > H_L \geqslant 0.1$	−3.50	15.42	20.01	20.32	-6.55×10^{-4}	3.42×10^{-3}	6.80×10^{-3}	6.83×10^{-3}
	$0.1 > H_L \geqslant 0.05$	−5.93	20.15	25.86	26.54	-2.39×10^{-4}	1.93×10^{-3}	3.91×10^{-3}	3.92×10^{-3}
	$0.05 > H_L > 0$	−4.70	21.62	26.61	27.03	-1.03×10^{-3}	2.07×10^{-3}	2.58×10^{-3}	2.79×10^{-3}

flow, applying the holdup correlations sorted by flow pattern FPHC, developed by García et al. (2005). The parameters a_1, b_1, a_2, b_2, c, d and t, of the improved correlations are presented in Table 8.

The statistical parameters E_1–E_8 for each correlation are presented in Table 9.

The mixture friction factor improved correlations for slug flow have an average error of −0.7% and an average absolute error of 10.7%. 94.3% of the points (1313 experimental points) are in the band between ±30% and 86.1% of the points (1200 experimental points) are in the band between ±20%. The improved correlation for dispersed bubble flow has an average error of −4.1% and an average absolute error of 10.3%. 82.1% of the points (55 experimental points) are in the band between ±20%. The average error for stratified flow improved correlations is −5.2% and the average absolute error is 21.3%. 73.9% of the 532 points (393 points) are in the band between ±30%. The average error for annular flow improved correlations is −4.7% and the average absolute error is 19.5%. 76.4% of the 191 points (146 points) are in the ±30% range.

6. Performance comparison of correlations and models for pressures drop from various sources against the 2560 experimental data

In this section, we evaluate the performance of our correlations against 2560 pressure drop experimental data. We also compare 11 other correlations, six homogeneous models and four mechanistic models reported in the literature. Table 10 presents the acronyms used to identify the correlations developed in this work, as well as the acronyms for Lockhart and Martinelli (1949), Reid et al. (1957), Hoogendoorn (1959), Dukler et al. (1964), Beggs and Brill (1973), Kadambi (1981), Müller-Steinhagen and Heck (1986), Ortega et al. (2001) and

Table 10
Acronyms for the 24 tested models

Model or correlation	Acronyms
McAdams et al. (1942)	MHM
Lockhart and Martinelli (1949)	LMC
Reid et al. (1957)	REC
Hoogendoorn (1959)	HOC
Cicchitti et al. (1960)	CHM
Dukler et al. (1964)	DUC
Wallis (1969)	WHM
Beggs and Brill (1973)	BBC
Oliemans (1976)	OLHM
Kadambi (1981)	KAC
Beattie and Whalley (1982)	BWHM
Müller-Steinhagen and Heck (1986)	MHC
Xiao et al. (1990)	XMM
Ouyang (1998)	OMM
Ouyang (1998)	OHM
Gómez et al. (2000)	GMM
Ortega et al. (2001)	ORC
Padrino et al. (2002)	PMM
Chen et al. (2002)	CHC
García et al. (2003)	FFUC
García et al. (2003)	FFPC
Friction factor improved universal correlations (Eqs. (3)–(5), Table 6)	FFIUC
Friction factor improved flow pattern correlations (Eqs. (3)–(5), Table 8)	FFIPC

Chen et al. (2002) correlations; McAdams et al. (1942), Cicchitti et al. (1960), Wallis (1969), Oliemans (1976), Beattie and Whalley (1982) and Ouyang (1998) homogeneous models, and Xiao et al. (1990), Ouyang (1998), Gómez et al. (2000) and Padrino et al. (2002) mechanistic models.

In the application of homogeneous flow models of McAdams et al. (1942), citeyearbib12, Wallis (1969) and Beattie and Whalley (1982), we are only considering the frictional pressure gradient. The ϕ_L parameter, in the separated flow model of Lockhart and Martinelli (1949), is evaluated with the Chisholm (1967) correlation. The liquid holdup for Dukler et al. (1964) correlation is predicted from Bankoff's correlation as is suggested by Oballa et al. (1997).

