Finite Horizon Robustness Analysis of LTV Systems Using Integral Quadratic Constraints

Peter Seiler
University of Minnesota

M. Moore, C. Meissen, M. Arcak, and A. Packard
University of California, Berkeley

University of Stuttgart
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Research Summary

Jordan Hoyt
Parul Singh
Sanjana Vijayshankar
Wind Energy

Raghu Venkataraman
Harish Venkataraman
Small UAVs

Abhineet Gupta
Aeroelasticity

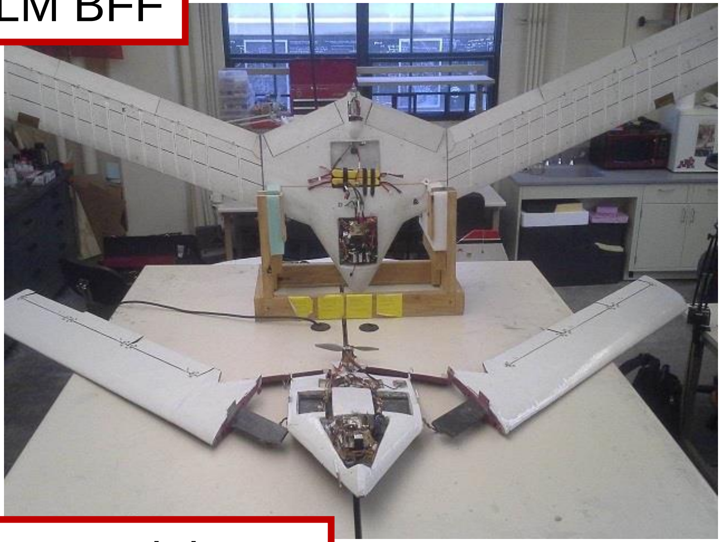
Robust Control Design and Analysis

Chris Regan
Brian Taylor
Curt Olson

Performance Adaptive Aeroelastic Wing (PAAW)

- Goal: Suppress flutter, control wing shape and alter shape to optimize performance
  - Funding: NASA NRA NNX14AL36A
  - Technical Monitor: Dr. Jeffrey Ouellette
  - Two years of testing at UMN followed by two years of testing on NASA's X-56 Aircraft

LM BFF



UMN Mini-Mutt

LM/NASA X-56

The FlexOp Project

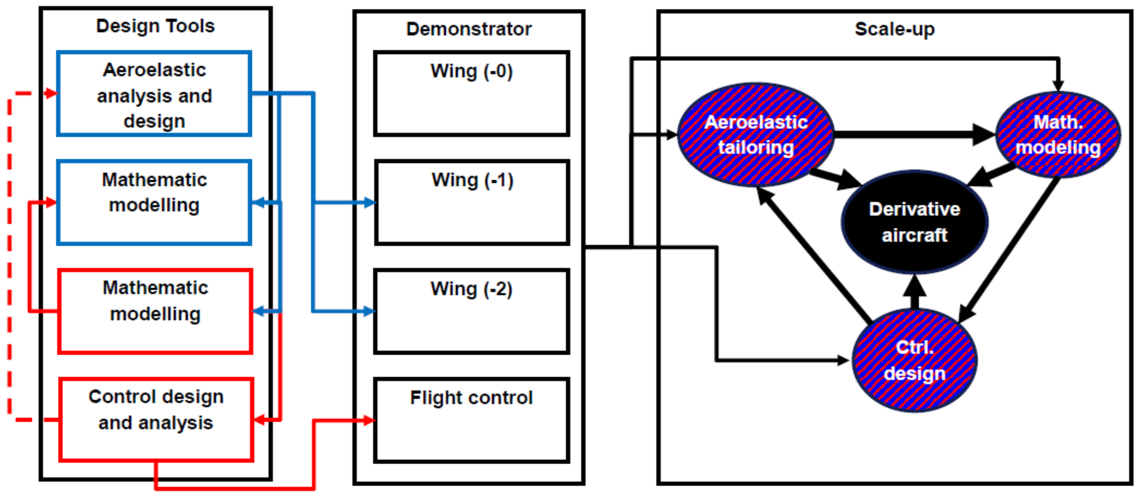
Flutter Free FLight Envelope eXpansion for ecOnomical Performance improvement

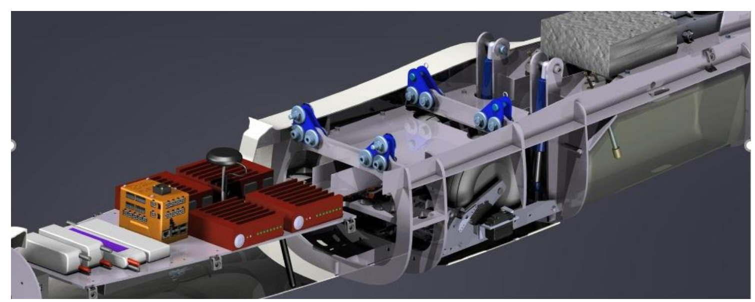
Balint Vanek (vanek@sztaki.hu), coordinator
Institute for Computer Science and Control, HAS

Approach

***Move towards methods and tools enabling multidisciplinary design analysis and optimization in the aeroservoelastic domain**

***Validate the developed tools with the demonstrator**





facc AIRBUS DLR INASCO MTA SZTAKI TUM TU Delft Delft University of Technology University of BRISTOL RWTH AACHEN UNIVERSITY

The FlexOp Project

Flutter Free FLight Envelope eXpansion for ecOnomical Performance improvement

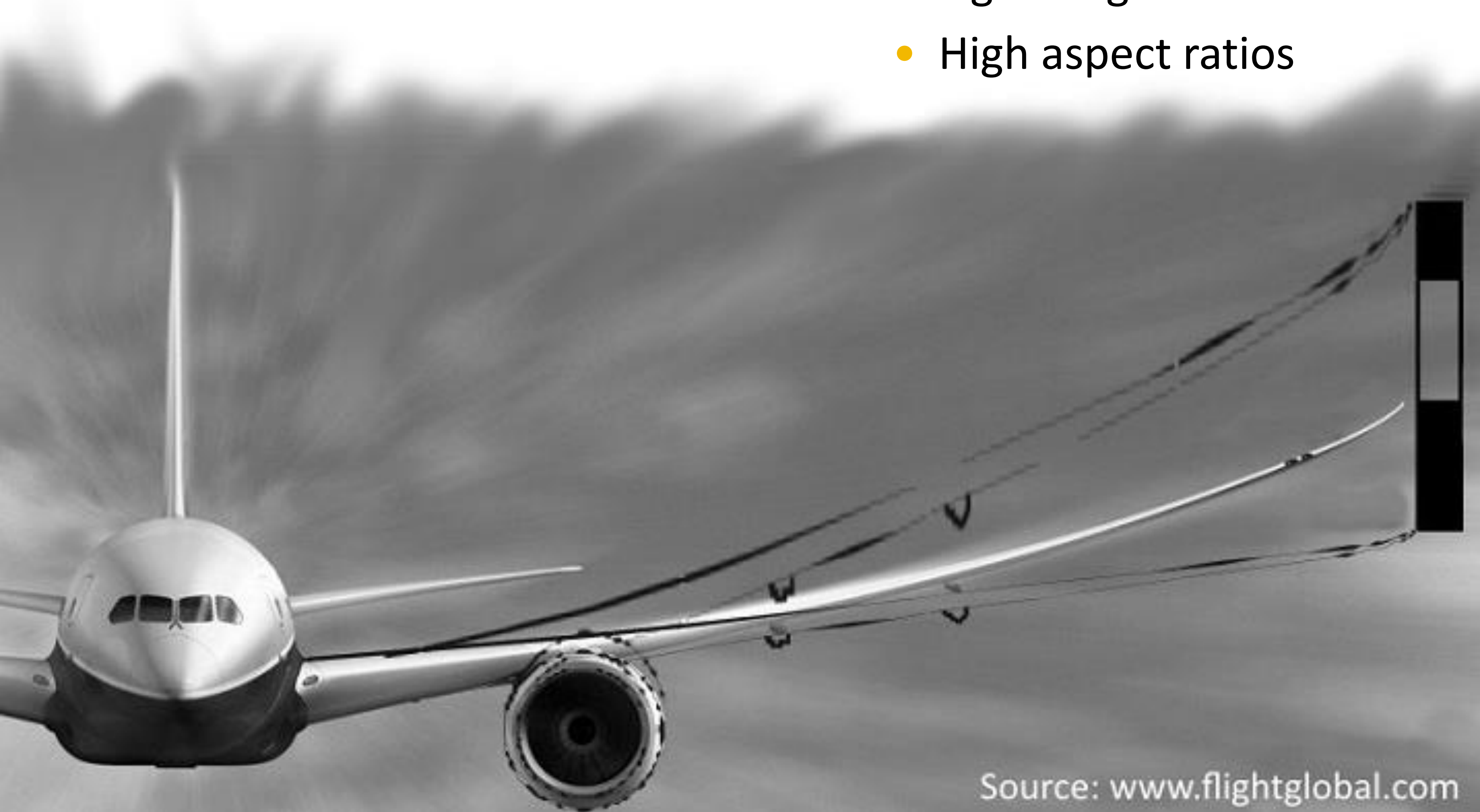
The FLEXOP Project

Flutter Free FLight Envelope eXpansion for ecOnomical Performance improvement

Aeroservoelasticity (ASE)

