ABSTRACT

The analysis of a rear-end collision warning/avoidance (CW/CA) system algorithm will be presented. The system is designed to meet several criteria:

1. System warnings should result in a minimum load on driver attention.
2. Automatic control of the brakes should not interfere with normal driving operation.
3. The system should perform well in a variety of driving conditions.

The resulting CA algorithm will use a tire-road friction estimate. The benefit of combining a tire-road friction estimator with a CA system will be studied.

INTRODUCTION

The development of collision avoidance systems is motivated by their potential for increased vehicle safety. Half of the more than 1.5 million rear-end crashes that occurred in 1994 could have been prevented by collision avoidance systems [1]. Collision avoidance systems can react to situations that humans can not or do not, due to driver error. Therefore, they are able to reduce the severity of accidents.

A rear-end collision avoidance system will be developed in the following manner. Human factors concerns will be reviewed. This is necessitated by the requirement that the brake control and warning algorithm be as unobtrusive as possible. Next, previous algorithms published by Mazda and Honda will be reviewed. This will allow the human factors concerns to be discussed in the context of actual CW/CA algorithms.

Finally, the new CW/CA algorithm will be proposed. This will include specifying a nominal criteria for warning and braking. It should be noted that only longitudinal CA is being considered, so no lateral control will be developed. The method of delivering the warnings will also be specified (i.e. visual, audio, etc.). This nominal criteria will then be modified based on driver inputs and tire-road friction information. The complete algorithm will then be tested in simulation.

HUMAN FACTORS CONSIDERATIONS

CW/CA systems must be accepted by drivers. In general, this means that the system must be useful to the driver and must not interfere with normal driving habits. This has several interpretations. First, warnings given by the system should result in a minimum load on driver attention. An increase in warning frequency produces a tradeoff between two harmful driver responses [2]. Frequent warnings may desensitize the driver and cause future warnings to be ignored. Rare warnings can distract the driver during critical situations. Therefore, the method of warning the driver and the frequency at which warnings are given must be chosen carefully. One potential solution is to give constant visual feedback to the driver. Unlike random warnings, constant visual feedback in the form of graduated light displays or relative distance displays may not be obtrusive to the driver. Therefore, the driver may not be desensitized by this type of warning. This type of warning should actually cause the driver to become accustomed to the CW/CA system so that they should not be startled when a critical warning is given.

Furthermore, automatic control of the brakes should not interfere with normal driving operation. A driver who is attempting an avoidance maneuver, such as steering, may be startled and possibly lose vehicle control if the system automatically applies the brakes [3]. Therefore, a very conservative CA system may be able to prevent all possible collisions. However, it will also be more likely to disrupt the driver by applying the brakes at inappropriate times. A more reasonable goal is to design an unobtrusive algorithm which prevents some collisions and reduces the severity of all other impacts.

Finally, the effect of individual driving styles must be considered [2]. Each driver has different following tendencies, from passive to aggressive. A conservative
system which has been designed for a passive driver may give many warnings to an aggressive driver. As stated above, frequent warnings tend to desensitize the driver. Hence, the aggressive driver will eventually ignore the warnings given by a passive system. If the system is instead designed with the “average” driver in mind, the warning frequency will tend to alienate passive and aggressive drivers. Hence, any algorithm needs a method of allowing the driver to customize the warning and braking frequencies.

PREVIOUS ALGORITHMS

Most CW/CA systems in existence use a similar algorithm. The systems use relative distance, relative velocity and vehicle velocity information to warn the driver or control the vehicle. Specifically, a warning critical distance is defined as a function of vehicle velocity and relative velocity. A warning is given to the driver when the vehicle spacing is less than this warning critical distance. A braking critical distance can be similarly defined. The system applies the brakes when the spacing is less than the braking critical distance. Systems published by Mazda and Honda will now be discussed.