The comparison of the accuracy of pressure gradient prediction of the correlations and the models from different authors against 2560 experimental points is carried out using the Ripley range factor R recommend by Govan (1988).

$$R = \exp\left[t_{0.01}E_{10}\sqrt{1+\frac{1}{n}}\right] \tag{14}$$

$$E_9 = \frac{1}{n}\sum_{i=1}^{n} e'_i \tag{15}$$

$$E_{10} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(e'_i - E_{10})^2} \tag{16}$$

where $t_{0.01}$ is the t-distribution with $n-1$ degrees of freedom, n is the number of the experimental data. E_9 is the average error and E_{10} is the standard deviation calculated as $e'_i = \ln[(\Delta p/L)_{\text{pred}}/(\Delta p/L)_{\text{exp}}]$. The range factor concept is useful when a measure of the prediction level of confidence of a given model is required (Govan, 1988).

The evaluation results are shown in Table 11. The statistical parameters E_1–E_8 (Eqs. (6)–(13)) for each model and correlation are also included, where $r_i = \left[\frac{(\Delta p/L)_{\text{pred}}-(\Delta p/L)_{\text{exp}}}{(\Delta p/L)_{\text{exp}}}\right] \times 100$ and $e_i = (\Delta p/L)_{\text{pred}} - (\Delta p/L)_{\text{exp}}$.

Table 11
Accuracy comparison of the pressure gradient prediction of the 24 correlations and models from different authors against 2560 experimental points

Model or correlation	Statistical parameters									
	R	E_1 (%)	E_2 (%)	E_3 (%)	E_4 (%)	E_5 (Pa/m)	E_6 (Pa/m)	E_7 (Pa/m)	E_8 (Pa/m)	$E_9 \times 10^2$ ( )	$E_{10} \times 10^2$ ( )
FFIPC	2.49	−3.1	18.1	36.9	37.0	−91.5	303.0	737.3	743.0	−8.2	35.4
FFIUC	2.49	−1.3	19.5	39.0	39.1	−100.4	328.1	818.1	824.2	−6.7	35.4
FFPC	2.68	−5.5	22.0	42.3	42.7	−92.2	386.3	1070.4	1074.4	−11.9	38.2
WHM	2.86	−8.1	26.1	48.8	49.5	−401.0	507.2	1163.6	1230.8	−16.1	40.7
FFUC	2.87	−6.6	24.7	45.6	46.1	−213.9	416.6	1077.6	1098.7	−14.4	40.8
OLHM	2.91	5.2	29.3	57.6	57.8	−240.8	545.7	1296.2	1318.4	−3.4	41.4
BWHM	3.08	19.4	35.2	65.0	67.8	229.4	601.0	1488.5	1506.1	8.5	43.6
DUC	3.24	−6.8	24.8	56.6	57.0	−97.4	435.7	1089.0	1093.4	−16.0	45.6
BBC	3.29	56.4	63.1	191.2	199.4	1878.9	1973.3	22994.8	23071.5	31.6	46.2
LMC	3.33	20.2	39.3	85.6	88.0	10.1	526.5	1469.8	1469.9	6.1	46.7
MHC	3.68	7.9	41.0	85.9	86.2	−543.4	641.8	1422.8	1523.1	−6.3	50.5
PMM	3.98	−16.5	27.8	40.6	43.8	1.5	532.5	1301.7	1301.7	−29.1	53.6
XMM	4.04	5.0	39.0	108.9	109.1	−170.7	521.9	1197.5	1209.6	−12.9	54.1
ORC	4.57	63.7	75.7	130.7	145.5	452.1	711.7	2444.2	2485.7	30.9	58.9
OHM	5.94	−25.0	32.8	43.4	50.0	−313.8	492.0	1204.0	1244.2	−46.5	69.1
MHM	7.48	−7.6	37.7	66.4	66.9	−647.2	732.6	1531.7	1662.9	−29.8	78.1
KAC	7.93	−55.1	62.9	51.7	75.5	−1638.8	1726.8	3088.8	3496.8	−109.8	80.3
GMM	9.75	35.1	67.0	636.3	637.3	−114.5	686.3	2488.7	2491.3	−3.7	88.3
CHC	10.23	1.6	61.2	136.6	136.6	−401.5	1383.6	3657.0	3679.0	−37.8	90.2
CHM	11.05	257.1	264.5	722.0	766.4	2258.1	2504.2	9497.1	9762.0	61.9	93.2
REC	11.72	173.8	196.6	599.0	623.7	1355.7	1916.7	7313.3	7438.0	30.0	95.5
OMM	11.85	90.8	132.4	305.7	318.9	−281.3	541.6	1075.8	1112.0	3.4	95.9
HOC	14.49	10.9	54.2	175.1	175.4	3.9	966.9	2994.9	2994.9	−30.1	103.7