Efficient aircraft design

- Lightweight structures
- High aspect ratios

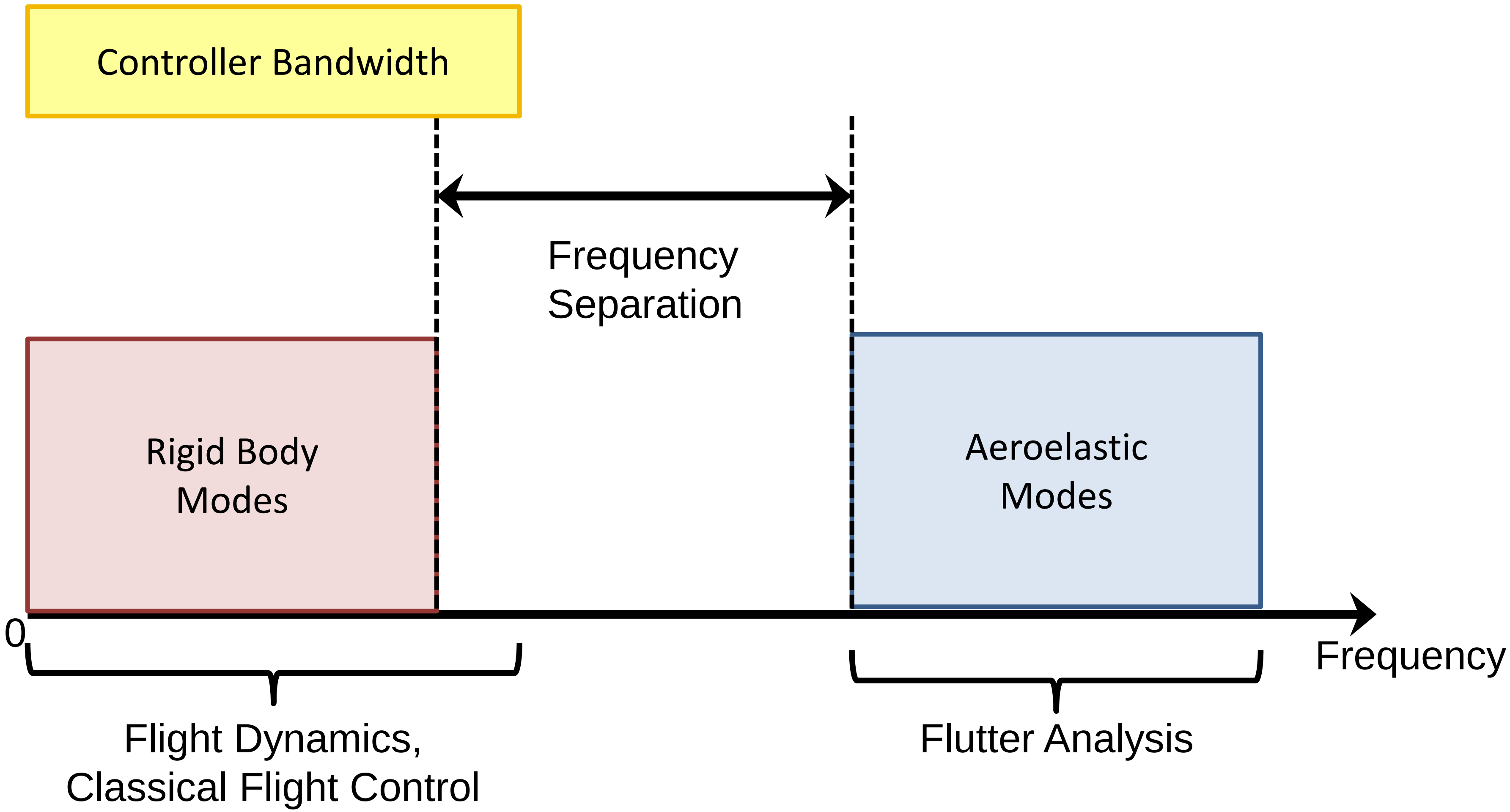


Source: www.flightglobal.com

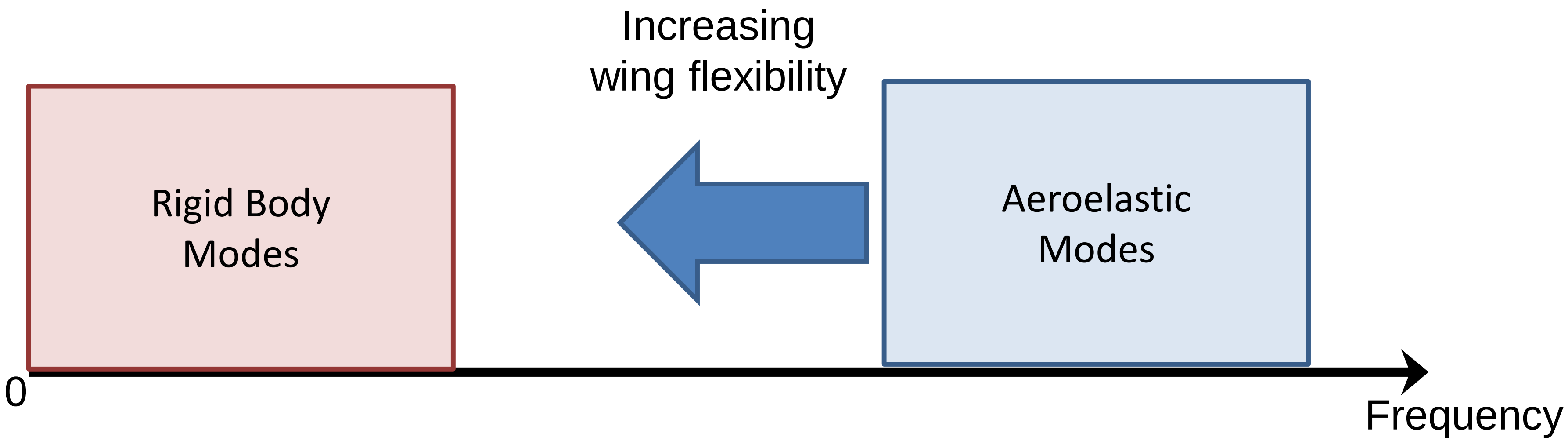
Flutter

Source: NASA Dryden Flight Research

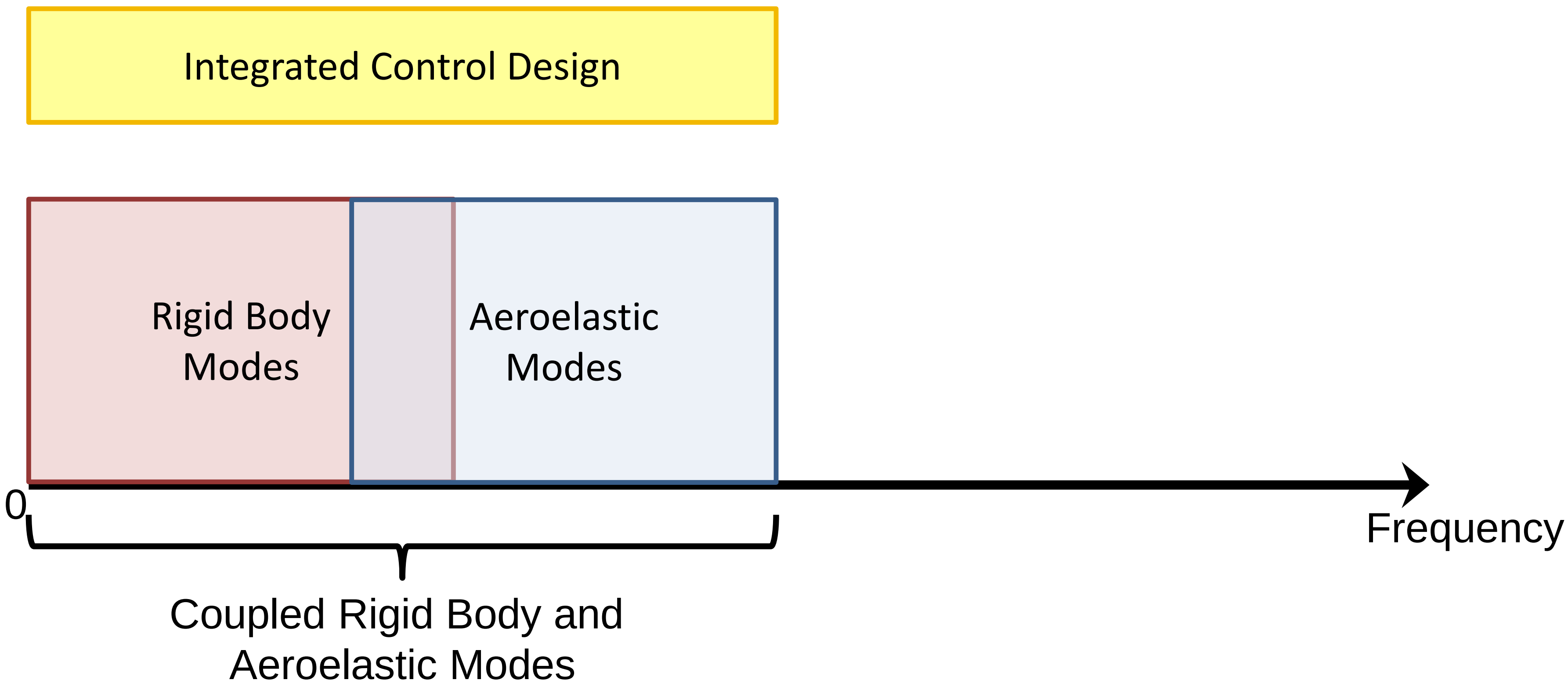
Classical Approach



Flexible Aircraft Challenges

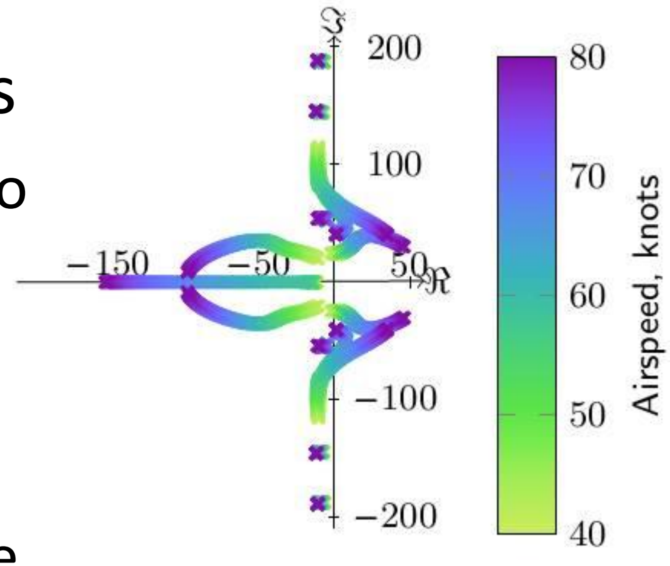


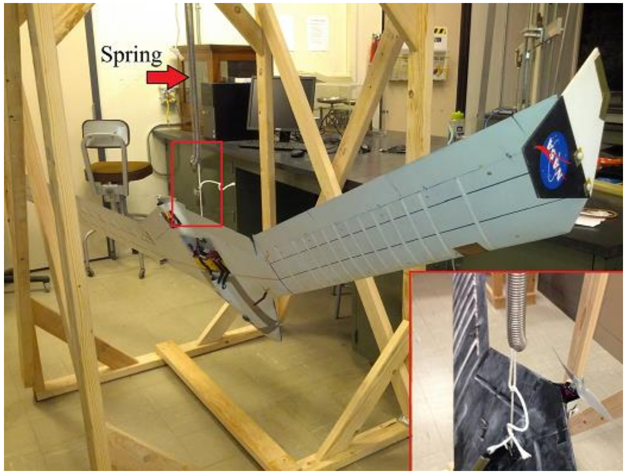
Flexible Aircraft Challenges



Modeling and Control for Flex Aircraft

1. Parameter Dependent Dynamics
   - Models depend on airspeed due to structural/aero interactions
   - LPV is a natural framework.
2. Model Reduction
   - High fidelity CFD/CSD models have many (millions) of states.
3. Model Uncertainty
   - Use of simplified low order models OR reduced high fidelity models
   - Unsteady aero, mass/inertia & structural parameters





