OPPORTUNITIES FOR EXTRACTING CORRELATIONS FROM NUMERICAL AND REAL EXPERIMENTS USING DIGITAL TECHNOLOGY

Daniel D. Joseph
University of Minnesota, AEM, 107 Akerman Hall, 110 Union Street, Minneapolis, MN 55455, USA


1. INTRODUCTION

In this document I have explored some of the ways in which direct numerical simulation (DNS) of solid-liquid flow can be interrogated for useful results. The value added by DNS is that initial value problems for particulate flows can be solved as exactly as numerical methods will allow. The signal feature of DNS is that the particles are moved by forces computed from the fluid motion, as they should be; the modeling of forces needed by other approaches to particulate flow is not needed or done by DNS. In our work the calculation of forces is implicit because the mutual forces disappear in the variational formulation for the total solid-liquid momentum. In this we avoid the explicit computation of forces, on the one hand, and the modeling of forces on the other.

There are many ways in which direct simulations may be interrogated for useful results. It has to be understood that DNS does not replace theory even though it gives rise to exact numerical solutions of the initial value problem for particulate flow. It is more appropriate to think of DNS as a surrogate for experiments. The simulations have
some great advantages; you can suppress physical effects one at a time in simulations which is something that cannot be done in experiments. Virtual experiments also have the potential to replace real experiments in generating data which form the basis for correlations of the type used in engineering practice; the generation of Richardson-Zaki correlations in our study of fluidization of 1204 spheres is an example. Quantities needed for theory, like slip velocities are ever so much easier to determine in simulations than in experiments.

Direct simulations lead to better understanding of flow fundamentals in situations otherwise opaque. The fluidization by lift of slurries in horizontal conduits is a concept generated by direct simulations. Liquid-solid flows are a nonlinear dynamical system which give rise to typical bifurcations like that discussed by Choi and Joseph (2001), Joseph and Ocando (2002) and Wang and Joseph (2003), to periodic and even chaotic solutions. It is not possible to study such bifurcation by analytical methods or two-fluid models. DNS may be the only method to study bifurcations of particulate flow.

There are many technologies in which depend critically on solid-liquid flows. Industries which utilize such technologies typically control operations with simplified models of particulate flow which can be run on PCs. It is widely believed and presently true that DNS is too slow and expensive to guide field operations. Such beliefs should always be revisited because the rapid expansion of software and hardware has a proven capacity for upward revision.

2. USE OF DNS RESULTS IN DEVELOPING PRACTICAL MODELS

We have been trying to find the structure of data arising from DNS which can be used to form useful models. There are different kinds of models; effective media and models which require the modeling of forces on particles. Modeling forces is a big problem; for example, there are no good models of lift forces in slurries. At the risk of being tiresome I want again to call attention to the huge difference between the modeling of forces as is done in modeling and the computing of forces as is done by DNS.

There are two kinds of models that require modeling forces; models in which the fluid motions are resolved by direct methods and the particles are moved by Newton’s laws using modeled forces and two-fluid models.

Effective media, two-fluid models do not require the modeling of forces. Our study of the Rayleigh-Taylor instability arising in the direct numerical simulation of 6400 circular particles (Pan, Joseph and Glowinski 2001) is a good example of an effective media two-fluid model. In that study we regarded the sedimenting suspension and the entrained fluid as another effective fluid. To realize the model it was necessary to come up with an effective density and viscosity; modeling forces was not required. It seems likely to me that the fluidization of 300 particles by lift as studied by Choi and Joseph (2001) can be modeled as an effective fluid, with an effective viscosity, density and zero surface tension. The waves, which propagate on the top of the fluidized suspension, look like waves which develop in two-fluid situations.