MAZDA’S ALGORITHM

Mazda’s algorithm uses the following braking critical distance definition [4]:

\[
d_{br} = \frac{1}{2} \left( \frac{v^2}{\alpha_1} - \frac{(v - v_{rel})^2}{\alpha_2} \right) + v \cdot \tau_1 + v_{rel} \cdot \tau_2 + d_o \tag{1}
\]

where \( v \) is the CW/CA vehicle velocity, \( v_{rel} \) is the relative velocity between vehicles (\( v_{rel} = v - v_{preceeding} \)), \( \alpha_1 \) is the maximum deceleration of the vehicle, \( \alpha_2 \) is the maximum deceleration of the preceding vehicle, \( \tau_1 \) and \( \tau_2 \) are delay times, and \( d_o \) is a headway offset. A plot of this critical distance as a function of velocity and relative velocity is shown in Figure 1. The following parameter values were used: \( \alpha_1 = 6 \text{ m/s}^2 \), \( \alpha_2 = 8 \text{ m/s}^2 \), \( \tau_1 = 0.1 \text{ s} \), \( \tau_2 = 0.6 \text{ s} \), and \( d_o = 5 \text{ m} \). The critical distance in this plot is equal to 0 in this plot when \( v_{rel} \) is greater than \( v \). In this case, a vehicle moving in the opposite direction has been detected. It is usually assumed that a vehicle in the opposite lane is detected and a warning is not given.

The sum of the terms in parenthesis and \( d_o \) is the minimum distance needed to prevent a collision if both vehicles begin braking with their respective maximum decelerations. These terms can be derived based on the kinematics of the two vehicles braking to a full stop. If the vehicles start at this distance and brake with their maximum decelerations, they will come to a stop with their bumpers touching. To make the system more conservative, the two delay terms are added. They account for system and driver delays.

Honda’s algorithm uses the following warning critical distance definition [3]:

\[
d_w = 2.2 \cdot v_{rel} + 6.2 \tag{2}
\]

where \( v_{rel} \) is the relative velocity between vehicles. Furthermore, the algorithm uses the following braking critical distance:

\[
d_{br} = \begin{cases} 
\frac{\tau_2 v_{rel} + \tau_1 \alpha_1 - 0.5 \alpha_1 \tau_1^2}{\alpha_2} & \frac{v_2}{\alpha_2} \geq \tau_2 \\
\frac{\tau_2 v - 0.5 \alpha_1 (\tau_2 - \tau_1)^2 - \frac{v_2}{2\alpha_2}}{\alpha_2} & \frac{v_2}{\alpha_2} < \tau_2 
\end{cases} \tag{3}
\]

where \( v \) is the vehicle velocity, \( v_{rel} \) is the relative velocity between vehicles, \( \alpha_1 \) is the maximum deceleration of the vehicle, \( \alpha_2 \) is the maximum deceleration of the preceding vehicle, \( v_2 \) is the preceding vehicle velocity, \( \tau_1 \) is the system delay, and \( \tau_2 \) is the braking time. A plot of this braking critical distance as a function of velocity and relative velocity is shown in Figure 2. The following parameter values were used: \( \alpha_1 = \alpha_2 = 7.8 \text{ m/s}^2 \), \( \tau_1 = .5 \text{ s} \), and \( \tau_2 = 1.5 \text{ s} \).
A comparison of the critical distance plots shows that Honda’s algorithm results in a much less conservative system. Honda developed their system with the intention that it would not be conservative. It is possible for a driver to begin a steering collision avoidance maneuver much later than a braking collision avoidance maneuver. Therefore, a conservative CA system might apply the brakes while the driver was attempting a steering collision avoidance maneuver. This could startle the driver and possible cause them to lose control of the vehicle. Honda’s system will be less likely to interfere with normal driver habits. As a result, it may not avoid all extreme case collisions, but it should reduce the impact speed of extreme case collisions.