Current PAAW Aircraft

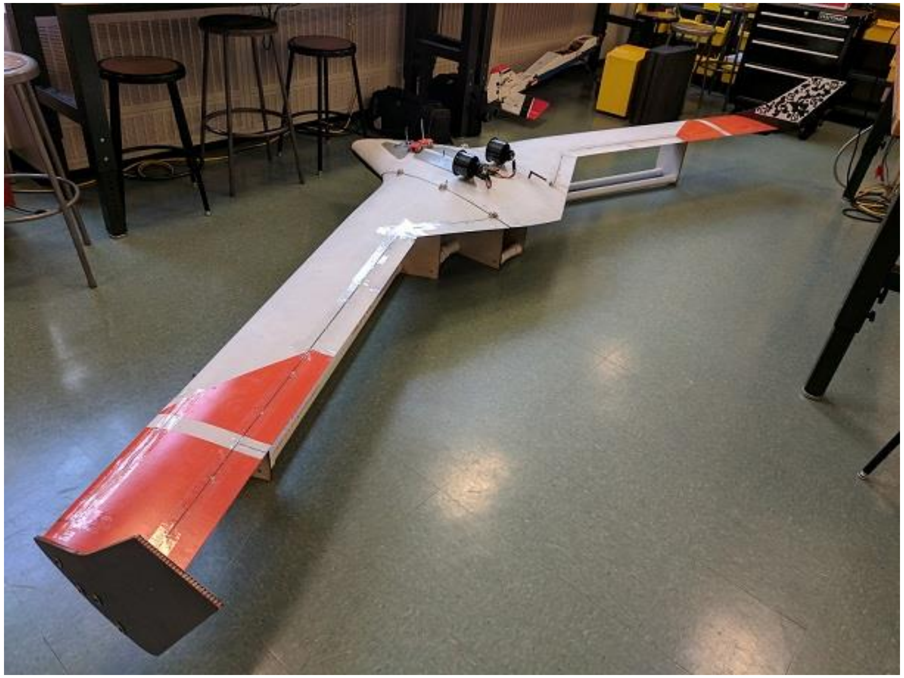
mAEWing1

10 foot wingspan

~14 pounds

Laser-scan replica of BFF

4 aircraft, >50 flights



mAEWing2

14 foot wingspan

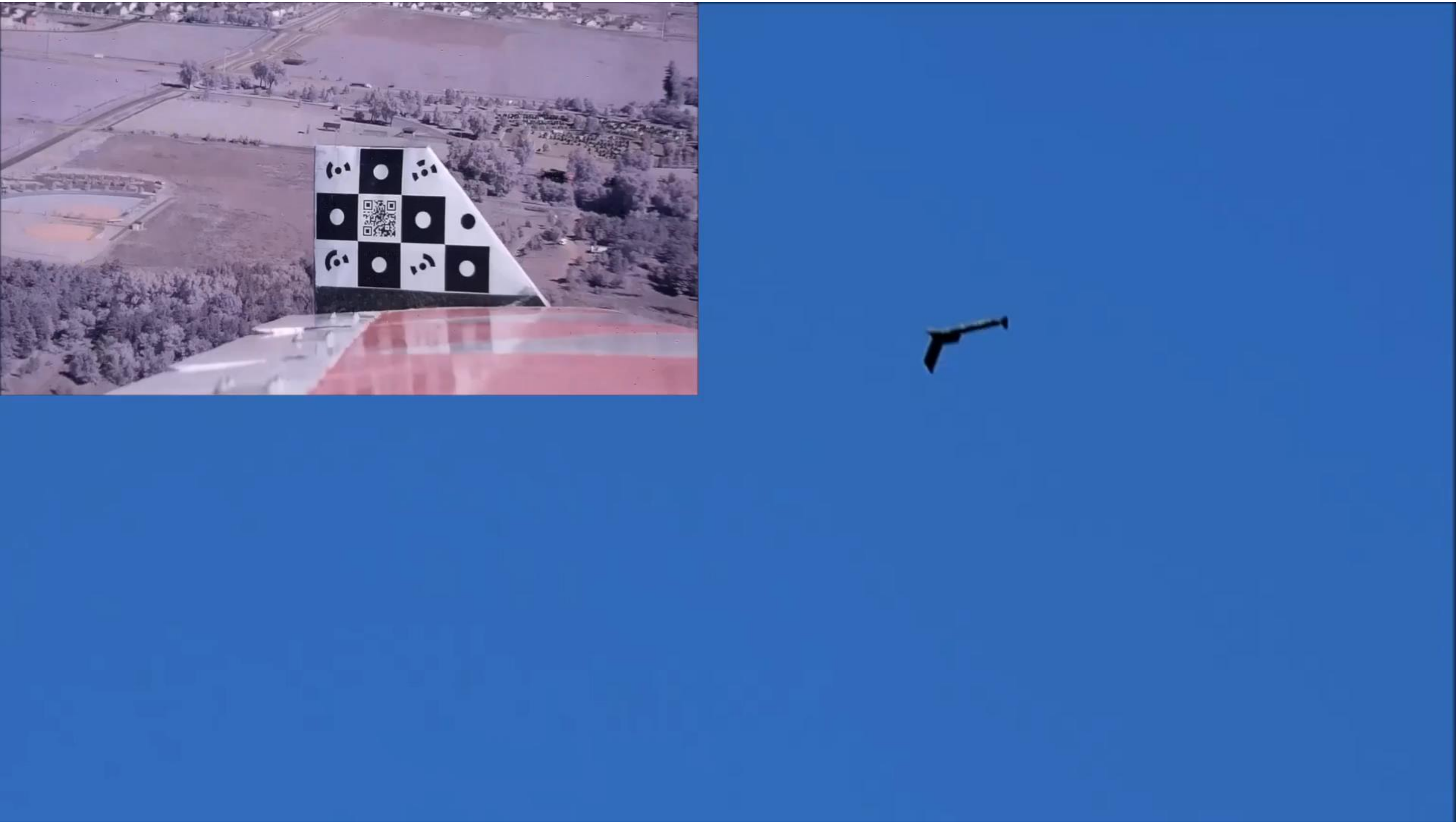
~42 pounds

Half-scale X-56

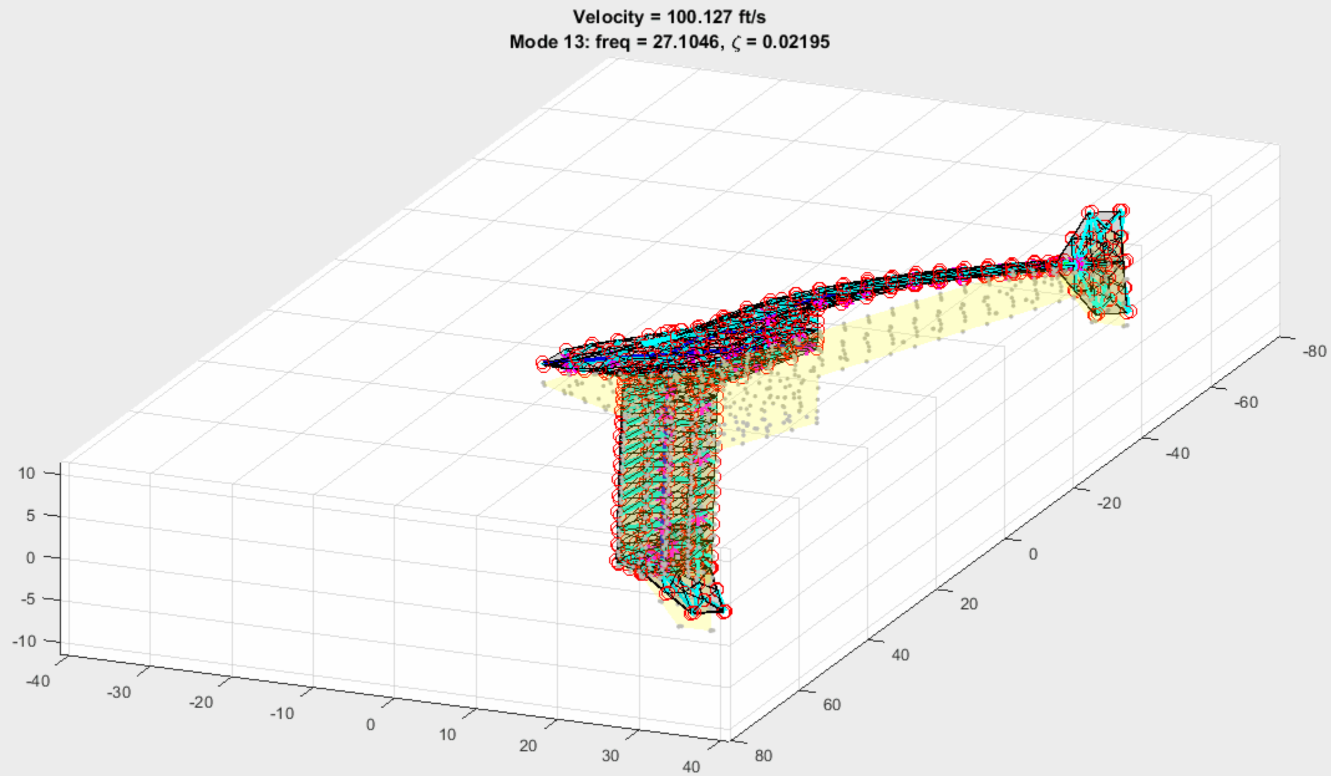
Currently ground testing

mAEWing1 and 2

Open-Loop Flutter



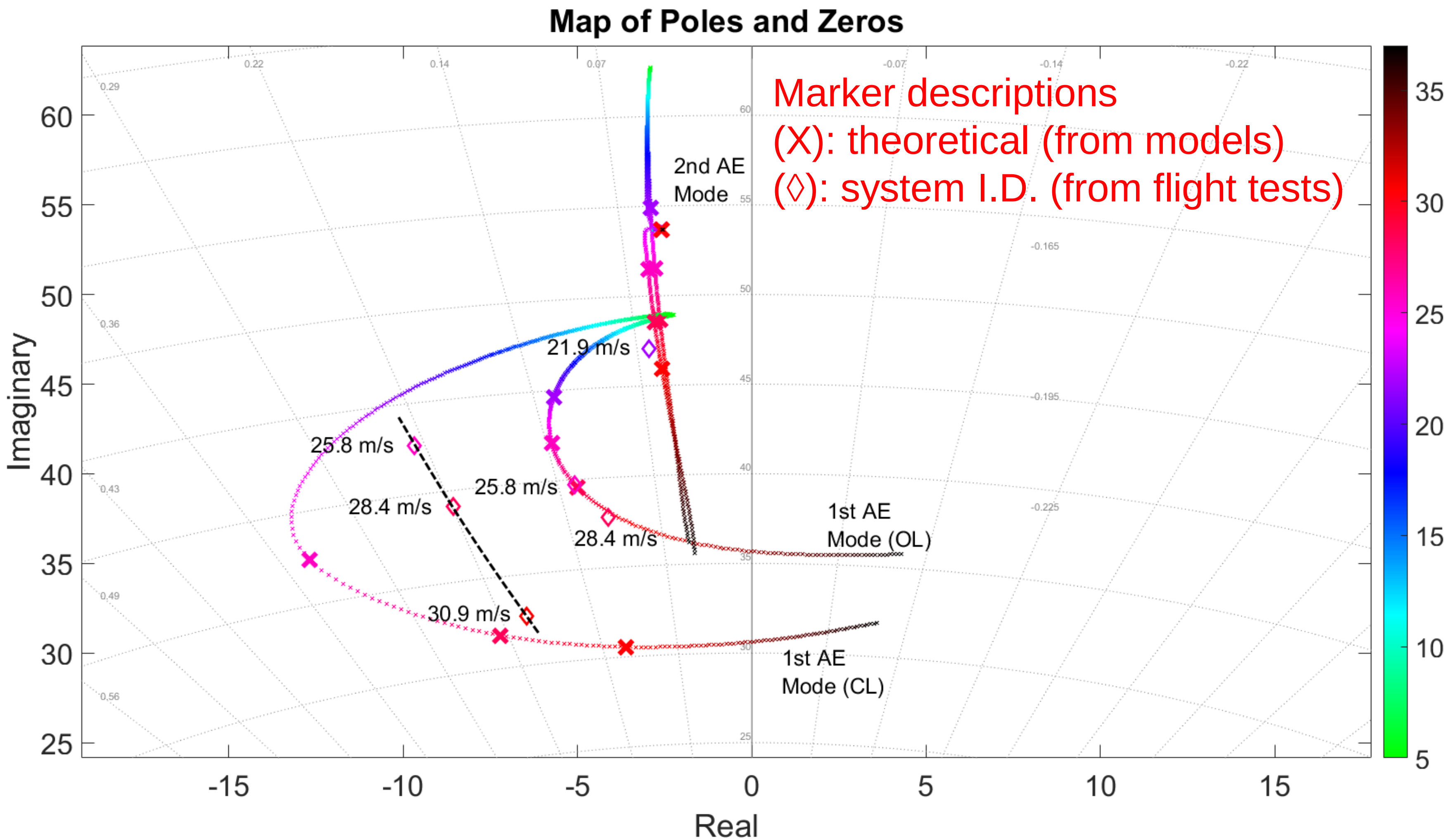
Animated Mode Shape



The BFF mode (genesis at SWB1) at a velocity near the flutter point. The coupling of SWB1 and short period is apparent

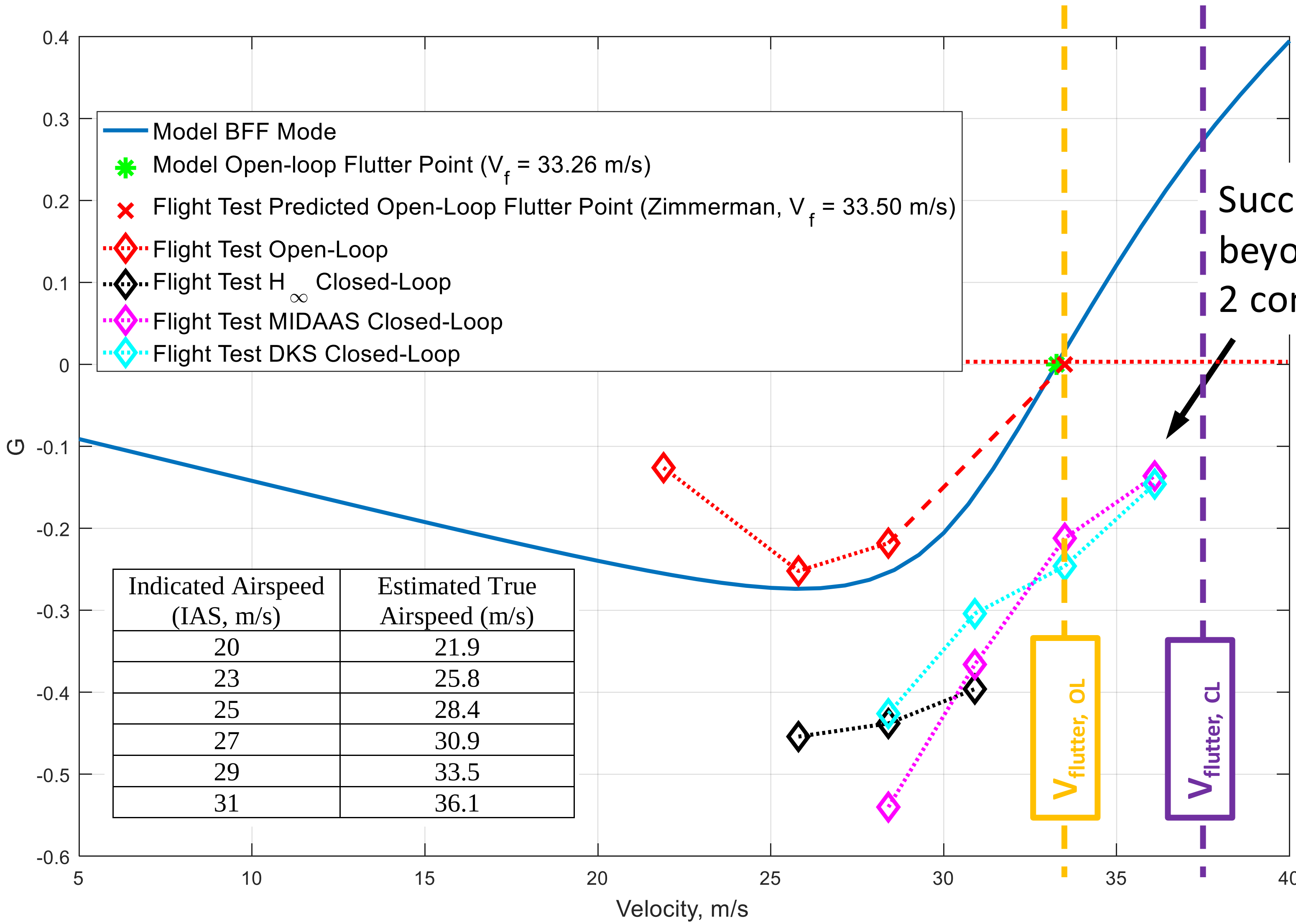
In Flight Mode Shape

Pole Map for H-Inf Controller



Hinf Design procedure due to Julian Theis ('16 AIAA, '18 Phd) with re-tuning by Kotikalpudi, et al ('18 Aviation).

Flight Test Summary



Indicated Airspeed (IAS, m/s)	Estimated True Airspeed (m/s)
20	21.9
23	25.8
25	28.4
27	30.9
29	33.5
31	36.1

Successful flight beyond flutter with 2 controllers!

