The Eleventh Yale Workshop on Adaptive and Learning Systems

June 4-6, 2001
Center for Systems Science
Yale University
New Haven, Connecticut
the effect of the ratio of SPs on overall performance. We have conducted a set of experiments to examine this point and our results are reported in Figure 5. The interesting result we observe is that as far as the DPs are concerned, when we have 176% or 208% of SPs we have achieved the major gain in delay reduction. Going beyond those values does not significantly reduce the delay for DPs.

Figure 5: Average round-trip delay for smart (top) and dumb (bottom) packets, and average delay for all packets (center) as a function of the percentage of smart packets. These experimental results were obtained while links cpn10-cpn2, cpn2-cpn4 and cpn4-cpn6 were loaded with obstructing traffic.

6 Conclusions

We have summarized the basic principles of CPN. Then we have derived analytical results for best and worst case performance of one particular class of packets: the smart packets. We then describe in some detail the design and implementation of a methodology, Automatica, Vol. 19, pp. 495-502, 1983.


6 Conclusions

We have summarized the basic principles of CPN. Then we have derived analytical results for best and worst case performance of one particular class of packets: the smart packets. We then describe in some detail the design and implementation of a methodology, Automatica, Vol. 19, pp. 495-502, 1983.


Mesh Stability of Helicopters

Karl Hedrick, Aniruddha Pant, Pele Seiler From linear system theory [2], if \( y = h \ast u \), then we have the following relationship:

\[ ||h(t)||_\infty \leq ||h(t)||_1 \\ (1) \]

Using a sliding control law, Hedrick and Swaroop [4] found an LTI convolution kernel, \( h(t) \), which relates the errors in a vehicle following chain by:

\[ e_{i+1} = \mu + e_i. \]

Thus string stability of the chain of vehicles can be determined by analyzing the one-norm of the error propagation impulse response, \( h(t) \). Since this norm represents the maximum amplification of any error as it propagates down the chain, it provides a useful metric for string stability. If this norm is less than one, then all input errors will be attenuated in the oo-norm sense as they propagate down the chain. If this norm is greater than one, then the system is string unstable and there exists an input error which will be amplified as it propagates.

If \( h(t) \) does not change sign, the string stability condition, \( ||h(t)||_1 \leq 1 \), is equivalent to the following frequency domain condition: The magnitude of the associated transfer function, \( H(j\omega) \), should be less than one at all frequencies, i.e. \( ||H(j\omega)||_\infty \leq 1 \). In [4], they found that the sliding control law resulted in a stable string system if reference vehicle information was used.

The SISO input/output norm results are easily generalized to the MIMO case. Let \( f : \mathbb{R} \rightarrow \mathbb{R}^n \) and define \( ||f||_\infty = \max_{\omega \in \mathbb{R}} ||f(\omega)||_\infty \). If \( h(t) \) is the convolution kernel for an n-input, n-output MIMO system, and \( y = h \ast u \), then the input-output relationship is given by:

\[ \|y(t)\|_\infty \leq \left( \max_{\omega} \|h_1(\omega)\|_1 \cdot |u(t)|_1 \right) \] \[ (2) \]

This can also be related to an equivalent frequency domain condition if none of the entries of the convolution kernel changes sign. Let \( H(j\omega) \) be the n x n transfer function matrix for the LTI system given by \( h(t) \). If none of the \( h_1(\omega) \) change sign, then:

\[ \|y(t)\|_\infty \leq \left( \max_{\omega} \sum_{j=1}^{n} \|H_{ij}(j\omega)\|_\infty \cdot |u_j(t)|_\infty \right) \] \[ (3) \]
For the problem under consideration, \( H(s) \in \mathbb{C}^{4 \times 4} \). The inputs are the desired Cartesian positions and yaw of the helicopter. The outputs are the realized Cartesian positions and yaw of the helicopter.