The warning critical distance was defined using driver test data. Drivers were told to perform a normal steering CA maneuver when approaching an obstacle. The warning distance is defined to be less than the distance at which drivers began their steering maneuvers. As a result, the warnings should not interfere with driver maneuvers and should not desensitize the driver.

In a second test, drivers were told to perform an emergency steering maneuver. The braking time parameter, $\tau_b$, was chosen that the braking critical distance was less than the distance at which the emergency steering maneuvers were started. Therefore, the brakes will not be applied when the driver is attempting a steering maneuver.

**ALGORITHM PROPOSAL**

The algorithm has several components. A non-dimensional warning value is used to evaluate driving situations. This warning value uses braking and warning critical distances which are functions of vehicle velocity and relative velocity. Finally, scaling factors are used to account for variable driver habits and different road conditions. These components will now be discussed in more detail.

**NON-DIMENSIONAL WARNING VALUE**

The algorithm is based on a non-dimensional warning value:

$$w = \frac{d - d_{br}}{d_w - d_{br}}$$

where $d$ is the actual vehicle spacing, $d_{br}$ is the braking critical distance and $d_w$ is the warning critical distance. A graduated light display (Figure 3) will be used to give a continuous set of warnings to the driver. The situation $w>1$ corresponds to $d < d_{br}$. Thus if $w=1$, the light meter displays green lights, denoting a safe driving situation. The light meter then displays an increasing number of yellow lights for $a < w < 1$, where $a$ is the “audio warning” parameter. In this situation, $d$ is still well above $d_{br}$. Next, visual (red lights) and audio warnings will be given if $0 < w < a$, i.e. $d$ is very close to $d_{br}$. Since $d$ is approaching $d_{br}$, strong warnings need to be given to the driver. Finally, if $w < 0$ then $d < d_{br}$; the system should apply the brakes!

**WARNING DISTANCE DEFINITION**

The algorithm uses a conservative warning distance and a non-conservative braking distance. The non-conservative braking distance should minimize intrusions on the driver. The conservative warning distance is used so that the graduated light display can give a wide range of feedback to the driver. This should ensure that the driver will not be startled by the warnings during critical situations. On the other hand, the initial graduated light display warnings are mild enough that they should not desensitize the driver. A modified form of the Mazda critical distance is proposed for the warning distance:

$$d_w = \frac{1}{2} \left( \frac{v^2}{\alpha} - \left( \frac{v - v_{rel}}{\alpha} \right)^2 \right) + v \cdot \tau + d_o$$

where $v$ is the vehicle velocity, $v_{rel}$ is the relative velocity between vehicles, $\alpha$ is the maximum deceleration of both vehicles (which are assumed to be equal), $\tau$ accounts for the system and driver delays, and $d_o$ is a headway offset.

**BRAKING DISTANCE DEFINITION**

As stated above, a conservative braking distance should be used so that the system will be unobtrusive to
the driver. One possible method is to define a “black box” braking distance:

\[ d_{br} = k_1 \cdot v + k_2 \cdot v_{rel} + k_3 \] (6)

The parameters \( k_1, k_2, \) and \( k_3 \) are chosen strictly from test data to ensure that the brakes are not applied during normal driving operation. This braking distance is referred to as “black box” because the form has no relation to kinematic analysis. This form was not analyzed due to the lack of intuition associated with the constants.

Another definition, the “time-to-collision” braking distance, will be used in the proposed algorithm. This definition is based on the following proposition: If the time-to-collision (obtained from kinematic analysis) is less than the total warning reaction delay (human + system), the brakes are not applied during normal driving operation. This form was not analyzed due to the lack of intuition associated with the constants.