Outline

- Motivation for LTV Analysis
- Nominal LTV Performance
- Robust LTV Performance
- Examples
- Conclusions

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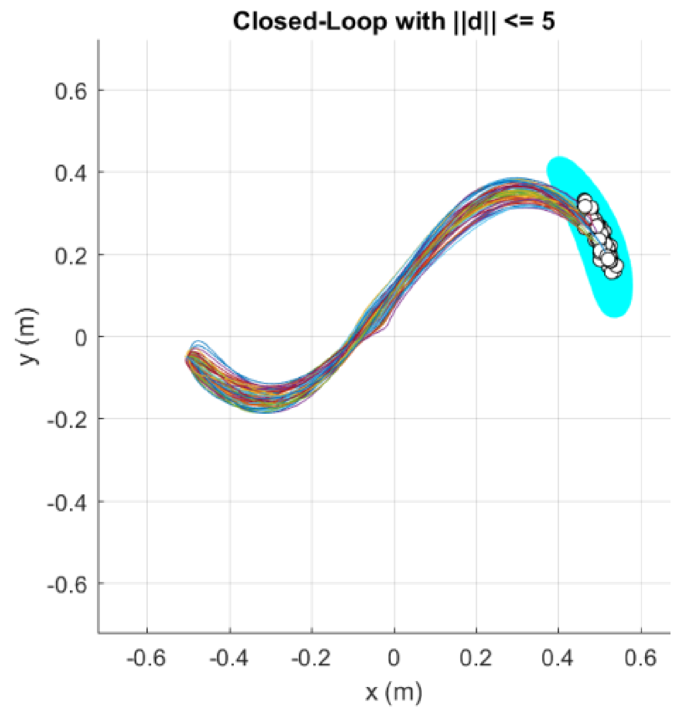
Analysis Objective

Goal: Assess the robustness of linear time-varying (LTV) systems on finite horizons.

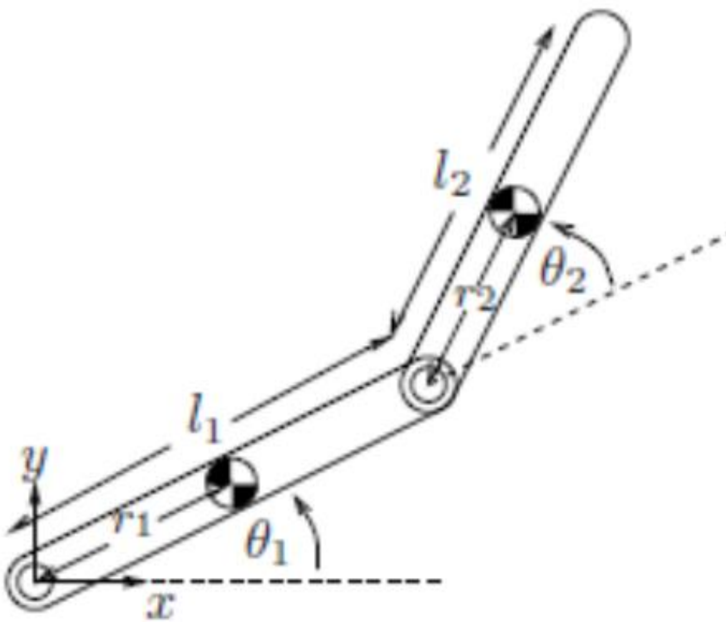
Approach: Classical Gain/Phase Margins focus on (infinite horizon) stability and frequency domain concepts.

Instead focus on:

- Finite horizon metrics, e.g. induced gains and reachable sets.
- Effect of disturbances and model uncertainty (D-scales, IQCs, etc).
- Time-domain analysis conditions.



Two-Link Robot Arm



Two-Link Diagram [MZS]

Nonlinear dynamics [MZS]:

$$\dot{\eta} = f(\eta, \tau, d)$$

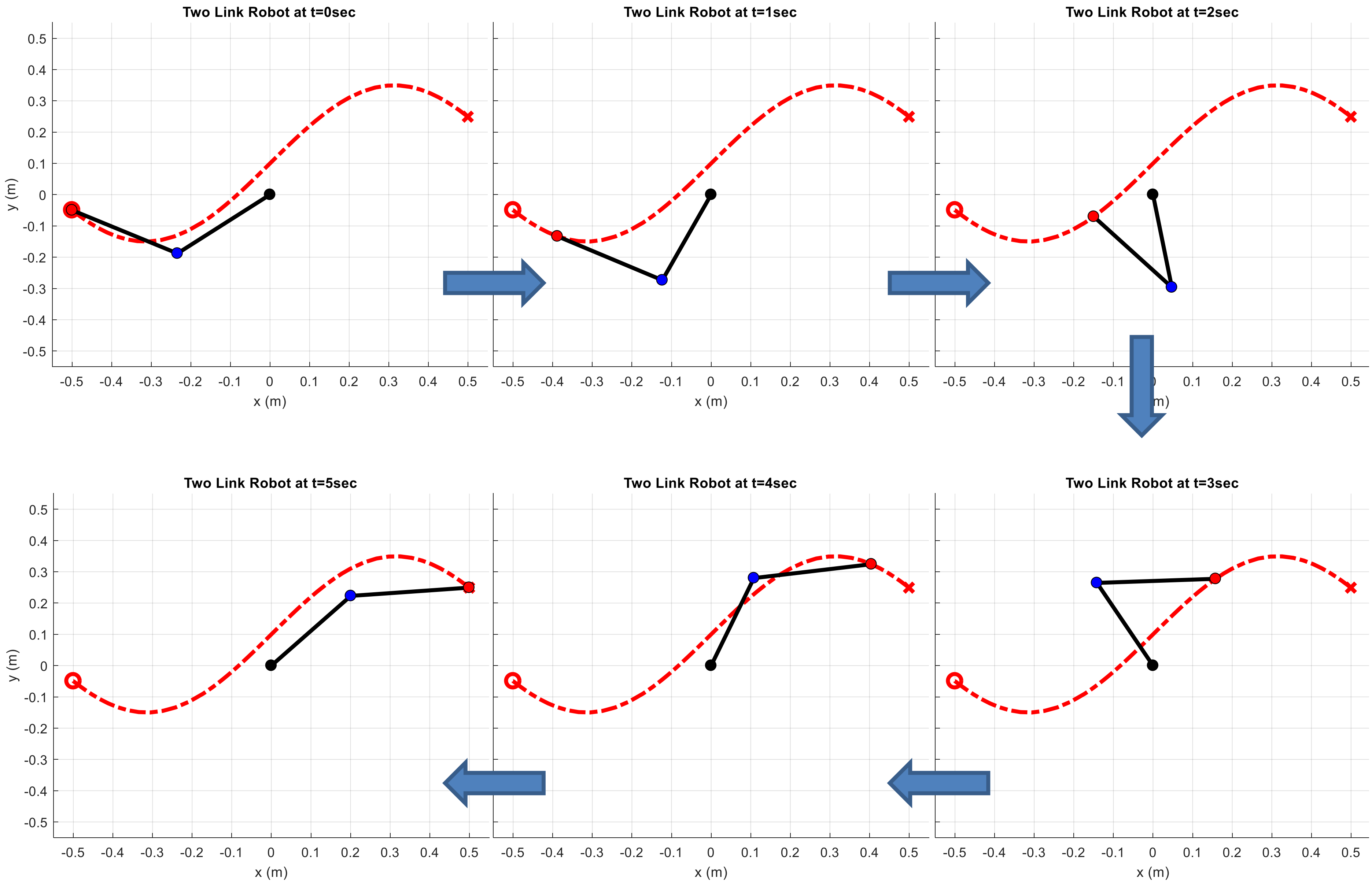
where

$$\eta = [\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2]^T$$
$$\tau = [\tau_1, \ \tau_2\ ]^T$$
$$d = [d_1, \ d_2\ ]^T$$

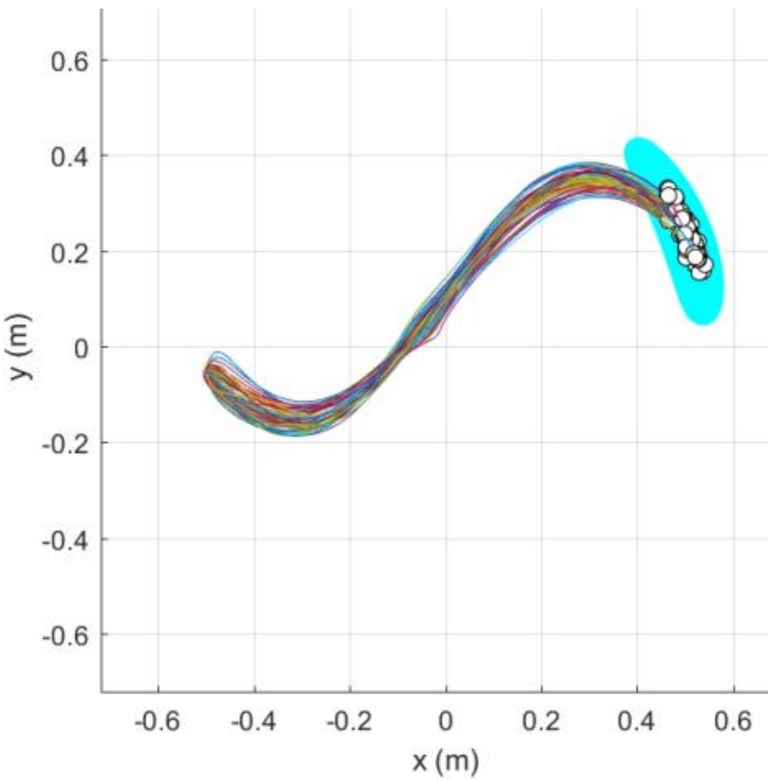
τ and d are control torques and disturbances at the link joints.

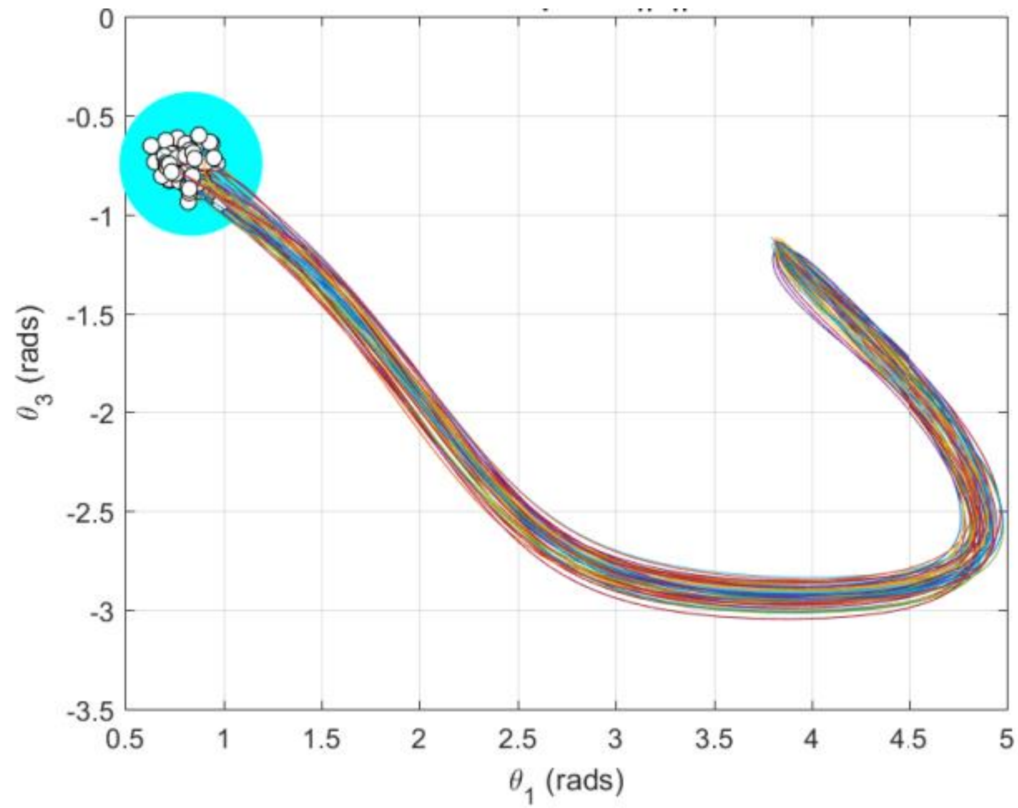
[MZS] R. Murray, Z. Li, and S. Sastry. *A Mathematical Introduction to Robot Manipulation*, 1994.

Nominal Trajectory (Cartesian Coords.)



Effect of Disturbances / Uncertainty





Cartesian Coords.