### 3 Regulated Helicopter Model

In this section we briefly describe a linear model for a Yamaha R-50 agricultural helicopter \([9]\). We then describe the linear controller which is used to track desired position trajectories. The linear model is given by:

\[
\ddot{x} = Ax + Bu
\]

where the state and input vectors are given by:

\[
x = [\dot{x}_v \dot{y}_v \dot{z}_v \dot{\psi}_v \dot{t}_v] \quad u = [u_{\text{rot}} \text{d}_{\text{rot}} \phi_{\text{rot}} \text{d}_{\text{rot}}]'\]

\( u_v, w, t \) are the \( x, y, z \) body-fixed velocities, respectively. \( \Phi, \Theta \) are the roll and pitch of the helicopter while \( p \) \( q \) are the roll and pitch rate. \( r \) is the yaw rate. \( a_{\text{rot}}, b_{\text{rot}} \) \( t_{\text{rot}} \) are actuator states. The first two inputs, \( u_{\text{rot}}, w_{\text{rot}} \), control the flapping coefficients of the helicopter. These inputs primarily control the pitch and roll of the helicopter. The final two inputs, \( a_{\text{rot}} \) \( b_{\text{rot}} \), control the main rotor thrust and tail rotor thrust. A predictor-error method was used to obtain the parameters in the \( A, B \) matrices from experimental input-output data. We refer the interested reader to \([9]\) for additional modeling details. We note that if the helicopter remains close to hovering, we can treat the \( x, y, z \) body-fixed velocities as global \( x, y, z \) velocities. Integration of these variables will then yield global position. Similarly, integration of the body-fixed yaw rate will yield the global heading if the helicopter is close to hovering.

A control law was designed by Shim \([9]\) to stabilize the helicopter dynamics and steer the vehicle along a desired trajectory. The desired trajectory is given by:

\[
\dot{y}_d = C_1 z_d + C_2 \text{d}_d \quad (4)
\]

Thus the goal is to force \( y_1 = [x_v y_v z_v \psi_v]' = C_1 \text{d}_d \) to track \( y_d \). In words, we are trying to steer the helicopter along a desired position and heading trajectory. This goal is accomplished using 4 proportional derivative controllers. Specifically, \( u_{\text{rot}}, u_{\text{rot}}, u_{\text{rot}}, u_{\text{rot}} \) are strongly coupled to global \( x, y, z, \) and global heading, respectively. The four proportional derivative controllers were designed to control each loop. The final control law had the following state space form:

\[
\dot{x} = K_1 C_2 x + K_3 d \quad (5)
\]

The closed loop model is then given by:

\[
\dot{x} = (A - BK_1 C_2) x + BK D_3 + \text{d}_d \quad (1)\]

The closed loop system has 4 inputs and 4 outputs. It can be represented in transfer function form as:

\[
Y_i(s) = H(s) R_i(s) \quad (6)
\]

Ideally this transfer function would be diagonal, but in reality there is coupling between the four modes which may degrade performance.

#### 4 Proposed Structure for Mesh Controller

In this section we propose a structure for designing mesh controllers for the helicopters. The lowercase letters represent variables which are functions of time. The corresponding uppercase letters represent Laplace transforms of the time functions, e.g. \( X(s) = C(x(t)) \). The procedure we describe produces inherently mesh stable controllers. This is an improvement on the previous design techniques where the controllers are designed and then checked for mesh stability. We consider three helicopters following the previous one, see figure 1. The mesh controller design structure is shown in the figure 2. The block Regulated Helicopter represents the stabilized dynamics of a helicopter. The tracking controller for an individual helicopter is designed as explained in Section 3. We assume that the regulated helicopter follows the desired trajectory faithfully. In the previous section we designed the following MIMO transfer function relation,

\[
X_i(s) = H(s) X_{d,i} \quad (7)
\]

Note that the notation has been changed slightly, in the previous section this equation was represented as, \( Y_i(s) = H(s) R_i(s) \). Here \( X_i \in \mathbb{C}^4 \) are the regulated outputs of the helicopter i.e. longitudinal, lateral, vertical positions and yaw. In addition we assume that \( H(0) = 14 \times 4 \); in other words, there is no steady state error between the desired and actual position. Define:

\[
x_{d,i} := x_{d,i} - x_i \quad x_{d,i} := x_{d,i} - x_i \quad E_f^i := E_f - \delta \quad \delta = \frac{x_{d,i} - x_i}{\delta} \quad (8)
\]

Here \( x_{d,i}, x_{d,i} \) are the desired positions of \( i^{th} \) vehicle with respect to the preceding and the leader vehicle respectively. \( E_f^i \) are the corresponding errors. For a safe formation flight \( E_f^i \) are the critical errors to protect against a crash. However it has been shown \([8]\) that if we design controllers based only on the preceding error information, then we get error amplification as we go down the chain. So taking a cue from that work we implement \( X_o \) as, see figure 2.

\[
X_{id} = p(K(s) E_f^i + X_o d_i) + (1 - p) (K(s) E_r + X_o d_i) \quad (8)
\]