The kinematic analysis assumes the CW/CA vehicle is initially at point \( x_{10} \) and has velocity \( v_1 \). The lead vehicle has initial position \( x_{20} \) and velocity \( v_2 \). At time \( t=0 \), the lead vehicle brakes with a deceleration of \( a_2 \) while the CW/CA vehicle continues at the same velocity. Under these conditions, the vehicles have the following paths as functions of time:

\[ x_1(t) = x_{10} + v_1 \cdot t \]
\[ x_2(t) = x_{20} + v_2 \cdot t - 0.5 \cdot a_2 \cdot t^2 \] (7)

At the time of collision, \( t_c \), this leads to:

\[ x_2(t_c) - x_1(t_c) = d - v_{rel} \cdot t_c - 0.5 \cdot a_2 \cdot t_c^2 = 0 \] (8)

where \( d = x_{20} - x_{10} \) and \( v_{rel} = v_1 - v_2 \). In accordance with the proposition above, assume the time-to-collision, \( t_c \), is equal to the total warning delay, \( \tau_{sys} + \tau_{hum} \). The system delay is given by \( \tau_{sys} \) and the human response delay is given by \( \tau_{hum} \). The braking distance definition is then given by:

\[ d_{br} = v_{rel} \left( \tau_{sys} + \tau_{hum} \right) + 0.5 \cdot a_2 \left( \tau_{sys} + \tau_{hum} \right)^2 \] (9)

If the actual vehicle spacing drops below this braking distance definition, then the time-to-collision is less than the total warning delay. Therefore, the system will apply the brakes. Notice that \( v_{rel} \) will be measured and \( \tau_{sys} \) will be given by the brake system hardware. Thus, \( a_2 \) and \( \tau_{hum} \) are the tunable parameters for the braking distance definition.

**ALGORITHM MODIFICATIONS**

Finally, the algorithm is modified in two ways. First, a tire-road friction coefficient estimation will be used to scale the critical distances. This scaling factor will be a function of the estimated friction coefficient, \( f(\mu) \). Second, a personalized algorithm will be obtained by letting the driver scale the braking and warning distances. The driver scaling will be done via a dashboard knob and will be denoted by \( g(\text{driver}) \). The non-dimensional warning value, \( w \), will actually be calculated using the scaled distance definitions:

\[ d_{w, \text{scaled}} = d_w \cdot f(\mu) \cdot g(\text{driver}) \]
\[ d_{br, \text{scaled}} = d_{br} \cdot f(\mu) \cdot g(\text{driver}) \] (10)

where \( f() \) is the friction scaling function, \( \mu \) is the estimated value of the tire-road friction coefficient, and \( g(\text{driver}) \) is the driver scaling function. A tire-road friction coefficient estimation scheme which uses only shaft angular velocity sensors, proposed by Yi, could be used with this CW/CA algorithm [5].

The driver scaling function must be bounded if it is desired to limit driver influence, \( g_{min} < g() < g_{max} \). As a first attempt, a piece-wise linear friction scaling function is proposed. The piece-wise linear function is simple, limits the number of parameters that need to be tuned, and can still provide the needed scaling. This function will have the following form if \( \mu_{min} < \mu < \mu_{norm} \):

\[ f(\mu) = \frac{f(\mu_{norm}) - f(\mu_{min})}{\mu_{norm} - \mu_{min}} \cdot (\mu - \mu_{min}) \] (11)

where \( \mu_{norm} \) is the normal friction coefficient and \( \mu_{min} \) is the smallest friction coefficient to be considered. Also, \( f(\mu) = f(\mu_{min}) \) if \( \mu < \mu_{min} \) and \( f(\mu) = f(\mu_{norm}) \) if \( \mu > \mu_{norm} \). We can set \( f(\mu_{norm})=1 \) because there should be no distance scaling when the friction coefficient is equal to its normal value. This leaves \( \mu_{norm}, \mu_{min} \), and \( f(\mu_{min}) \) as the tunable parameters for this scaling function.