Joint Angles

Overview of Analysis Approach

Nonlinear dynamics:

$$\dot{\eta} = f(\eta, \tau, d)$$

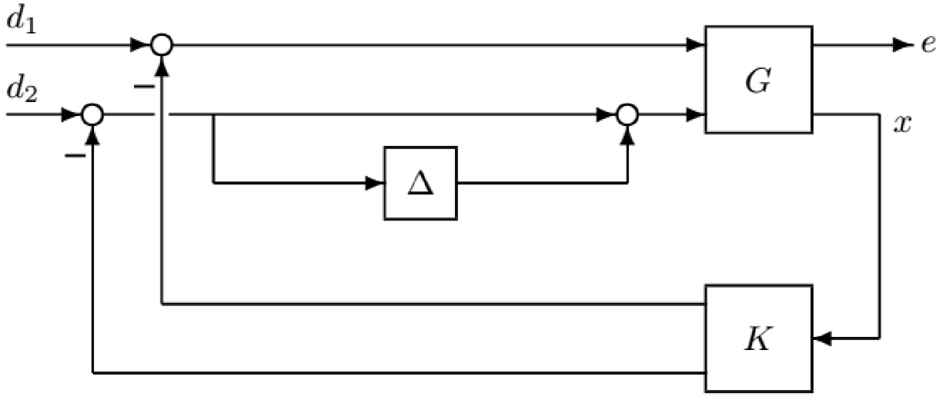
Linearize along a (finite –horizon) trajectory $(\bar{\eta}, \bar{\tau}, d = 0)$

$$\dot{x} = A(t)x + B(t)u + B(t)d$$

Compute bounds on the terminal state *x(T)* or other quantity *e(T) = C x(T)* accounting for disturbances and uncertainty.

Comments:

- The analysis can be for open or closed-loop.
- LTV analysis complements the use of Monte Carlo simulations.



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Finite-Horizon LTV Performance

Finite-Horizon LTV System *G* defined on [0,T]

$$\dot{x}(t) = A(t)x(t) + B(t)d(t)$$
$$e(t) = C(t)x(t) + D(t)d(t)$$

Induced L_2 Gain

$$\|G\|_{2,[0,T]} := \sup\left\{ \frac{\|e\|_{2,[0,T]}}{\|d\|_{2,[0,T]}} \;\middle|\; x(0)=0, 0 \neq d \in \mathcal{L}_{2,[0,T]} \right\}$$

L_2-to-Euclidean Gain

$$\|G\|_{E,[0,T]} := \sup\left\{ \frac{\|e(T)\|_2}{\|d\|_{2,[0,T]}} \;\middle|\; x(0)=0, 0 \neq d \in \mathcal{L}_{2,[0,T]} \right\}$$

The L_2-to-Euclidean gain requires *D(T)=0* to be well-posed.

The definition can be generalized to estimate ellipsoidal bounds on the reachable set of states at *T.*

General (Q,S,R,F) Cost

Cost function *J* defined by *(Q,S,R,F)*

$$J(d) := x(T)^T F x(T) + \int_0^T \begin{bmatrix} x(t) \\ d(t) \end{bmatrix}^T \begin{bmatrix} Q(t) & S(t) \\ S(t)^T & R(t) \end{bmatrix} \begin{bmatrix} x(t) \\ d(t) \end{bmatrix} dt$$

Subject to: LTV Dynamics with *x(0)=0*

Example: Induced L_2 Gain

Select *(Q,S,R,F)* as:

$$Q(t) := C(t)^T C(t),\ S(t) := C(t)^T D(t),\ R(t) := D(t)^T D(t) - \gamma^2 I_{n_d},\ \text{and}\ F := 0.$$

Cost Function *J* is:

$$J(d) = \|e\|_{2,[0,T]}^2 - \gamma^2 \|d\|_{2,[0,T]}^2$$

$$\Rightarrow\quad J(d) \leq 0 \text{ for all } d \in \mathcal{L}_2[0,T] \text{ if and only if } \|G\|_{2,[0,T]} \leq \gamma.$$

General (Q,S,R,F) Cost

Cost function *J* defined by *(Q,S,R,F)*

$$J(d) := x(T)^T F x(T) + \int_0^T \begin{bmatrix} x(t) \\ d(t) \end{bmatrix}^T \begin{bmatrix} Q(t) & S(t) \\ S(t)^T & R(t) \end{bmatrix} \begin{bmatrix} x(t) \\ d(t) \end{bmatrix} dt$$

Subject to: LTV Dynamics with *x(0)=0*

Example: L_2-to-Euclidean Gain

Select *(Q,S,R,F)* as:

$$Q(t) := 0,\ S(t) := 0,\ R(t) := -\gamma^2 I_{n_d},\ \text{and } F := C^T(T)C(T).$$

Cost Function *J* is:

$$J(d) = \|e(T)\|_2^2 - \gamma^2 \|d\|_{2,[0,T]}^2$$

$\Rightarrow$ $J(d) \leq 0$ for all $d \in \mathcal{L}_2[0,T]$ if and only if $\|G\|_{E,[0,T]} \leq \gamma$.

Strict Bounded Real Lemma

Theorem 1. *Assume $R(t) \prec 0$ for all $t \in [0,T]$. The following are equivalent:*

1. $\exists \epsilon > 0$ *such that* $J(d) \leq -\epsilon \|d\|^2_{2,[0,T]}$ $\forall d \in \mathcal{L}_2[0,T]$.

2. *There exists a differentiable function Y on $[0,T]$ such that $Y(T) = F$ and*

$$\dot{Y} + A^T Y + YA + Q - (YB+S)R^{-1}(YB+S)^T = 0$$

   This is a **Riccati Differential Equation (RDE)**.

3. *There exists $\epsilon > 0$ and a differentiable function P on $[0,T]$ such that $P(T) \succeq F$ and*

$$\dot{P} + A^T P + PA + Q - (PB+S)R^{-1}(PB+S)^T \preceq -\epsilon I$$

   This is a strict **Riccati Differential Inequality (RDI)**.

This is a generalization of results contained in:

*Tadmor, Worst-case design in the time domain. *MCSS*, 1990 .

*Ravi, Nagpal, and Khargonekar. H_∞ control of linear time-varying systems. *SIAM JCO*, 1991.

*Green and Limebeer. *Linear Robust Control*, 1995.

*Chen and Tu. The strict bounded real lemma for linear time-varying systems. JMAA, 2000.

Proof: 3→1

By Schur complements, the RDI is equivalent to:

$$\begin{bmatrix} \dot{P} + A^T P + PA & PB \\ B^T P & 0 \end{bmatrix} + \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \preceq -\tilde{\epsilon} I$$

This is an LMI in *P*. It is also equivalent to a dissipation inequality with the storage function $V(x,t) := x^T P(t) x$.

$$\dot{V} + \begin{bmatrix} x \\ d \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} \leq -\tilde{\epsilon} \begin{bmatrix} x \\ d \end{bmatrix}^T \begin{bmatrix} x \\ d \end{bmatrix}$$

Integrate from *t=0* to *t=T*:

$$V(x(T),T) - V(x(0),0) + \int_0^T \begin{bmatrix} x(t) \\ d(t) \end{bmatrix}^T \begin{bmatrix} Q(t) & S(t) \\ S(t)^T & R(t) \end{bmatrix} \begin{bmatrix} x(t) \\ d(t) \end{bmatrix} dt \leq -\tilde{\epsilon} \, \| [\begin{smallmatrix} x \\ d \end{smallmatrix}] \|_{2,[0,T]}^2$$

Apply *x(0)=0* and *P(T)≥F:*

$$J(d) \leq -\epsilon \|d\|_{2,[0,T]}^2$$

Strict Bounded Real Lemma

Theorem 1. *Assume $R(t) \prec 0$ for all $t \in [0,T]$. The following are equivalent:*

1. *$\exists \epsilon > 0$ such that $J(d) \leq -\epsilon \|d\|^2_{2,[0,T]}$ $\forall d \in \mathcal{L}_2[0,T]$.*
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This is a strict **Riccati Differential Inequality (RDI)**.

Comments:

*For nominal analysis, the RDE can be integrated. If the solution exists on [0,T] then nominal performance is achieved. This typically involves bisection, e.g. over γ, to find the best bound on a gain.

*For robustness analysis, both the RDI and RDE will be used to construct an efficient numerical algorithm.

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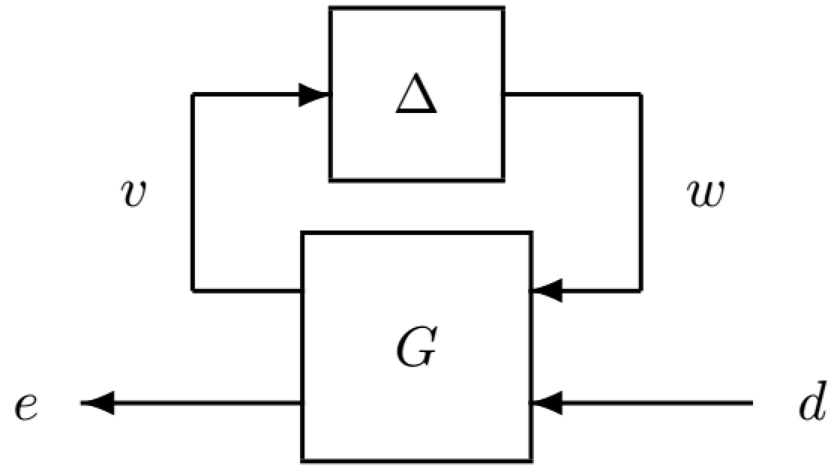
Uncertainty Model

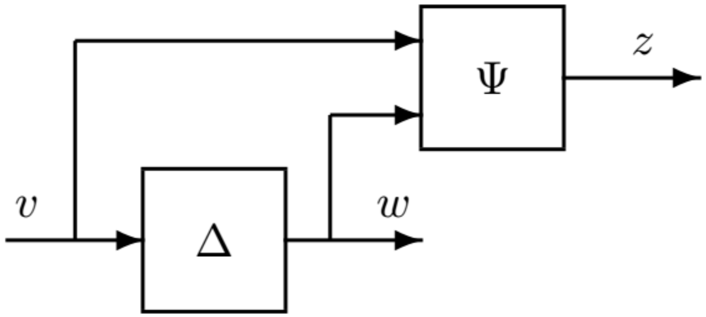
- Standard LFT Model, $F_u(G,\Delta)$, where G is LTV:

$$\begin{aligned}\dot{x}_G(t) &= A_G(t)\,x_G(t) + B_{G1}(t)\,w(t) + B_{G2}(t)\;d(t)\\ v(t) &= C_{G1}(t)\,x_G(t) + D_{G11}(t)\,w(t) + D_{G12}(t)\;d(t)\\ e(t) &= C_{G2}(t)\,x_G(t) + D_{G21}(t)\,w(t) + D_{G22}(t)\;d(t)\end{aligned}$$

Δ is block structured and used to model parametric / dynamic uncertainty and nonlinear perturbations.



Integral Quadratic Constraints (IQCs)



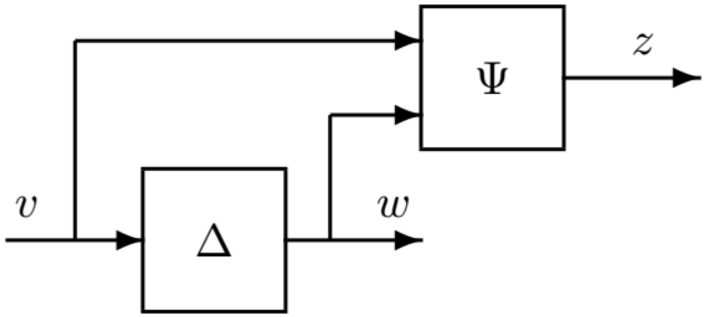
$\int_0^T z(t)^T M(t) z(t)\, dt \geq 0$

Definition 2. *Let* $\Psi \in \mathbb{RH}_\infty^{n_z \times (n_v + n_w)}$ *and* $M : [0, T] \to \mathbb{S}^{n_z}$ *with* M *piecewise continuous. A bounded, causal operator* $\Delta : \mathbf{L}_2^{n_v}[0, T] \to \mathbf{L}_2^{n_w}[0, T]$ *satisfies the time domain IQC defined by* (Ψ, M) *if the following inequality holds for all* $v \in \mathcal{L}_2^{n_v}[0, T]$ *and* $w = \Delta(v)$*:*

$$\int_0^T z(t)^T M(t) z(t)\, dt \geq 0 \qquad (1)$$

where z *is the output of* Ψ *driven by inputs* (v, w) *with zero initial conditions* $x_\psi(0) = 0$.

Integral Quadratic Constraints (IQCs)



$$\int_0^T z(t)^T M(t) z(t)\, dt \geq 0$$

Comments:

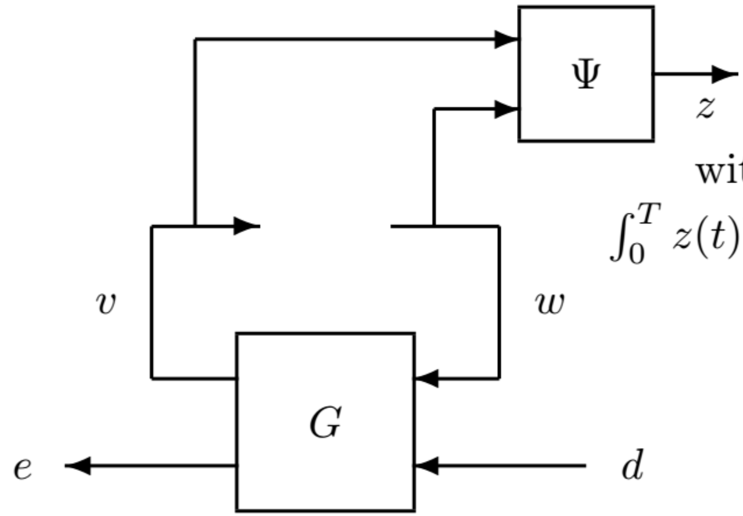
*A library of IQC for various uncertainties / nonlinearities is given in [MR]. Many of these are given as frequency domain inequalities.