If we assume that \( K(s) \equiv 0 \) then the above expression is easy to interpret. It says the desired position of a helicopter is convex combination of its desired position with respect to leader and preceding helicopters. Here \( p \) represents the coupling to the preceding vehicle. As stated above we would like \( p \) as large as possible for safety. Using the above definitions we get following relations which will be used in further simplification.

\[
X_{1,i-1} - X_{d,i} = \delta \quad \delta \quad \delta \quad (9)
\]

#### 5 Robustness to Disturbances

The analysis of the previous section assumes a linear model for the regulated helicopter as well as perfect tracking of the desired position command at steady state. In reality, the helicopter dynamics are nonlinear and the assumption of linearity is only justified by a small operating range or a feed-back linearizing controller at the regulation layer. However, the regulation layer cannot achieve perfect tracking of the desired profile generated by the mesh controller. Furthermore, external disturbances acting on the UAVs, such as wind gusts, will cause additional errors. In this section, we justify the two-layer control structure. We will show that a mesh stable controller leads to the additional property that the effect of any such disturbances will be damped out as they propagate. Consider the regulated helicopter model with a disturbance reflected...
The helicopter position consists of the desired part tracked by the regulated helicopter plus a term, \( d_i \), representing external disturbances and imperfect tracking of the regulation layer. We use the same mesh controller given in Equation 8.

Using analysis entirely analogous to the preceding section, we obtain the following relation:

\[
E_i(s) = pG(s)E_{i-1}(s) + G(s)(D_{i-1}(s) - D_i(s)) 
\]

where \( pG(s) \) is given in Equation 10 and \( G(s) = \left[I + HK\right]^{-1} \). Equation 12 shows that the \( p^6 \) error consists of a term which is propagated, via \( pG(s) \), from other errors in the string. The \( 4 \)th error also contains a term due to variations in the disturbances acting on the helicopters of the string. Note that if there is a disturbance which acts uniformly on the string, \( d_\text{av} \), then the second term of Equation 12 is small.

It is reasonable to expect large disturbances will occasionally act on one portion of the string. The relation given above implicitly shows that the effect of disturbances on other members of the mesh is also propagated via \( pG(s) \). Suppose that there is a large wind gust acting on the second \((i = 2)\) helicopter and other disturbances are negligible. The disturbance affects the error, \( E_2 \), through the transfer function \( G(s) = pG(s) + GD_3 \). However, it propagates to other errors though \( pG(s) \). It is easy to show that for \( t > 2 \), \( E_i = (pG)^{i-2} E_2 + (pG)^{i-3} GD_3 \).

Without being too rigorous, we note that \( \|pG\|_\infty < 1 \) when the mesh stable controller is used. This causes the magnitude of \( (pG)^{i-3} \) to geometrically decay with \( i \) at each frequency. Thus, disturbances acting on the string will decay as they propagate. If a mesh unstable controller is used, then \( \|pG\|_\infty > 1 \). This mesh unstable controller causes the magnitude of \( (pG)^{i-3} \) to geometrically grow with \( i \) at some frequency. In this simple example, if the disturbance acting on the second helicopter has the right frequency, its amplitude will grow geometrically down the string. For a large string, the result could be catastrophic.

In summary, the results above can be generalized to random disturbances propagating in the string. The key point is that we designed the mesh controller under the assumption of a linearized regulated helicopter with perfect steady state tracking.

\[ X_i = H(s)X_0 + D(s) \]  

\[ (11) \]

7 Results and Simulations

In this section we will present results of simulations performed using the theory presented in previous sections. As a first cut \( K(s) \) was assumed to be zero. The error propagation characteristics for different values of \( p \) are shown in Figure 4. We can see that for \( p = 1 \), the maximum of the error for the second vehicle \( e_2 \) is greater than the maximum of the error for the first follower \( e_1 \). This is consistent with our theoretical result which says that the vehicle following based on only preceding vehicle information implies string instability. On the other hand for the other subplots in the figure we can see that the second error is lower than the first error and thus we have the error damping characteristic. It should be noted however that for formation safety one would want the controller as strongly connected to the closest vehicles as possible i.e. \( p \) should be as high as possible as long as we are string stable.

8 Conclusions

We proposed a general structure for designing mesh controllers for formation flying. The mesh stability property of the structure was demonstrated using linear transfer function theory as well as a simple simulation experiment. In future, we expect to perform experimental testing of the ideas presented in this paper.

References

[10] Aniruddha Pant, Peter Seiler, John Koo, Karl Hedrick, Formation flying of Unmanned Aerial Vehicles, accepted for publication in 2001 ACC.