1. Consider the nonlinear system \( \dot{x} = f(x, u) \) where \( f(x, u) = -2x^3 + xu \)

(a) Find an equilibrium point \((\bar{x}, \bar{u})\) with \(\bar{x} = 1\).

(b) Linearize the system around \((\bar{x}, \bar{u})\). Express your answer as a linear ODE.

(c) Sketch the approximate solution of \( \dot{x} = f(x, u) \) if \( u(t) = \bar{u} + 0.4 \) for all \( t \geq 0 \) and \( x(0) = \bar{x} \). Label the settling time and steady-state value of \( x \) on your sketch.

2. Sketch the response \( x(t) \) vs. \( t \) for the system, initial conditions, and input given below. Label the steady-state value of \( x \) and the approximate settling time. Also label the approximate peak value of \( x \). Specify whether the system is overdamped or underdamped.

\[
\ddot{x} + 5\dot{x} + 4x = 8u, \quad (1)
\]
\[
x(0) = 0, \quad \dot{x}(0) = 0, \quad u(t) = \begin{cases} 0 & t < 0 \text{ sec} \\ 1 & t \geq 0 \text{ sec} \end{cases} \quad (2)
\]

The following table may be useful to sketch the response for an underdamped system.

<table>
<thead>
<tr>
<th>( \zeta )</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e^{-\pi \zeta / \sqrt{1-\zeta^2}} )</td>
<td>0.729</td>
<td>0.527</td>
<td>0.372</td>
<td>0.254</td>
<td>0.163</td>
<td>0.095</td>
<td>0.046</td>
<td>0.015</td>
<td>0.002</td>
</tr>
</tbody>
</table>

3. Let \( P \) denote the system described by \( \dot{x} - 3x = u \). Consider the feedback system shown in Figure 1 where \( K_p \) and \( K_f \) are constant gains. \( d \) is a disturbance at the input of \( P \).

(a) What is the ordinary differential equation (ODE) that models the closed-loop system? Your ODE should have \( x \) as the state and include both the reference \( r \) and disturbance \( d \) as inputs.

(b) Choose the gains \( K_p \) and \( K_f \) so that the closed-loop: i) is stable with time constant \( T = 1 \) sec, and ii) has zero steady state error for a unit step reference \((r(t) = 1 \text{ for } t \geq 0)\). You may assume \( d = 0 \) in this part.

(c) For the gains selected in part (b), what is the steady-state value of \( x \) for a unit step disturbance \((d(t) = 1 \text{ for } t \geq 0)\)? You may assume \( r = 0 \) in this part.

(d) Is it possible to reduce the effect of the disturbance in part (c) by changing the gain \( K_f \)? If so, explain how you would change the gain \( K_f \) to reduce the effect of the disturbance.

4. Consider the system \( 2\dot{x} + 6x = 8u \). Let \( u \) be given by a PI control law, \( u(t) = K_pe(t) + K_i \int_0^t e(\tau)d\tau \), where \( e = r - x \) is the error between the reference and the system output \( x \).

(a) Derive an ordinary differential equation that models the closed-loop system with input \( r \) and output \( x \).

(b) Choose the gains \( K_p \) and \( K_i \) so that the closed-loop system is stable and has:

i. damping ratio equal to \( \zeta = 0.5 \)

ii. settling time \( t_s \leq 3 \text{ sec} \).

iii. zero steady state error for a unit step reference.

(c) Using the gains selected in part (b), what is the peak value of \( x \) for a unit step reference? Note: The value will not be an integer or simple fraction but it can be easily computed from the available information.