**PARAMETER SELECTIONS**

The proposed algorithm has a total of 11 tunable parameters:

1. (3) Warning Distance Parameters: \( d_0, \tau, \alpha \)
2. (2) Braking Distance Parameters: \( \tau_{hum}, \alpha_2 \)
3. (1) “Audio warning” parameter: a
4. (3) Friction Scaling Parameters: \( \mu_{min}, \mu_{norm}, f(\mu_{min}) \)
5. (2) Driver Scaling Parameters: \( g_{min} \) and \( g_{max} \)

The number of parameters can be reduced by tying together the warning and braking distance definitions. Both definitions use parameters to define lead car deceleration. The parameters could be set equal: \( \alpha = \alpha_2 \). Furthermore, \( \tau \) in the warning distance definition represents the total warning response delay. Therefore,
this parameter can be eliminated by letting $\tau = \tau_{\text{hum}} + \tau_{\text{sys}}$.

The parameter set is now reduced to nine. The values for all parameters, especially $\tau_{\text{hum}}$, should be tuned based on driver test data. However, for testing purposes, the following warning/braking parameter values will be used: $\tau_{\text{hum}} = 1$ sec, $\alpha = 6$ m/s$^2$, $d_0 = 5$ m. Also, it is assumed that $\tau_{\text{sys}} = 0.2$ sec. A plot of the warning and braking critical distances as functions of velocity and relative velocity are shown in Figure 4 and Figure 5, respectively. Figure 4 actually shows the Honda warning surface beneath the Modified warning surface. This plot emphasizes the fact that the Modified algorithm uses a conservative warning distance while the Honda algorithm uses a very non-conservative warning distance. In fact, the Modified warning distance plot looks similar to the Mazda critical distance plot (Figure 1), as expected. Also, the Modified braking distance plot looks similar to the Honda critical distance plot (Figure 2). In summary, the plots show that the algorithm is composed of a conservative warning distance and non-conservative braking distance. The conservative warning distance is designed so that the graduated light display will give a wide range of feedback to the driver. The non-conservative braking distance is designed to be unobtrusive.

**SIMULATION RESULTS**

The Honda and Modified algorithms will be compared using a single test case. These simulations will be used to show the benefit of using friction estimation in conjunction with a CA algorithm. The Honda algorithm is used for comparison because it is uses a non-conservative braking distance, similar to the Modified algorithm. The Mazda algorithm uses a conservative brake distance. Hence, it would perform better at avoiding collisions. However, this performance would come at the cost of intruding on normal driving maneuvers. In the test case, the CW/CA car and a lead car are both traveling at 27.8 m/s with a separation of 50 m. The lead car suddenly applies the brakes and decelerates at 6 m/s$^2$. The CW/CA vehicle maintains its velocity, which simulates a driver who is unaware of the critical nature of the situation. This test case will be studied for normal and degraded road conditions. A Bakker-Pacejka “Magic” Formula tire model is used in the simulation. Various road conditions are simulated by multiplying the tire force by a road-condition factor (i.e., scaling the peak factor of the “Magic” Formula model). For the normal conditions, this factor is equal to 1.0. For the degraded road conditions, this factor is equal to 0.3.

**NORMAL ROAD CONDITIONS**

Figure 6 shows the vehicle response when the Honda algorithm is used. The top plot shows the vehicle spacing (range), warning distance and braking distance used by this algorithm. The bottom plot shows the output of the algorithm. In this case, algorithm = 0 means the system is doing nothing. Algorithm = 1 means the system is offering a warning. Finally, algorithm = 2 means that the system has applied the brakes and shut the throttle angle. The brakes are applied until the driver takes over by pushing the brake pedal or until the vehicle comes to a stop. In this simulation, the driver is completely out of the loop, so the CW/CA system brings the vehicle to a rest. Notice that the first plot shows that the vehicles collide at ~6 seconds (range drops to zero). Therefore, this system
did not prevent the collision from occurring. On the other hand, the middle plot shows that the relative velocity at impact is 3.9 m/s. If the Honda CW/CA system was not implemented, the CW/CA vehicle would continue moving at 27.8 m/s while the lead vehicle decelerated at 6 m/s\(^2\). In this case, the impact velocity would be 24.5 m/s. Therefore, this algorithm has reduced the impact energy by 96%.