*Time-domain IQCs that hold over finite horizons are called hard.

*This generalizes D and D/G scales for LTI and parametric uncertainty. It can be used to model the I/O behavior of nonlinear elements.

[MR] Megretski and Rantzer. System analysis via integral quadratic constraints, TAC, 1997.

Robustness Analysis



The robustness analysis is performed on the extended (LTV) system of *(G, Ψ)* using the constraint on *z*.

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ e(t) \end{bmatrix} = \begin{bmatrix} \mathcal{A}(t) & \mathcal{B}_1(t) & \mathcal{B}_2(t) \\ \mathcal{C}_1(t) & \mathcal{D}_{11}(t) & \mathcal{D}_{12}(t) \\ \mathcal{C}_2(t) & \mathcal{D}_{21}(t) & \mathcal{D}_{22}(t) \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ d(t) \end{bmatrix}$$

Robustness Analysis: Induced L_2 Gain

Theorem 3. *Assume Δ satisfies the IQC defined by (Ψ, M). If there exists $\epsilon > 0$, $\gamma > 0$ a differentiable function P on $[0,T]$ and such that $P(T) \succeq 0$ and for all $t \in [0,T]$*

$$\begin{bmatrix} \dot{P} + \mathcal{A}^T P + P\mathcal{A} & P\mathcal{B}_1 & P\mathcal{B}_2 \\ \mathcal{B}_1^T P & 0 & 0 \\ \mathcal{B}_2^T P & 0 & -\gamma^2 I \end{bmatrix} + (\cdot)^T \begin{bmatrix} \mathcal{C}_2 & \mathcal{D}_{21} & \mathcal{D}_{22} \end{bmatrix} + (\cdot)^T M \begin{bmatrix} \mathcal{C}_1 & \mathcal{D}_{11} & \mathcal{D}_{12} \end{bmatrix} \preceq -\epsilon I$$

then $\|F_u(G,\Delta)\|_{2,[0,T]} < \gamma$.

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then $\|F_u(G,\Delta)\|_{2,[0,T]} < \gamma$.

Proof:

The Differential LMI (DLMI) is equivalent to a dissipation ineq. with storage function $V(x,t) := x^T P(t) x$.

$$\dot{V}(x,t) + z(t)^T M z(t) - (\gamma^2 - \epsilon) d(t)^T d(t) + e(t)^T e(t) \leq 0$$

Integrate and apply the IQC + boundary conditions to conclude that the induced L_2 gain is $\leq\gamma$.

Robustness Analysis: Induced L_2 Gain

Theorem 3. *Assume Δ satisfies the IQC defined by (Ψ, M). If there exists $\epsilon > 0$, $\gamma > 0$ a differentiable function P on $[0,T]$ and such that $P(T) \succeq 0$ and for all $t \in [0,T]$*

$$\begin{bmatrix} \dot{P} + \mathcal{A}^T P + P\mathcal{A} & P\mathcal{B}_1 & P\mathcal{B}_2 \\ \mathcal{B}_1^T P & 0 & 0 \\ \mathcal{B}_2^T P & 0 & -\gamma^2 I \end{bmatrix} + (\cdot)^T \begin{bmatrix} \mathcal{C}_2 & \mathcal{D}_{21} & \mathcal{D}_{22} \end{bmatrix} + (\cdot)^T M \begin{bmatrix} \mathcal{C}_1 & \mathcal{D}_{11} & \mathcal{D}_{12} \end{bmatrix} \preceq -\epsilon I$$

then $\|F_u(G,\Delta)\|_{2,[0,T]} < \gamma$.

Comments:

*A similar result exists for L_2-to-Euclidean or, more generally (Q,S,R,F) cost functions.

*The DLMI can be expressed as a Riccati Differential Ineq. (RDI) by Schur Complements.

*The RDI is equivalent to a related Riccati Differential Eq. (RDE) condition by the strict Bounded Real Lemma.

Robustness Analysis: Induced L_2 Gain

Theorem 3. *Assume Δ satisfies the IQC defined by (Ψ, M). If there exists $\epsilon > 0$, $\gamma > 0$ a differentiable function P on $[0,T]$ and such that $P(T) \succeq 0$ and for all $t \in [0,T]$*

$$\begin{bmatrix} \dot{P} + \mathcal{A}^T P + P\mathcal{A} & P\mathcal{B}_1 & P\mathcal{B}_2 \\ \mathcal{B}_1^T P & 0 & 0 \\ \mathcal{B}_2^T P & 0 & -\gamma^2 I \end{bmatrix} + (\cdot)^T \begin{bmatrix} \mathcal{C}_2 & \mathcal{D}_{21} & \mathcal{D}_{22} \end{bmatrix} + (\cdot)^T M \begin{bmatrix} \mathcal{C}_1 & \mathcal{D}_{11} & \mathcal{D}_{12} \end{bmatrix} \preceq -\epsilon I$$

then $\|F_u(G,\Delta)\|_{2,[0,T]} < \gamma$.

Comments:

*The DLMI is convex in the IQC matrix *M* but requires gridding on time *t* and parameterization of *P*.

*The RDE form directly solves for *P* by integration (no time gridding) but the IQC matrix *M* enters in a non-convex fashion.

Numerical Implementation

An efficient numerical algorithm is obtained by mixing the LMI and RDE conditions.

Sketch of algorithm:

1. Initialize: Select a time grid and basis functions for *P(t).*
2. Solve DLMI: Obtain finite-dimensional optim. by enforcing DLMI on the time grid and using basis functions.
3. Solve RDE: Use IQC matrix *M* from step 2 and solve RDE. This gives the optimal storage *P* for this matrix *M*.
4. Terminate: Stop if the costs from Steps 2 and 3 are similar. Otherwise return to Step 2 using optimal storage P as a basis function.

Outline

- Motivation for LTV Analysis
- Nominal LTV Performance
- Robust LTV Performance
- **Examples**
- Conclusions

Example 1: LTI Plant

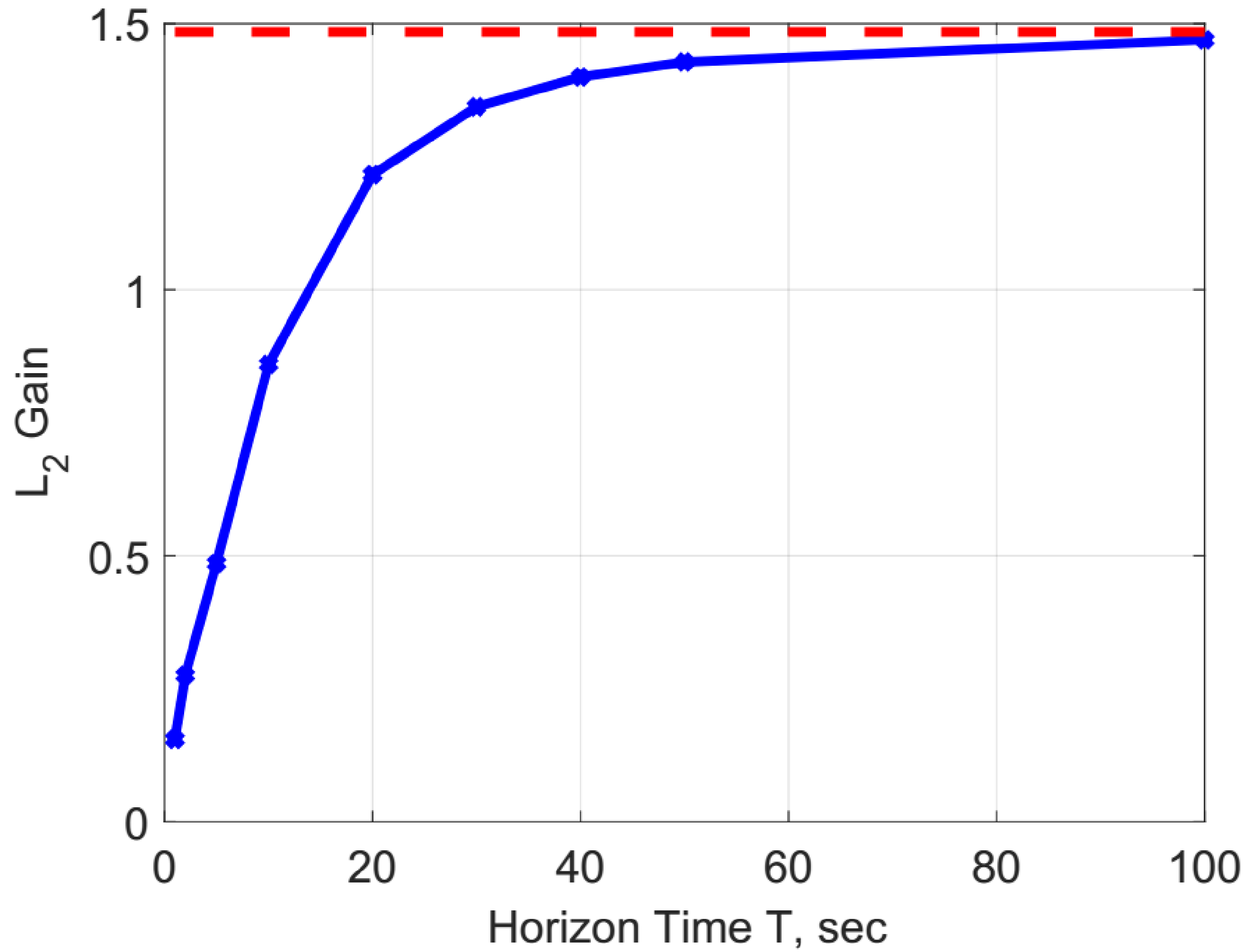
- Compute the induced L_2 gain of *Fu(G,Δ)* where *Δ* is LTI with $\|\Delta\| \leq 1$ and *G* is:

$$A_G := \begin{bmatrix} -0.8 & -1.3 & -2.1 & -2.5 \\ 2 & -0.9 & -8.4 & 0.7 \\ 2 & 8.6 & -0.5 & 12.5 \\ 2.1 & -0.3 & -12.6 & -0.6 \end{bmatrix} \quad B_G := \begin{bmatrix} -0.6 & 1 \\ 0 & 0.2 \\ 0 & 0.4 \\ -1.3 & -0.2 \end{bmatrix}$$

$$C_G := \begin{bmatrix} -1.4 & 0 & 0.5 & 0 \\ 0 & -0.1 & 1 & 0 \end{bmatrix} \quad D_G := \begin{bmatrix} -0.3 & 0 \\ 0 & 0 \end{bmatrix}$$

- By (standard) mu analysis, the worst-case (infinite horizon) L_2 gain is 1.49.
- This example is used to assess the finite-horizon robustness results.

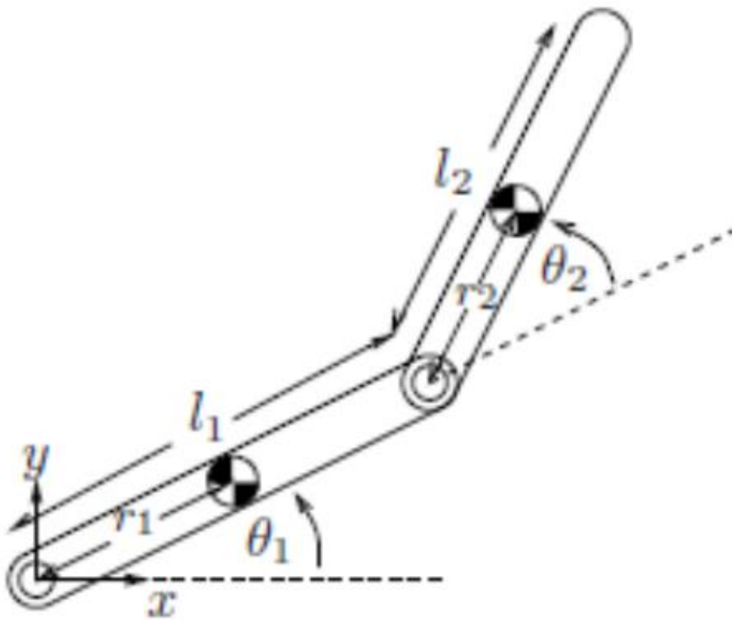
Example 1: Finite Horizon Results

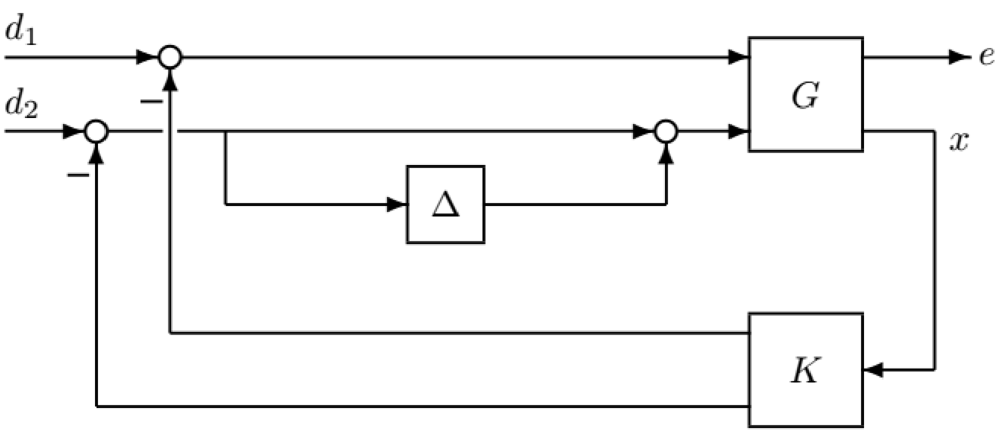


Total comp. time is 466 sec to compute worst-case gains on nine finite horizons.