In the general evaluation the FFIPC and FFIUC correlations, developed in this work, have the best performances in the pressure gradient prediction. The improved correlations (FFIPC) which have been sorted by flow type have the best performance with an average absolute error of 18.1%. The universal improved correlations (FFIUC), in which flow patterns are ignored, have the second best performance with an average absolute error of 19.5%. The mixture friction factor correlation FFPC that do not account for the relative velocity of the phases, developed by García et al. (2003), obtain the third best performance with an average absolute error of 22.0%. The Wallis (1969) homogeneous model shows the fourth best performance with an average absolute error of 26.1%. The mixture friction factor correlation FFUC gets the fifth best performance with an average absolute error of 24.7%. The models and correlations developed by Reid et al. (1957), Hoogendoorn (1959), Cicchitti et al. (1960), Beggs and Brill (1973), Kadambi (1981), Ouyang (1998), Ortega et al. (2001), Gómez et al. (2000) and Chen et al. (2002) obtain average absolute errors higher to 50%.

For flow of high viscosity oils in pipes, it is interesting to evaluate the pressure gradient prediction for those experiments where oil viscosity $\mu_L \geqslant 400$ mPa s. The comparison of the accuracy of pressure gradient prediction of the correlations and the models against high viscosity data are shown in Table 12.

The improved correlations FFIPC, developed in this work, again show the first best performance in the pressure gradient prediction against the experimental data of gas and high viscosity liquid flow, with an average absolute error of 10.9%. The Lockhart and Martinelli (1949) correlation shows the second best performance with an average absolute error of 15.6%. The improved correlations FFIUC attain the third best performance in the pressure gradient prediction against the experimental data of gas and high viscosity liquid flow, with an average absolute error of 13.7%. The mixture friction factor correlations FFPC, developed by García et al. (2003), obtain the fourth best performance with an average absolute error of 13.8%. The models and correlations developed by McAdams et al. (1942), Reid et al. (1957), Hoogendoorn (1959), Cicchitti et al. (1960), Oliemans (1976), Kadambi (1981), Beattie and Whalley (1982) and Chen et al. (2002) obtain average absolute errors higher to 50%.

Table 12
Accuracy comparison of the pressure gradient prediction of the 24 correlations and models from different authors against high viscosity data ($\mu_L \geqslant 400$ mPa s)

Model or correlation	Statistical parameters										
	R	E_1 (%)	E_2 (%)	E_3 (%)	E_4 (%)	E_5 (Pa/m)	E_6 (Pa/m)	E_7 (Pa/m)	E_8 (Pa/m)	$E_9 \times 10^2$ ( )	$E_{10} \times 10^2$ ( )
FFIPC	1.62	−3.1	10.9	17.8	18.0	−305.1	467.4	927.6	976.9	−4.8	18.3
LMC	1.62	12.0	15.6	22.9	25.8	264.4	451.7	627.0	681.0	9.5	18.3
FFIUC	1.72	−0.6	13.7	20.5	20.5	−243.5	557.7	961.0	991.6	−2.6	20.6
FFPC	1.82	−7.7	13.8	17.9	19.5	−324.7	548.5	947.6	1002.1	−10.3	22.8
ORC	1.96	−15.7	17.1	17.7	23.7	−784.3	806.2	1345.2	1558.8	−19.9	25.6
XMM	3.03	−14.6	21.9	32.2	35.4	−774.5	919.3	1624.2	1800.9	−23.4	42.1
FFUC	3.05	−17.8	22.2	27.0	32.4	−839.1	990.4	1678.1	1877.8	−27.1	42.4
BBC	3.31	8.4	32.2	35.6	36.6	409.7	1431.6	1948.9	1991.9	−0.1	45.5
OMM	4.38	−24.8	25.7	28.4	37.8	−1139.8	1160.5	1800.7	2133.7	−40.4	56.2
DUC	4.41	−33.3	34.3	31.3	45.8	−1215.7	1275.5	1948.2	2299.2	−54.4	56.4
OHM	4.45	−25.3	26.1	28.5	38.2	−1148.1	1168.2	1802.4	2139.7	−41.3	56.8
WHM	4.56	−26.0	26.7	28.8	38.8	−1160.4	1180.0	1811.2	2153.7	−42.6	57.7
BWHM	5.95	76.8	96.5	82.0	112.6	2788.1	3536.0	3419.6	4419.8	39.9	67.8
KAC	6.33	−92.1	92.1	5.0	92.7	−4340.1	4340.1	3086.6	5341.0	−276.0	70.2
MHC	6.49	−25.9	27.0	31.0	40.4	−1150.4	1174.8	1892.7	2217.5	−47.1	71.1
PMM	7.50	−5.5	25.5	38.2	38.6	−67.5	1209.1	2097.2	2098.3	−23.9	76.6
CHC	7.70	−90.6	90.6	11.2	91.7	−4144.6	4144.6	2923.0	5086.2	−271.3	77.6
OLHM	8.08	22.4	58.9	74.3	77.6	409.5	1926.0	2576.2	2608.8	−4.4	79.4
MHM	8.12	−90.0	90.0	8.4	90.8	−4160.3	4160.3	2918.0	5096.3	−259.0	79.6
GMM	19.40	−19.6	42.7	50.6	54.3	−809.0	1614.7	2336.9	2474.1	−45.1	112.8
REC	22.88	824.5	824.5	1307.9	1548.0	19748.0	19750.0	25071.3	31967.4	148.2	119.0
CHM	27.47	1039.6	1039.6	1616.6	1924.5	25161.3	25162.4	31889.5	40687.6	163.6	126.0
HOC	176.09	219.0	260.1	465.0	514.3	4819.4	6670.3	9668.5	10812.3	14.1	196.6