The Modified algorithm actually does worse at preventing collisions than the Honda algorithm. However, this means that the Berkeley algorithm is less likely to bother the driver. Extensive testing must be done to find the largest braking distance possible that is still unobtrusive to the driver.

DEGRADED ROAD CONDITIONS

The same simulations as above will be conducted except that road conditions will be degraded. This degradation could come from many factors such as ice or snow. The results, given in Figure 8, are qualitatively the same as in Figure 6, except that the CW/CA deceleration capabilities are reduced. The warning and braking distances are essentially the same in both cases. The Honda algorithm does not vary as tire-road friction condition changes. The impact speed is greatly increased under degraded road conditions. The impact speed in this case is 20.6 m/s and the impact energy is only reduced by 29%.

Figure 6: Honda’s Algorithm in Normal Conditions

The Modified algorithm response to the same situation is shown in Figure 7. Comparison of the top plot of Figure 6 and Figure 7 shows that the Modified warning and braking distances are more conservative than Honda’s algorithm. The more conservative braking distance means that the Modified system applies the brakes later and thus will have a larger impact speed than the Honda system. In fact the impact speed is 11.5 m/s (see middle plot), which means that impact energy has been reduced by 78% (compared to the case where no CW/CA system is present). Notice that the algorithm command in the bottom plot is continuous and not discrete. This is to emphasize that a continuous set of warnings are given to the driver via the graduated light display for 0\(<\)w\(<\)1. For w\(<\)0, the system applies the brakes.

Figure 7: Modified Algorithm in Normal Conditions

It is assumed that the Modified algorithm has full knowledge of the tire-road friction coefficient. A friction estimation scheme is needed to provide the algorithm with this knowledge. The Modified algorithm uses the knowledge of degraded road conditions to increase the warning and braking distances. The top plot of Figure 9 shows that the warning and braking distances are significantly larger than their normal road condition counterparts (shown in Figure 8). Consequently, the Modified algorithm applies the brakes sooner during degraded road conditions, which gives the vehicle more time to slow down. As a result, the impact speed increases only to 19.3 m/s and the impact energy is reduced by 38%. These results can be improved even more by increasing one of the scaling factors, \(f(h_{\text{min}})\). The tradeoff is that the algorithm will be more likely to interfere with normal driver habits.
Figure 9: Modified Algorithm in Degraded Conditions

It should be noted that the Modified algorithm performs worse in the normal road conditions. As stressed before this is only due to the parameter selection. The Modified algorithm could be tuned to perform better at the expense of driver intrusion. Therefore, the main purpose of the simulations is to show the improvement that is possible when friction estimation is used.

SUMMARY

It was desired to design a rear-end collision avoidance system. However, this problem was complicated by human factors considerations. Specifically, the frequency of warnings, timing of brake application, and adaptability of the system to various conditions (e.g. weather and driving styles) needed to be considered. Algorithms proposed by Mazda and Honda were described and used for comparison with the proposed algorithm.

The Modified algorithm addressed each of these concerns. A non-dimensional warning parameter was defined. This parameter used a conservative warning distance and a non-conservative braking distance. Then, the warning parameter was used to design a continuous driver warning scheme. The continuous warning scheme should not desensitize nor startle the driver. Furthermore, the non-conservative braking distance was chosen to reduce brake control intrusion on normal driving maneuvers. Finally, the algorithm was modified to include scaling functions which account for the variation in tire road friction and driving styles.

Then, the Modified and Honda algorithms were simulated in normal and degraded road conditions. The Modified algorithm, which was designed to use a tire-road friction estimator, was assumed to have full knowledge of the road conditions. The Berkeley algorithm adapted its critical distance definitions when the road conditions were degraded. Thus, the impact of weather condition variations on this CW/CA algorithm was reduced. These simulations showed the benefit of combining a tire-road friction estimator with a CW/CA algorithm.

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