Example 2: Two-Link Robot Arm

- Assess the worst-case L2-to-Euclidean gain from disturbances at the arm joints to the joint angles.
- LTI uncertainty with $\|\Delta\| \leq 0.8$ injected at 2nd joint.
- Analysis performed along nominal trajectory in with LQR state feedback.

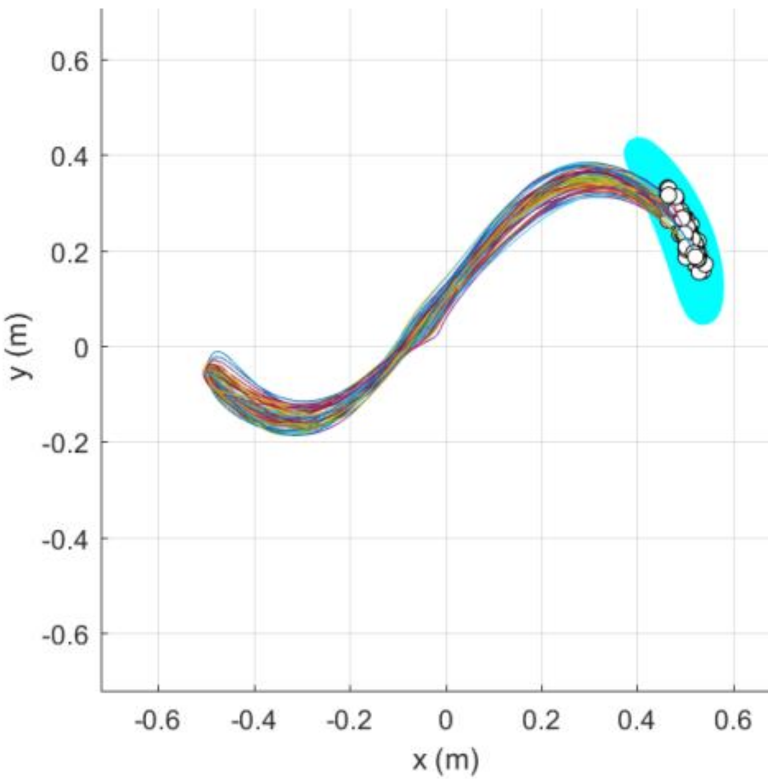




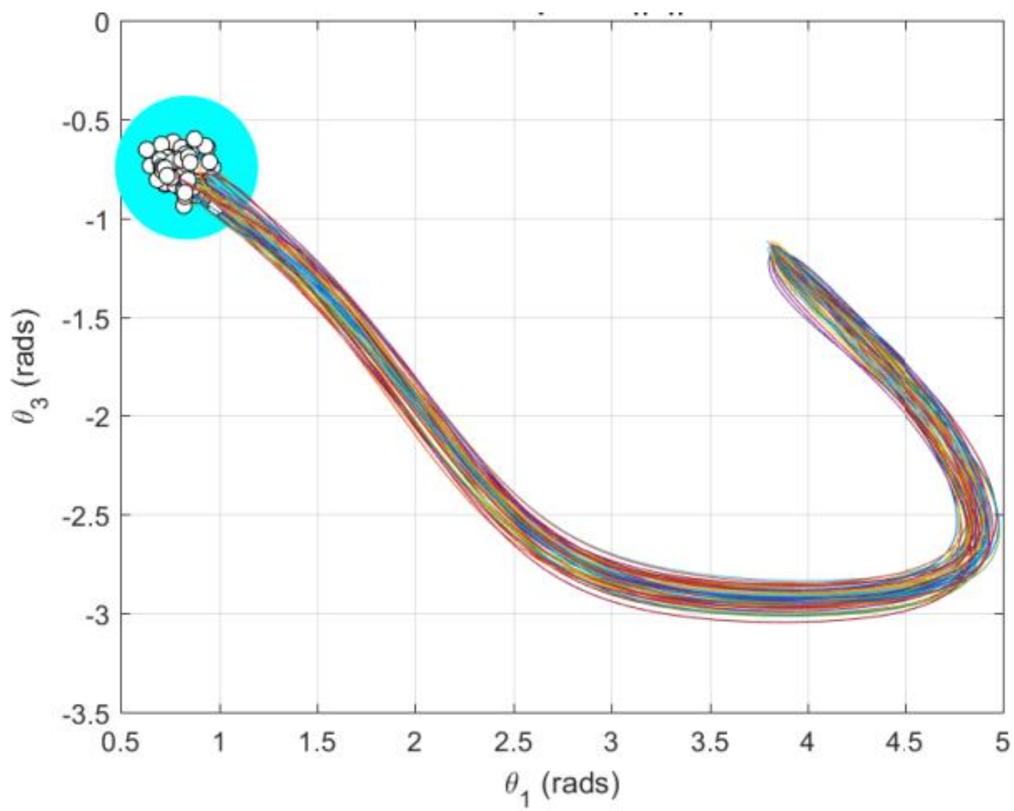
Example 2: Results

Bound on worst-case L_2-to-Euclidean gain = 0.0592.

Computation took 102 seconds.



Cartesian Coords.



Joint Angles

Outline

- Motivation for LTV Analysis
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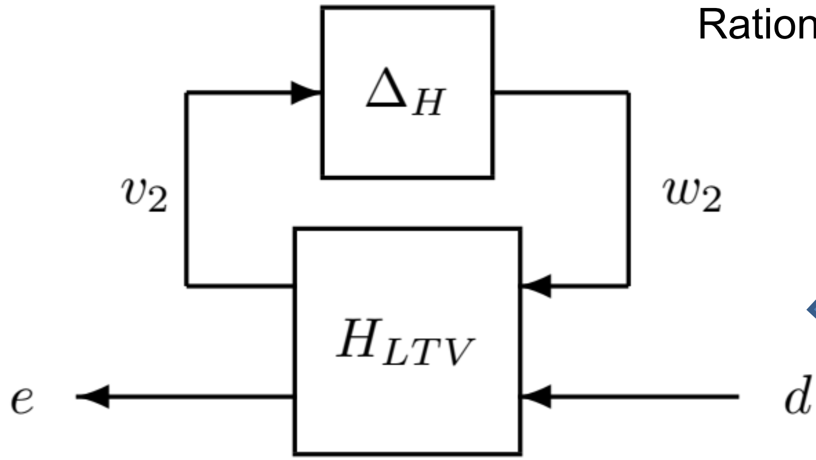
Conclusions

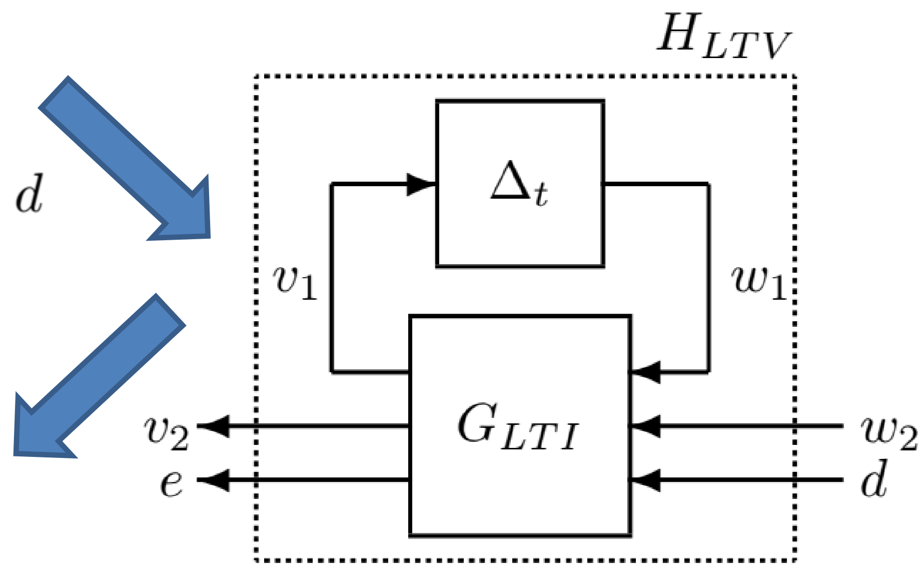
- Main Result: Bounds on finite-horizon robust performance can be computed using differential equations or inequalities.
  - These results complement the use of nonlinear Monte Carlo simulations.
  - It would be useful to construct worst-case inputs / uncertainties analogous to μ lower bounds.
  - An LTVTools toolbox is in development with β-code of the proposed methods.
- References

  - Moore, Finite Horizon Robustness Analysis Using Integral Quadratic Constraints, MS Thesis, 2015.
  - Seiler, Moore, Meissen, Arcak, Packard, Finite Horizon Robustness Analysis of LTV Systems Using Integral Quadratic Constraints, arXiv + submission to Automatica.
  - Related work by Biertümpfel and Pfifer with application to rocket launchers submitted to the 2018 IEEE Conference on Control Technology and Applications.

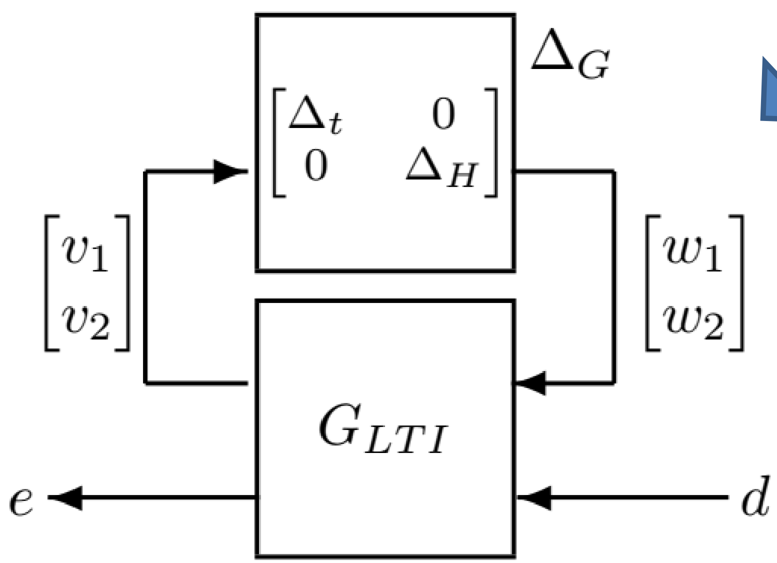
Extensions: Rational Dependence on Time

Ref: Seiler, IQC-Analysis of Uncertain LTV Systems With Rational Dependence on Time, submitted to the ‘18 CDC.