Additionally, we evaluate the pressure gradient prediction of the different models and correlations when the experimental data are sorted by flow pattern. The following data are used: 1493 slug flow data points, 67 dispersed bubble data points, 693 stratified flow data points, and 264 annular flow data points. The experimental data points (43 points) corresponding to transitions were not considered in this evaluation. The statistical parameters E_1–E_{10} for each flow pattern are presented in Tables 13–16.

The FFIPC correlations developed in this work, which have been sorted by flow type, show the best performance for slug flow, dispersed bubble flow and annular flow with average absolute errors of 11.9%, 10.2% and 25.3%, respectively. In general, the performance of the universal improved correlations FFIUC developed in this work, which do not depend on flow pattern, is excellent. For stratified flow the universal improved correlations FFIUC have the best performance with an average absolute error of 33.1%. For slug flow and annular flow the universal improved correlations FFIUC have the second best performance with average absolute errors of 11.8% and 28.4%, respectively. In general, most of the models and correlations present high average absolute errors when they are evaluated against the annular flow and stratified flow experimental data.

7. Summary and conclusions

Data from 2183 of the 2560 gas–liquid flow experiments in horizontal pipelines were compiled and processed in order to develop mixture friction factor improved correlations that consider the effect of the relative velocity of the phases. This database gathers the widest range of operational conditions and fluid properties so far compiled to develop mixture friction factor correlations for gas–liquid flow in horizontal pipelines.

Friction factor correlations obtained in this work are composite analytical expressions that fit the transition region between laminar and turbulent flow with a logistic dose curve. Logistic dose curves lead to rational fractions of power laws which reduce to the power laws for laminar flow when the Reynolds number is low and to turbulent flow when the Reynolds number is large.