Maybe effective media two-fluid models in which model assumptions can be tested by DNS ought to be restricted to special situations like those mentioned in the prior paragraph. One two-fluid model of particulate flow which covers all situations is not
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likely to be an achievable goal since closures which work for some situations will not
work for others. In the literature one finds formulas for single particle lift which apply in
special circumstances mainly for low Reynolds numbers. Rigorously derived
mathematical formulas for the lift and drag on particles moving at finite Reynolds
numbers do not exist and such formulas are put forward only as empirical results
following out of experimental data, from real experiments and now from numerical
experiments.

It can be said that two-fluid models and perturbations of Stokes flow have not
worked all that well. An alternative to these methods is the method of correlations used
in this book. This method leads from data to formulas. Data from experiments and
numerical experiments are processed in the same way. This method makes maximum
use of computers and storage tapping opportunities provided by new technology. I think
that curve fitting plus computers and storage gives rise to new and great opportunities for
particulate flow and multiphase science. The secrets are in the data and with digital
technology we can interrogate this data.

Generating correlations from experiments is an old method on which many
industrial applications are based but it has come to have a bad name, viewed as empirical
and not fundamental. The great example is the Richardson-Zaki correlation which is the
cornerstone of fluidized bed practice. My enthusiasm for correlations has to do with the
surprising emergence of correlations from the simplest kind of post-processing of our
numerical experiments. We have done lift correlations for single particles and for the bed
expansion of many particles in slurries. The procedure we follow is to plot the results of
our simulations in log-log plots of the relevant variables. The surprise for us is that these
plots frequently come up as straight lines giving rise to power laws. For example, a
single particle will lift-off in a Poiseuille flow at a certain Reynolds number \( R = \frac{Ud}{\nu} \)
for a given settling Reynolds number \( R_\sigma = \frac{\rho_f (\rho_p - \rho_f)gd^3}{\eta^2} \). When we plotted the lift off
criterion from about 20 points we found that \( R = aR_\sigma^m \) with an intercept \( a \) and
slope \( m \) in the log-log plot. The straight lines are impressively straight and we generated such
correlations for lift to equilibrium, for the bed expansion of many particles and in non-
Newtonian fluids. The existence of such power laws is an expression of self-similarity,
which has not been predicted from analysis or physics. The flow of dispersed matter
appears to obey those self-similar rules to a large degree.

We can get power laws when only two variables are at play; when there are three
variables or more, it would appear that we get different power laws separated by
transition regions. This is certainly the case for the Richardson-Zaki correlation; it has
one power law relating the fluidization velocity to the solids fraction at low Reynolds
number, and another at high Reynolds with a Reynolds number-dependent transition
between. We got such correlations between three variables for slurries, and from
experiments (Choi and Joseph 2001, Patankar, Ko, Choi and Joseph 2001, Pan, Joseph,
Bai, Glowinski and Sarin 2002, Joseph and Ocampo 2002, Patankar, Joseph, Wang,

The direction of our work is to develop simulations to get efficient computation
leading to 3D correlations. This will happen. Then we will get real engineering
correlations from numerical experiments. I like this approach since it uses numerical
simulations in a natural way evolving from their intrinsic properties rather than trying to
fit them into a more familiar frame using models. I think that processing of data for
correlations, from experiments, field data or simulations is a great new opportunity of
the computer age and ought to be vigorously pursued.

The problem faced by models is how to get the various interaction terms right. Much
of the time the guesses made for these interaction terms are poor and the
predictive power of the model is not there. Better models must also make use of
correlations for the interaction terms. For example, the Richardson-Zaki correlation
gives an excellent correlation for bed expansion, but leaves the modeling of the drag
force needed for a mechanist’s model to imagination.

Let it be said that the active pursuit of correlations is an excellent direction for
future research using computers in a new way with direct applications to both
engineering practice and model construction.

REFERENCES

instability of a sedimenting suspension of thousands of circular particles. J. Fluid
in Newtonian and viscoelastic fluids by direct numerical simulation. J. Fluid Mech.,
438, 67-100.
Power law correlations for sediment transport in pressure driven channel flows, Int.
J. Multiphase Flow, 28(8), 1269-1292.