. $w_1 = \Delta_t(v_1)$ is defined by $w_1(t) = t \cdot v_1(t)$



Dynamic IQC for Time Operator Δ_t

Swapping Lemma

Lemma 1. *Let* $F \in \mathbb{RH}_\infty^{n_F \times n_{v1}}$ *have a realization with state matrices* (A_F, B_F, C_F, D_F)*. Then*

$$F\Delta_\delta = \Delta_\delta F - F_C \Delta_{\dot{\delta}} F_B$$

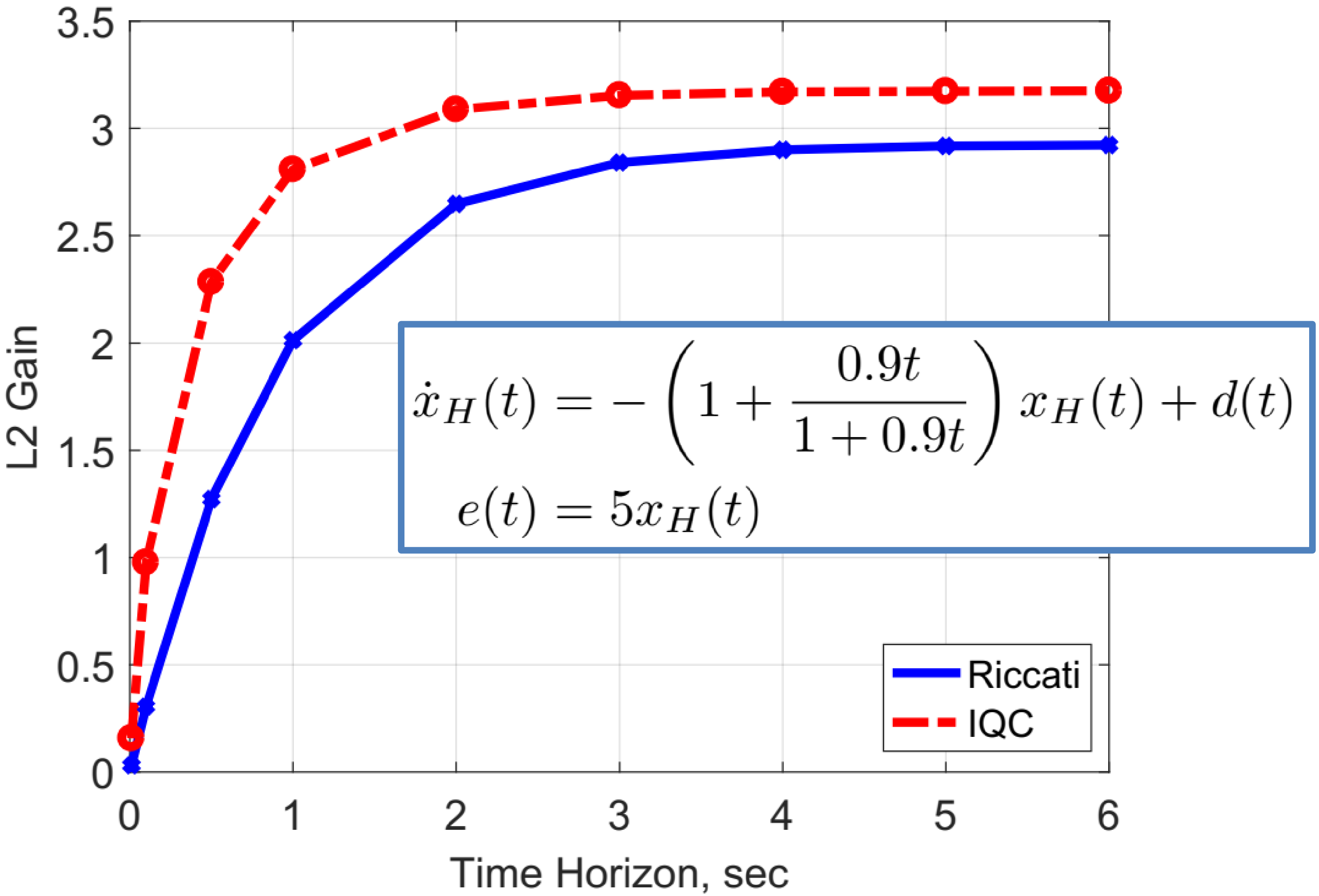
where the state realizations of F_B *and* F_C *are given by* $F_B := (A_F, B_F, I, 0)$ *and* $F_C := (A_F, I, C_F, 0)$.

IQC for Δ_t

$$\Psi := \begin{bmatrix} F & 0 \\ F_C F_B & F \end{bmatrix} \text{ and } M := \begin{bmatrix} 0 & X+Y \\ X+Y^T & 0 \end{bmatrix}$$

for any $X \succeq 0$ *and* $Y = -Y^T$

Example: Nominal Analysis



$$\dot{x}_H(t) = -\left(1 + \frac{0.9t}{1+0.9t}\right) x_H(t) + d(t)$$

$$e(t) = 5x_H(t)$$

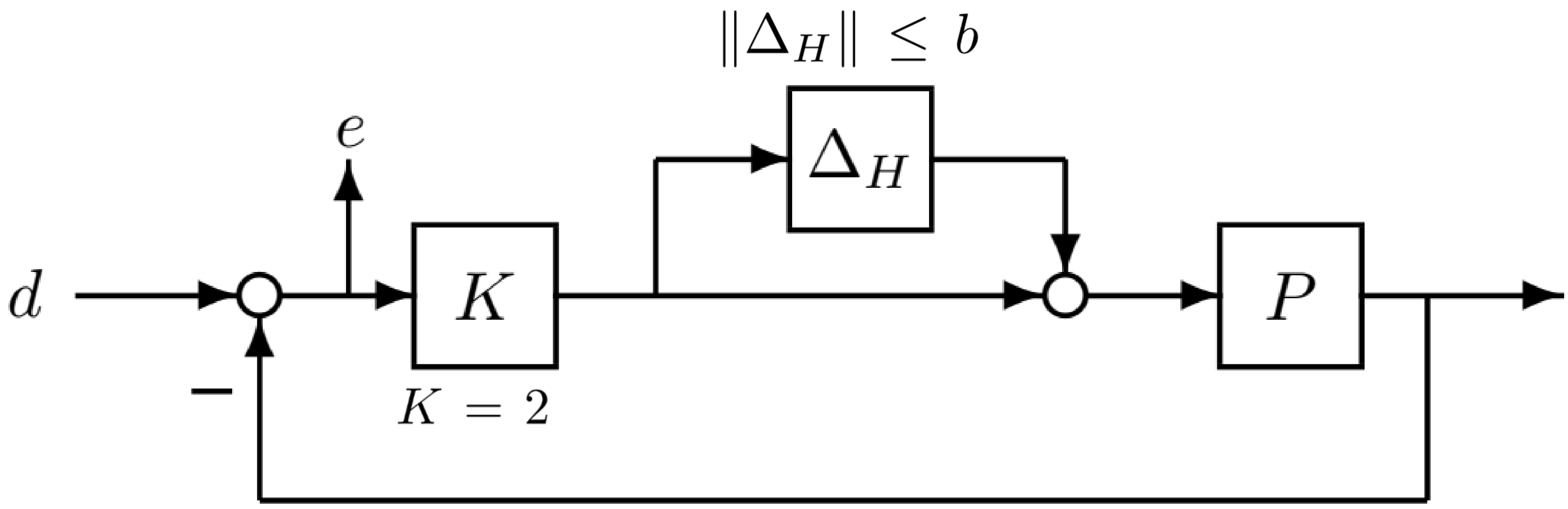
Example: Robust Analysis

Plant, P

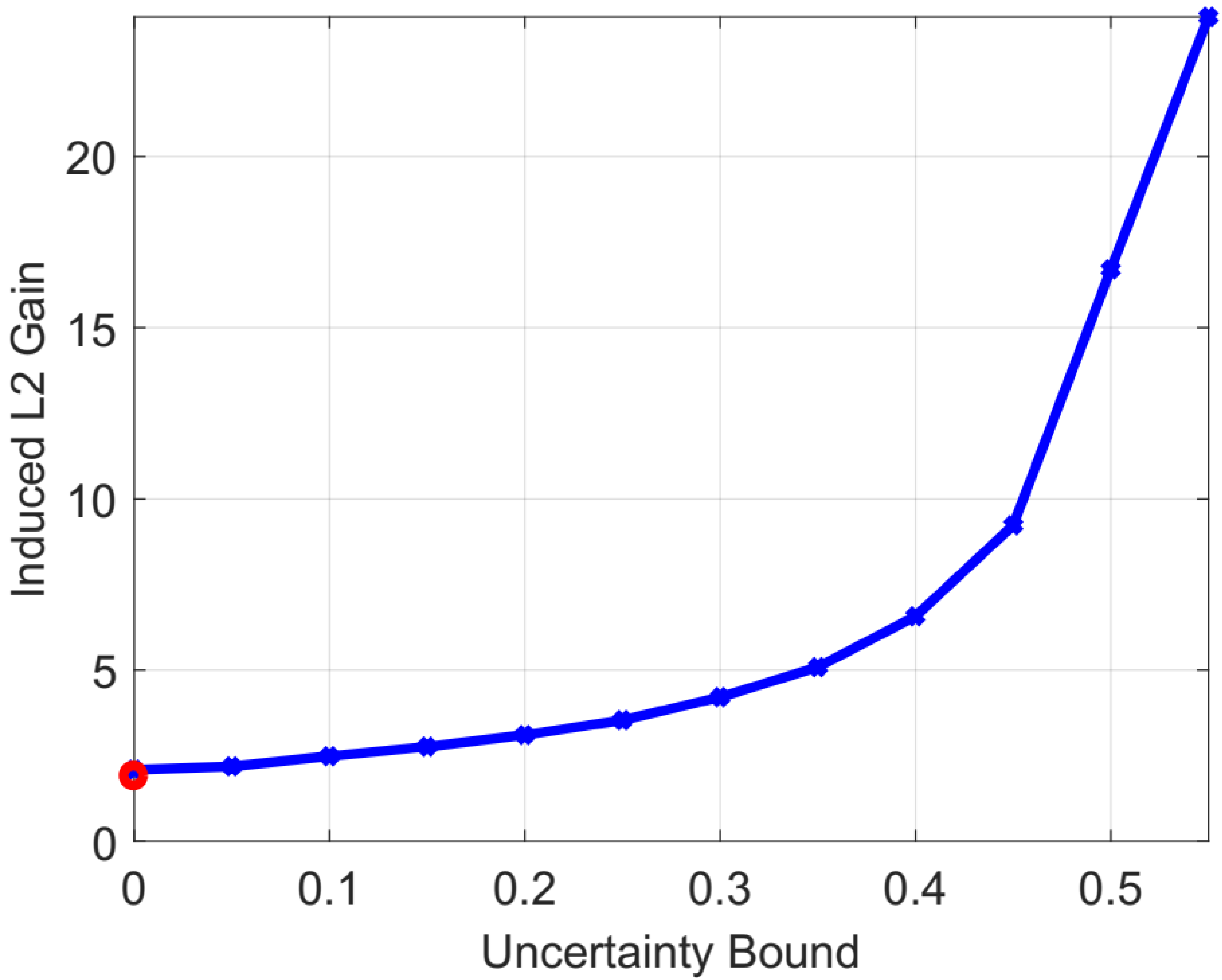
$$\dot{x}_P(t) = \begin{bmatrix} 0 & 1 \\ -4 & -2\zeta(t) \end{bmatrix} x_P(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 4 & 0 \end{bmatrix} x_P(t)$$

$$\zeta(t) := 0.2 + \tfrac{0.63t}{1+0.9t}$$



Example: Robust Analysis



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  - Grant No. NSF/CNS-1329390: “CPS: Breakthrough: Collaborative Research: Managing Uncertainty in the Design of Safety-Critical Aviation Systems”. Prog. Manager: D. Corman.
- NASA
  - NRA NNX14AL36A: "Lightweight Adaptive Aeroelastic Wing for Enhanced Performance Across the Flight Envelope," Tech. Monitor: J. Ouelette.
  - NRA NNX12AM55A: “Analytical Validation Tools for Safety Critical Systems Under Loss-of-Control Conditions.” Tech. Monitor: C. Belcastro.
  - SBIR contract #NNX12CA14C: “Adaptive Linear Parameter-Varying Control for Aeroservoelastic Suppression.” Tech. Monitor. M. Brenner.
- Eolos Consortium and Saint Anthony Falls Laboratory
  - http://www.eolos.umn.edu/ & http://www.safl.umn.edu/

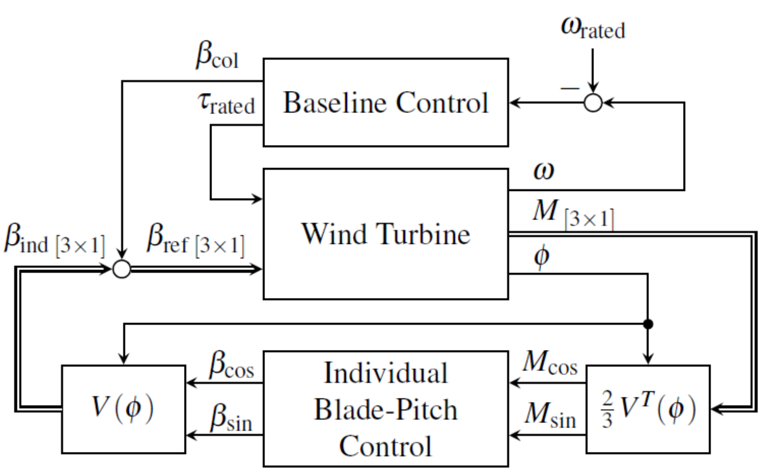
Clipper Liberty, 2012:
Modern utility-scale turbine.

- Rosemount, MN.
- Diameter: 96m
- Power: 2.5MW
- Eolos Consortium: http://www.eolos.umn.edu/
- Saint Anthony Falls Lab: http://www.safl.umn.edu/

Individual Blade Pitch Control

Goals:

- Reducing structural loads on the turbine to
- increase life time of turbine and components while
- keeping power production constant by
- adding an individual blade pitch controller



Controller architecture

Implementation 2017

C96 Liberty research turbine

Ref: Ossmann, Theis, Seiler, '16 ASME DSCC, Best Energy Paper Award