Table 13
Evaluation of the models and correlations using the 1493 slug flow data

Model or correlation	Statistical parameters									
	R	E_1 (%)	E_2 (%)	E_3 (%)	E_4 (%)	E_5 (Pa/m)	E_6 (Pa/m)	E_7 (Pa/m)	E_8 (Pa/m)	$E_9 \times 10^2$ ( )	$E_{10} \times 10^2$ ( )
FFIPC	1.64	0.1	11.9	18.1	18.1	−53.8	326.6	735.3	737.3	−1.5	19.2
FFIUC	1.66	−0.9	11.8	17.7	17.7	−44.7	324.1	722.7	724.1	−2.7	19.7
FFPC	1.74	−3.8	13.7	18.7	19.1	−112.5	389.4	936.0	942.7	−5.9	21.4
FFUC	1.74	−5.9	14.1	18.3	19.3	−183.8	398.7	954.1	971.7	−8.2	21.5
MHC	1.77	−18.2	21.9	18.5	26.0	−776.7	806.8	1524.6	1711.1	−22.4	22.0
WHM	1.81	−13.1	18.1	19.7	23.6	−468.0	561.2	1126.7	1220.1	−16.5	23.1
OHM	1.86	−11.0	16.8	21.0	23.7	−258.0	447.4	867.4	905.0	−14.3	24.1
OLHM	2.02	2.3	20.8	32.4	32.5	−253.0	634.4	1355.4	1378.9	−1.6	27.2
BBC	2.04	34.0	37.3	51.5	61.7	1460.6	1493.5	3844.6	4112.8	25.0	27.5
ORC	2.13	10.8	23.1	40.0	41.4	394.9	640.4	1717.4	1762.2	5.5	29.4
OMM	2.17	−15.0	21.6	24.9	29.1	−319.8	460.0	799.3	860.9	−20.4	30.0
BWHM	2.23	16.2	25.8	41.8	44.9	327.9	653.1	1399.0	1436.9	9.9	31.1
DUC	2.24	−2.5	18.7	29.7	29.8	71.8	467.7	1063.5	1065.9	−7.0	31.3
XMM	2.26	−8.8	20.9	41.2	42.1	−267.0	530.8	1100.6	1132.6	−14.5	31.6
PMM	2.33	−5.1	17.7	24.3	24.9	214.1	565.8	1307.8	1325.2	−9.5	32.8
LMC	2.45	4.9	25.3	51.1	51.3	−12.8	658.6	1724.7	1724.7	−2.2	34.8
REC	3.94	39.9	58.7	247.2	250.4	273.4	969.3	2575.8	2590.2	7.8	53.2
HOC	4.29	27.9	50.3	173.3	175.6	121.1	911.7	2178.0	2181.4	2.0	56.4
CHM	4.54	78.2	87.6	389.1	396.9	994.3	1306.8	3638.1	3771.7	24.1	58.6
MHM	5.96	−19.6	26.3	30.8	36.5	−742.4	820.7	1419.8	1602.3	−35.9	69.2
KAC	6.08	−60.8	65.5	34.5	70.0	−2081.0	2201.1	3385.6	3974.4	−117.8	70.0
GMM	7.29	−8.4	30.8	73.1	73.6	−269.1	805.8	2568.0	2582.1	−23.9	77.0
CHC	8.90	13.8	62.0	159.3	159.9	−223.4	1773.6	4456.7	4462.3	−24.5	84.7

Table 14
Evaluation of the models and correlations using the 67 dispersed bubble flow

Model or correlation	Statistical parameters									
	R	E_1 (%)	E_2 (%)	E_3 (%)	E_4 (%)	E_5 (Pa/m)	E_6 (Pa/m)	E_7 (Pa/m)	E_8 (Pa/m)	$E_9 \times 10^2$ ( )	$E_{10} \times 10^2$ ( )
FFIPC	1.49	−4.0	10.2	13.3	13.9	−331.1	603.9	992.6	1047.2	−5.2	15.0
DUC	1.54	−3.6	10.7	13.9	14.4	−226.9	554.2	779.4	812.2	−4.8	16.2
ORC	1.55	−9.8	11.9	13.2	16.5	−600.3	775.4	1110.5	1264.5	−11.5	16.4
FFUC	1.56	−6.9	12.1	14.6	16.1	−537.5	853.2	1362.1	1465.8	−8.5	16.7
BBC	1.58	11.2	16.6	18.2	21.4	698.5	911.4	1131.0	1332.1	9.2	17.1
OHM	1.58	−11.3	13.6	14.2	18.2	−724.9	912.9	1349.3	1534.3	−13.3	17.2
FFPC	1.59	−4.3	12.0	15.7	16.3	−345.6	852.3	1438.9	1480.4	−5.8	17.3
OMM	1.59	−11.3	13.7	14.4	18.4	−728.3	916.6	1355.5	1541.4	−13.5	17.4
LMC	1.60	−3.5	12.0	15.5	15.9	−233.1	611.2	894.5	924.8	−5.0	17.7
MHC	1.60	−13.4	15.3	14.4	19.7	−824.1	997.8	1496.3	1711.2	−15.8	17.7
CHM	1.62	−6.8	12.9	15.5	17.0	−349.3	870.9	1312.7	1359.1	−8.5	18.0
WHM	1.68	−13.0	15.1	15.4	20.2	−808.0	977.9	1493.0	1700.5	−15.7	19.4
FFIUC	1.68	0.7	14.5	19.5	19.6	−6.9	1026.5	1695.4	1695.4	−1.1	19.5
REC	1.69	−9.7	14.6	16.4	19.1	−542.9	981.3	1525.2	1620.3	−12.0	19.7
HOC	1.73	−8.5	13.8	16.9	19.0	−471.2	937.2	1477.2	1551.7	−10.8	20.5
OLHM	1.77	−12.1	16.9	17.7	21.5	−752.0	1143.1	1827.3	1978.2	−15.0	21.5
PMM	1.81	−2.0	16.5	21.3	21.4	118.3	1171.9	1750.2	1754.3	−4.4	22.2
BWHM	1.83	−3.8	16.1	20.8	21.1	−26.8	1065.9	1681.0	1681.2	−6.3	22.6
XMM	1.83	−17.0	18.9	17.1	24.2	−819.7	995.9	1269.7	1514.7	−20.9	22.7
MHM	4.39	−22.1	22.8	25.6	33.9	−1724.1	1754.9	2593.4	3121.4	−35.4	55.4
CHC	4.96	−4.5	29.8	38.5	38.7	−837.3	2168.1	3350.3	3454.8	−17.5	60.0
KAC	55.74	−13.9	108.3	167.8	168.4	−4183.7	4847.7	4332.1	6044.5	−128.8	150.5
GMM	94.73	−13.4	15.8	20.2	24.3	−738.8	912.0	1344.4	1536.7	−42.2	170.4

Table 15
Evaluation of the models and correlations using the 693 stratified flow data

Model or correlation	Statistical parameters									
	R	E_1 (%)	E_2 (%)	E_3 (%)	E_4 (%)	E_5 (Pa/m)	E_6 (Pa/m)	E_7 (Pa/m)	E_8 (Pa/m)	$E_9\times10^2$ ( )	$E_{10}\times10^2$ ( )
FFIUC	4.16	5.5	33.1	66.4	66.7	−15.4	53.9	151.2	151.9	−7.2	55.2
OLHM	4.24	25.6	47.2	93.3	96.8	−14.4	56.0	156.3	157.0	8.4	55.9
FFIPC	4.26	−6.8	29.8	62.4	62.8	−54.7	66.1	251.9	257.8	−19.6	56.1
BWHM	4.56	38.6	55.2	100.0	107.2	30.2	64.5	153.6	156.6	17.7	58.7
WHM	4.71	12.0	41.1	83.4	84.3	−29.6	59.4	163.6	166.3	−3.9	60.0
FFPC	4.76	−7.3	37.8	73.3	73.7	−44.7	65.4	181.6	187.1	−23.2	60.3
PMM	4.79	−30.8	41.3	58.9	66.4	−65.2	79.6	203.3	213.5	−52.5	60.6
FFUC	4.87	2.4	42.2	78.4	78.4	−29.5	66.5	168.7	171.2	−14.2	61.2
BBC	5.17	86.4	98.0	128.3	154.7	119.2	149.9	577.4	589.6	46.4	63.5
DUC	5.23	−7.0	36.1	96.6	96.9	−54.7	74.7	214.9	221.7	−24.9	64.0
LMC	5.31	61.2	76.0	135.9	149.0	61.6	96.8	199.4	208.7	27.8	64.6
MHC	5.95	71.5	90.0	142.4	159.4	18.7	89.3	176.2	177.2	33.3	69.0
ORC	6.68	173.9	184.8	193.0	259.9	174.9	207.8	408.4	444.3	81.1	73.5
XMM	7.60	18.2	62.2	162.4	163.4	63.0	130.4	410.9	415.7	−17.7	78.5
CHC	8.69	0.9	58.6	102.5	102.5	−64.7	112.8	402.6	407.8	−31.3	83.6
KAC	8.90	−42.4	52.2	63.4	76.3	−130.6	133.0	444.9	463.7	−83.0	84.6
GMM	8.97	89.3	112.1	169.4	191.6	59.1	109.1	190.4	199.4	35.5	84.9
OHM	9.12	−42.7	56.7	64.8	77.6	−84.8	91.2	187.3	205.6	−89.3	85.5
MHM	9.16	30.7	62.6	107.4	111.7	−33.7	83.9	244.3	246.6	−0.6	85.7
CHM	13.91	563.9	568.1	1018.5	1164.4	1247.6	1253.7	5291.5	5436.8	129.0	101.9
REC	24.50	376.8	400.7	870.8	948.9	845.4	890.6	3521.3	3621.5	68.8	123.8
OMM	32.00	373.2	406.5	476.6	605.5	164.0	286.9	406.5	438.4	85.9	134.1
HOC	33.40	−34.3	45.1	74.1	81.6	−115.6	117.6	444.4	459.3	−91.1	135.7

Table 16
Evaluation of the models and correlations using the 264 annular flow data

Model or correlation	Statistical parameters									
	R	E_1 (%)	E_2 (%)	E_3 (%)	E_4 (%)	E_5 (Pa/m)	E_6 (Pa/m)	E_7 (Pa/m)	E_8 (Pa/m)	$E_9\times10^2$ ( )	$E_{10}\times10^2$ ( )
FFIPC	2.30	−12.1	25.3	28.6	31.1	−281.9	646.3	1129.2	1164.0	−18.0	32.0
FFIUC	2.34	−22.5	28.4	25.5	34.0	−616.7	797.5	1350.7	1485.3	−30.7	32.8
FFPC	2.54	−9.4	30.0	33.3	34.6	122.8	958.3	2101.8	2105.4	−16.2	35.8
LMC	2.70	5.3	32.5	39.7	40.1	−0.9	845.6	1883.2	1883.2	−2.0	38.3
DUC	3.06	−28.0	32.0	26.7	38.7	−918.2	1005.0	1640.8	1881.1	−41.0	43.0
BWHM	3.13	−13.7	34.9	38.6	41.0	−125.4	1148.5	2453.2	2456.4	−24.3	43.9
FFUC	3.26	−32.2	42.4	33.8	46.7	−631.8	1182.7	2044.6	2140.4	−49.7	45.4
OLHM	3.34	−25.6	34.5	30.7	40.0	−614.0	1074.2	1924.2	2020.2	−39.2	46.4
WHM	3.36	−28.4	35.4	29.3	40.8	−742.2	1114.4	1937.7	2075.5	−42.9	46.7
ORC	3.59	105.0	111.8	84.6	135.0	1961.2	2376.1	6099.9	6408.6	61.5	49.1
MHC	4.43	−0.9	29.6	38.1	38.1	−539.4	976.4	2009.8	2081.2	−12.6	57.2
GMM	4.57	153.5	168.1	1949.3	1955.3	408.2	1220.9	4411.8	4430.7	20.6	58.4
KAC	4.79	−60.6	60.6	17.2	63.1	−1950.1	1950.1	3019.7	3596.7	−107.0	60.3
PMM	5.40	−51.0	52.8	25.7	57.1	−1101.4	1232.6	1810.9	2120.6	−88.1	64.9
MHM	5.50	−27.7	35.7	33.0	43.2	−948.2	1180.8	2180.9	2378.9	−48.1	65.6
BBC	5.79	122.8	134.9	535.2	549.2	9320.2	9862.8	70668.1	71282.4	39.4	67.5
XMM	7.11	58.1	89.0	178.1	187.4	55.8	1296.3	2236.0	2236.7	11.7	75.5
OHM	7.66	−60.9	66.5	36.9	71.3	−971.5	1555.0	2781.2	2946.7	−127.0	78.3
CHC	9.21	−51.2	69.2	69.9	86.7	−1578.1	1870.7	2800.9	3216.4	−110.5	85.4
OMM	13.96	−7.4	87.3	141.2	141.4	−957.2	1454.1	2183.5	2384.8	−68.4	101.4
CHM	13.98	414.5	417.4	705.4	818.6	9615.9	9860.6	21931.7	23954.5	105.1	101.5
HOC	24.71	−29.4	47.2	47.8	56.1	−1791.0	1915.8	3634.9	4053.7	−76.5	123.4
REC	41.16	355.8	400.6	756.1	835.9	7139.9	7998.6	17858.8	19238.2	50.9	143.0

We propose universal (independent of flow type) composite power laws, which consider the effect of the relative velocity of the phase through liquid holdup evaluated with the universal composite holdup García et al. (2005) correlations (UCHC); were determined. The parameters a_1, b_1, a_2, b_2, c, d and t, of the composite power laws were fit simultaneously to experiment subsets classified for different liquid holdup ranges. Upon comparing the a_1 and b_1 parameters, corresponding to the dominant power law for small Reynolds numbers (laminar flow), with the equivalent parameters for singles-phase flow (16 and −1), the similarity is evident. For the data group with liquid holdup greater 0.4, the values coincide. Insofar as liquid holdup decrease, the a_1 and b_1 parameter values are minor to their homologous in single phase flow, until reach values of 15.88 and −0.74, respectively, for the experimental data group with $0.05 > H_L > 0$. Upon comparing the a_2 and b_2 parameters, corresponding to the dominant power law for large Reynolds numbers (turbulent flow), with the Blasius equation parameters for smooth pipes singles-phase flow (0.079 and −0.25), the values change significantly and a clear tendency is not evidenced.

We classify the two phase flow experimental data in horizontal pipelines by flow regime and in turn the data subsets corresponding to slug flow, stratified flow and annular flow, were classified by liquid holdup ranges calculated with the holdup García et al. (2005) correlations sorted by flow pattern (FPHC).

For each subgroup the parameters a_1, b_1, a_2, b_2, c, d and t, of the composite power laws were fit simultaneously. In the improved correlations classified by flow pattern, the a_1 and b_1 parameters, corresponding to the dominant power law for small Reynolds numbers (laminar flow), change: (a_1) from 15.13 for annular flow and $1 > H_L \geqslant 0.1$ to 17.00 for slug flow and $1 > H_L \geqslant 0.5$; and (b_1) from −0.78 for annular flow and $0.05 > H_L > 0$ to −1.2 for stratified flow and $1 > H_L \geqslant 0.3$. The a_2 and b_2 parameters, corresponding to the dominant power law for large Reynolds numbers (turbulent flow), change: (a_2) from 0.07 for dispersed bubble flow to 1.52 for slug flow and $1 > H_L \geqslant 0.5$; and (b_2) from −0.06 for stratified flow and $0.05 > H_L > 0$ to −0.41 for annular flow and $0.05 > H_L > 0$.

The accuracy in the pressure gradient prediction for two phase flow in horizontal pipes of the improved correlations developed in this study was compared with the accuracy of other 21 correlations and models available in the literature. In general, the proposed composite power law correlations FFIPC, in which the flow pattern and the liquid holdup are considered, show the best performance, with an average absolute error of 18.1%. The universal improved correlations FFIUC, which consider the liquid holdup, show the second best performance with an average absolute error of 19.5%.

In the evaluation of the different models and correlations against high viscosity data ($\mu_L \geqslant 400$ mPa s), the improved correlations sorted by flow pattern FFIPC achieve the best performance with an average absolute error of 10.9%, followed in second place by the Lockhart and Martinelli (1949) correlation with an average absolute error of 15.6%. The universal improved correlations FFIUC attain the third better performance, with an average absolute error of 13.7%, followed in fourth place by the correlations sorted by flow pattern FFPC, developed by García et al. (2003), in which the liquid holdup are not considered, with an average absolute error of 13.8%. In gas and high viscosity liquid two-phase flow the dominant flow pattern is slug flow, which explains that correlations such as FFPC and FFUC (developed by García et al., 2003), that do not consider the effect of the relative velocity of the phases, have excellent performances.

In general, the average absolute errors in the pressure gradient predictions of the mechanistic models are high. These range from 27.8% for the Padrino et al. (2002) model to 132.4% for the Ouyang (1998) model. In the evaluation by flow patterns, the lowest error for annular flow was obtained by the Padrino et al. (2002) mechanistic model with an average absolute error of 52.8% and for stratified flow the lowest error was obtained by the Xiao et al. (1990) mechanistic model with an average absolute error of 62.2%.

Including the effect of the relative velocity of the phases through the liquid holdup, improves significantly the performance of the correlations (FFIPC and FFIUC). In the general evaluation against the whole database (2560 experiments), the FFIUC and FFIPC correlations improve the predictions accuracy in 5.2% and 3.7% on the average, respectively, with regard to the correlations developed by García et al. (2003), that do not consider the relative velocity of the phases (FFUC and FFPC). The most significant improvements are obtained in the predictions accuracy of the pressure gradient for annular and stratified flow. For annular flow the improved correlations FFIPC and FFIUC increase their accuracy in 14% and 4.7%, respectively, with regard to their homologous correlations FFUC and FFPC. For stratified flow the improved correlations FFIPC and FFIUC increase their accuracy in 9.1% and 8%, respectively. The accuracy increment of the

FFIPC correlations for slug flow and dispersed bubble flow with regard to their homologous correlations FFPC are less, 1.8% for both flow patterns.

Our results evidence that the effects associated with the hydrodynamics of the annular and stratified flow could be captured in more than the 70% for an appropriate combination of mixture Reynolds number and liquid holdup. However, other dimensionless parameters are required to further diminish the average absolute errors obtained in annular flow (25.3%) and stratified flow (29.8%). We anticipate that these dimensionless parameters should consider the effects of the gravity and superficial tension, such as Froude number, Morton number or Weber number.

The proposed universal (independent of flow type) and composite (for all Reynolds numbers) correlations are very useful for field operations for which the flow type is normally unknown. It is a best guess for the pressure gradient when the flow type is unknown or different flow types are encountered in a production pipe. In this sense, the friction factor universal correlations proposed in this work represent for two-phase flow what for one-phase flow is the celebrated Moody diagram.

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